66. $\begin{bmatrix} 2 & -1 & 4 & : & 21 \\ -4 & 3 & 1 & : & -14 \\ -1 & -4 & 7 & : & 12 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 0 & : & 5 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$

Answer: $(5, 1, 3)$

68. $\begin{vmatrix} 2 & 4 & 8 \\ 0 & 6 & -9 \\ 4 & -3 & 8 \end{vmatrix} = 2(48 - 27) + 4(-36 - 48) = -294$

70. $(2x - y)^2 = 4x^2 - 4xy + y^2$

72. $(2x - 4y)^3 = 8x^3 - 48x^2y + 96xy^2 - 64y^3$

Section 9.5 The Binomial Theorem

Solutions to Even-Numbered Exercises

2. ${}_9C_6 = \dfrac{9!}{6!3!} = \dfrac{9 \cdot 8 \cdot 7 \cdot 6!}{6!\,3 \cdot 2} = \dfrac{9 \cdot 8 \cdot 7}{6} = 84$

4. $\dbinom{20}{20} = {}_{20}C_{20} = \dfrac{20!}{20!0!} = 1$

6. ${}_{12}C_3 = \dfrac{12!}{9!3!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9!}{9!\,3 \cdot 2} = 220$

8. ${}_{18}C_{17} = \dfrac{18!}{17!1!} = \dfrac{18 \cdot 17!}{17!} = 18$

10. $\dbinom{10}{7} = \dfrac{10!}{7!3!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7!}{7!\,3 \cdot 2} = 120$

12. ${}_{13}C_8 = \dfrac{13!}{8!5!} = \dfrac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8!5!} = 1287$

14. ${}_{17}C_4 = 2380$

16. ${}_{52}C_{47} = 2{,}598{,}960$

18. ${}_{34}C_4 = 46{,}376$

20.

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20 (15)  6   1
```

${}_6C_4 = 15$, the 5th entry in the 6th row.

22.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5  10  10   5   1
    1   6  15  20  15   6   1
  1   7  21  35  35  21   7   1
1   8  28  56  70  56 (28)  8   1
```

${}_8C_6 = 28$, the 7th entry in the 8th row.

24. $(x + 1)^6 = {}_6C_0x^6 + {}_6C_1x^5(1) + {}_6C_2x^4(1)^2 + {}_6C_3x^3(1)^3 + {}_6C_4x^2(1)^4 + {}_6C_5x(1)^5 + {}_6C_6(1)^6$
$= x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$

26. $(a + 2)^4 = {}_4C_0a^4 + {}_4C_1a^3(2) + {}_4C_2a^2(2)^2 + {}_4C_3a(2)^3 + {}_4C_4(2)^4$
$= a^4 + 8a^3 + 24a^2 + 32a + 16$

28. $(y - 2)^5 = {}_5C_0y^5 - {}_5C_1y^4(2) + {}_5C_2y^3(2)^2 - {}_5C_3y^2(2)^3 + {}_5C_4y(2)^4 - {}_5C_5(2)^5$
$= y^5 - 10y^4 + 40y^3 - 80y^2 + 80y - 32$

30. $(x + y)^6 = {}_6C_0x^6 + {}_6C_1x^5y + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5xy^5 + {}_6C_6y^6$
$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

32. $(x + 3y)^4 = x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$

34. $$\begin{aligned}(2x - y)^5 &= {}_5C_0(2x)^5 - {}_5C_1(2x)^4y + {}_5C_2(2x)^3y^2 - {}_5C_3(2x)^2y^3 + {}_5C_4(2x)y^4 - {}_5C_5(2x)y^5\\ &= 32x^5 - 5(16x^4)y + 10(8x^3)y^2 - 10(4x^2)y^3 + 5(2x)y^4 - y^5\\ &= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5\end{aligned}$$

36. $(5 - 2y)^3 = 125 - 150y + 60y^2 - 8y^3$

38. $$\begin{aligned}(x^2 + y^2)^6 &= {}_6C_0(x^2)^6 + {}_6C_1(x^2)^5(y^2) + {}_6C_2(x^2)^4(y^2)^2 + {}_6C_3(x^2)^3(y^2)^3 + {}_6C_4(x^2)^2(y^2)^4\\ &\qquad + {}_6C_5(x^2)(y^2)^5 + {}_6C_6(y^2)^6\\ &= x^{12} + 6x^{10}y^2 + 15x^8y^4 + 20x^6y^6 + 15x^4y^8 + 6x^2y^{10} + y^{12}\end{aligned}$$

40. $$\begin{aligned}\left(\frac{1}{x} + 2y\right)^6 &= {}_6C_0\left(\frac{1}{x}\right)^6 + {}_6C_1\left(\frac{1}{x}\right)^5(2y) + {}_6C_2\left(\frac{1}{x}\right)^4(2y)^2 + {}_6C_3\left(\frac{1}{x}\right)^3(2y)^3 + {}_6C_4\left(\frac{1}{x}\right)^2(2y)^4 + {}_6C_5\left(\frac{1}{x}\right)(2y)^5 + {}_6C_6(2y)^6\\ &= 1\left(\frac{1}{x}\right)^6 + 6(2)\left(\frac{1}{x}\right)^5y + 15(4)\left(\frac{1}{x}\right)^4y^2 + 20(8)\left(\frac{1}{x}\right)^3y^3 + 15(16)\left(\frac{1}{x}\right)^2 + y^4 + 6(32)\left(\frac{1}{x}\right)y^5 + 1(64)y^6\\ &= \frac{1}{x^6} + \frac{12y}{x^5} + \frac{60y^2}{x^4} + \frac{160y^3}{x^3} + \frac{240y^4}{x^2} + \frac{192y^5}{x} + 64y^6\end{aligned}$$

42. $$\begin{aligned}3(x + 1)^5 - 4(x + 1)^3 &= 3[{}_5C_0x^5 + {}_5C_1x^4(1) + {}_5C_2x^3(1)^2 + {}_5C_3x^2(1)^3 + {}_5C_4x(1)^4 + {}_5C_5(1)^5]\\ &\qquad - 4[{}_3C_0x^3 + {}_3C_1x^2(1) + {}_3C_2x(1)^2 + {}_3C_3(1)^3]\\ &= 3[(1)x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1] - 4[(1)x^3 + 3x^2 + 3x + 1]\\ &= 3x^5 + 15x^4 + 26x^3 + 18x^2 + 3x - 1\end{aligned}$$

44. $$\begin{aligned}6(x + 2)^5 - 2(x - 1)^2 &= 6[x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32] - 2[x^2 - 2x + 1]\\ &= 6x^5 + 60x^4 + 240x^3 + 478x^2 + 484x + 190\end{aligned}$$

46. 5^{th} row of Pascal's Triangle: 1 5 10 5 1

$$\begin{aligned}(x + 2y)^5 &= (1)x^5 + 5x^42y + 10x^3(2y)^2 + 10x^2(2y)^3 + 5x(2y)^4 + (2y)^5\\ &= x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5\end{aligned}$$

48. 5^{th} row of Pascal's Triangle: 1 5 10 10 5 1

$$\begin{aligned}(3y + 2)^5 &= (3y)^5 + 5(3y)^4(2) + 10(3y)^3(2)^2 + 10(3y)^2(2)^3 + 5(3y)(2)^4 + (2)^5\\ &= 243y^5 + 810y^4 + 1080y^3 + 720y^2 + 240y + 32\end{aligned}$$

50. The term involving x^{10} in the expansion of $(x^2 + 3)^{12}$ is
${}_{12}C_5(x^2)^5(3)^7 = (792)(3^7)x^{10} = 1{,}732{,}104x^{10}$

52. The term involving x^2y^8 in the expansion of $(4x - y)^{10}$ is ${}_{10}C_8(4x)^2(-y)^8 = \dfrac{10!}{(10 - 8)!8!} \cdot 16x^2y^8 = 720x^2y^8$.
The coefficient is 720.

54. The term involving x^4y^4 in the expansion of $(2x - 3y)^8$ is
${}_8C_4(2x)^4(-3y)^4 = 70(2^4)(-3)^4x^4y^4 = 90{,}720x^4y^4$

56. The term involving z^6 in the expansion of $(z^2 - 1)^{12}$ is ${}_{12}C_9(z^2)^3(-1)^9 = \dfrac{12}{(12 - 9)!9!}z^6(-1) = -220z^6$. The coefficient is -220.

58. $\left(4\sqrt{t}-1\right)^3 = \left(4\sqrt{t}\right)^3 + 3\left(4\sqrt{t}\right)^2(-1) + 3\left(4\sqrt{t}\right)(-1)^2 + (-1)^3$

$= 64t\sqrt{t} - 48t + 12\sqrt{t} - 1 = 64t^{3/2} - 48t + 12t^{1/2} - 1$

60. $(u^{3/5}+2)^5 = (u^{3/5})^5 + 5(u^{3/5})^4(2) + 10(u^{3/5})^3(2)^2 + 10(u^{3/5})^2(2)^3 + 5(u^{3/5})(2)^4 + 2^5$

$= u^3 + 10u^{12/5} + 40u^{9/5} + 80u^{6/5} + 80u^{3/5} + 32$

62. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^4 - x^4}{h}$

$= \dfrac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 + x^4}{h}$

$= \dfrac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$

$= 4x^3 + 6x^2h + 4xh^2 + h^3, h \neq 0$

64. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h}$

$= \dfrac{\dfrac{x-(x+h)}{x(x+h)}}{h}$

$= \dfrac{\dfrac{-h}{x(x+h)}}{h}$

$= -\dfrac{1}{x(x+h)}, h \neq 0$

66. $(4-i)^5 = 1024 - 1280i + 640i^2 - 160i^3 + 20i^4 - i^5$

$= 1024 - 1280i - 640 + 160i + 20 - i$

$= 404 - 1121i$

68. $\left(5+\sqrt{-9}\right)^3 = (5+3i)^3$

$= 5^3 + 3 \cdot 5^2(3i) + 3 \cdot 5(3i)^2 + (3i)^3$

$= 125 + 225i - 135 - 27i$

$= -10 + 198i$

70. $\left(5-\sqrt{3}i\right)^4 = 5^4 - 4 \cdot 5^3\left(\sqrt{3}i\right) + 6 \cdot 5^2\left(\sqrt{3}i\right)^2 - 4 \cdot 5\left(\sqrt{3}i\right)^3 + \left(\sqrt{3}i\right)^4$

$= 625 - 500\sqrt{3}i - 450 + 60\sqrt{3}i + 9$

$= 184 - 440\sqrt{3}i$

72. $(2.005)^{10} = (2+0.005)^{10} = 2^{10} + 10(2)^9(0.005) + 45(2)^8(0.005)^2 + 120(2)^7(0.005)^3 + 210(2)^6(0.005)^4$

$+ 252(2)^5(0.005)^5 + 210(2)^4(0.005)^6 + 120(2)^3(0.005)^7 + 45(2)^8(0.005)^2$

$+ 10(2)(0.005)^9 + (0.005)^{10}$

$= 1024 + 25.6 + 0.288 + 0.00192 + 0.0000084 + \cdots$

≈ 1049.890

74. $(1.98)^9 = (2 - 0.02)^9 = 2^9 - 9(2)^8(0.02) + 36(2)^7(0.02)^2 - 84(2)^6(0.02)^3 + 126(2)^5(0.02)^4$

$- 126(2)^4(0.02)^5 + 84(2)^3(0.02)^6 - 36(2)^2(0.02)^7 + 9(2)(0.02)^8 - (0.02)^9$

$= 512 - 46.08 + 1.8432 - 0.043008 + 0.00064512$

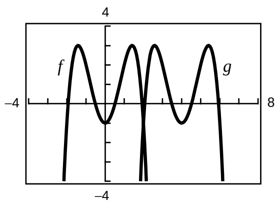
≈ 467.721

76. $f(x) = -x^4 + 4x^2 - 1$

$g(x) = f(x - 4)$

$= -(x - 4)^4 + 4(x - 4)^2 - 1$

$= -x^4 + 16x^3 - 92x^2 + 224x - 193$



The graph of g is the same as the graph of f shifted 4 units to the right.

78. $f(x) = 2x^2 - 4x + 1, g(x) = f(x + 3)$

$g(x) = f(x + 3)$

$= 2(x + 3)^2 - 4(x + 3) + 1$

$= 2[x^2 + 6x + 9] - 4x - 12 + 1$

$= 2x^2 + 8x + 7$

$g(x)$ if $f(x)$ shifted 3 units to the left.

80. (a) ${}_{25}C_6 = 177{,}100$

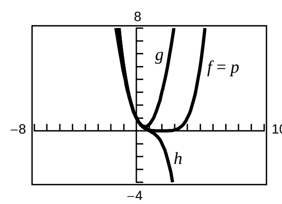
(b) $2({}_{25}C_2 + {}_{25}C_4) = 25{,}900$

(c) $\sum_{k=0}^{5}[({}_{10}C_k)({}_8C_{5-k})] = 8568$

(d) ${}_{18}C_5 = 8568$

(c) and (d) are equal.

82. $p(x) = 1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4 = f(x)$



$p(x)$ is the expansion of $f(x)$.

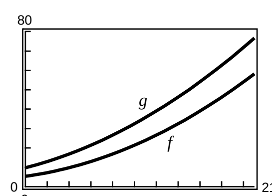
84. ${}_{10}C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^7 = 120\left(\frac{1}{64}\right)\left(\frac{2187}{16{,}384}\right) \approx 0.2503$

86. ${}_8C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^4 = 70\left(\frac{1}{16}\right)\left(\frac{1}{16}\right) \approx 0.273$

88. (a) $g(t) = f(t + 4) = 0.0834(t + 4)^2 + 0.7657(t + 4) + 5.3680$

$= 0.0834t^2 + 1.4329t + 9.7652$

(b)



90. False. The coefficient of x^{10} is 1,732,104 and the coefficient of x^{14} is 192,456.

92.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

94. The expansions of $(x + y)^n$ and $(x - y)^n$ are almost the same except that the signs of the terms in the expansion of $(x - y)^n$ alternate from positive to negative.