96. $0 = (1 - 1)^n = {}_nC_0 - {}_nC_1 + {}_nC_2 - {}_nC_3 + \cdots + (\pm {}_nC_n) = 0$

98. ${}_nC_0 + {}_nC_1 + {}_nC_2 + {}_nC_3 + \cdots + {}_nC_n = (1 + 1)^n = 2^n$

100. $g(x) = f(x - 3)$

$g(x)$ is shifted three units to the right of $f(x)$.

102. $g(x) = -f(x)$

$g(x)$ is the reflection of $f(x)$ in the x-axis.

104. $4A - B = \begin{bmatrix} -8 & 16 & 3 \\ 17 & -5 & 6 \\ 21 & 2 & -2 \end{bmatrix}$

106. $6A + 10B = \begin{bmatrix} -12 & -22 & -30 \\ 60 & 4 & 32 \\ -26 & -20 & 66 \end{bmatrix}$

108. $BA = \begin{bmatrix} -32 & 4 & -11 \\ 7 & 8 & 4 \\ 24 & -13 & 2 \end{bmatrix}$

110. $\begin{bmatrix} 1.2 & -2.3 \\ -2 & 4 \end{bmatrix}^{-1} = \dfrac{1}{4.8 - 4.6}\begin{bmatrix} 4 & 2.3 \\ 2 & 1.2 \end{bmatrix} = \begin{bmatrix} 20 & 11.5 \\ 10 & 6 \end{bmatrix}$

Section 9.6 Counting Principles

Solutions to Even-Numbered Exercises

2. Even integers: 2, 4, 6, 8, 10, 12

6 ways

4. Greater than 7: 8, 9, 10, 11, 12

5 ways

6. Divisible by 3: 3, 6, 9, 12

4 ways

8. Two distinct integers whose sum is 10: $1 + 9$, $2 + 8$, $3 + 7$, $4 + 6$

4 ways

10. Monitors: 3

Keyboards: 2

Computers: 7

Total: $3 \cdot 2 \cdot 7 = 42$ ways

12. Math courses: 2

Science courses: 3

Social sciences and humanities courses: 5

Total: $2 \cdot 3 \cdot 5 = 30$ ways

14. $2^{10} = 1024$

16. Drivers seat: 3 choices

Next seat: 3 choices

Next seat: 2 choices

Next seat: 1 choice

Total: $3 \cdot 3 \cdot 2 \cdot 1 = 18$ ways

18. $24 \cdot 24 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 5{,}760{,}000$

20. (a) $9 \cdot 10 \cdot 10 \cdot 10 = 9000$

(b) $9 \cdot 9 \cdot 8 \cdot 7 = 4536$

(c) $4 \cdot 10 \cdot 10 \cdot 10 = 4000$

(d) $9 \cdot 10 \cdot 10 \cdot 5 = 4500$

22. $3(8{,}000{,}000) = 24{,}000{,}000$ telephone numbers

24. (a) $8! = 40{,}320$ ways

(b) $(5!)(3!) = 120(6) = 720$ ways

26. $_nP_r = \dfrac{n!}{(n-r)!}$

$_5P_5 = \dfrac{5!}{(5-5)!} = \dfrac{5!}{0!} = 120$

28. $_{20}P_2 = \dfrac{20!}{18!} = 20.19 = 380$

30. $_7P_4 = \dfrac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$

32. $_nP_5 = 18 \cdot {}_{n-2}P_4$

Note: $n \geq 6$ for this to be defined.

$$\frac{n!}{(n-5)!} = 18\left(\frac{(n-2)!}{(n-6)!}\right)$$

$$n(n-1)(n-2)(n-3)(n-4) = 18(n-2)(n-3)(n-4)(n-5)$$

$\left(\text{We can divide by } (n-2), (n-3), (n-4) \text{ since } n \neq 2, n \neq 3, \text{ and } n \neq 4.\right)$

$$n^2 - n = 18n - 90$$

$$n^2 - 19n + 90 = 0$$

$$(n-9)(n-10) = 0$$

$$n = 9 \text{ or } n = 10$$

34. $_{100}P_5 = 9{,}034{,}502{,}400$

36. $_{10}P_8 = 1{,}814{,}400$

38. $_{10}C_7 = 120$

40. $4! = 24$

42. $4! = 24$ ways

44. $\dfrac{8!}{3!5!} = 56$

46. $\dfrac{11!}{1!4!4!2!} = \dfrac{11!}{4!4!2!} = 34{,}650$

48. A B C D
A C B D
D B C A
D C B A

50. $_6C_3 = \dfrac{6!}{3!3!} = 20$

ABC, ABD, ABE, ABF, ACD, ACE, ACF, ADE, ADF, AEF, BCD, BCE, BCF, BDE, BDF, BEF, CDE, CDF, CEF, DEF

52. $_{14}C_{12} = 91$ ways

54. $_{50}C_6 = 15{,}890{,}700$ ways

56. $_{80}C_6 = 300{,}500{,}200$ ways

58. There are 22 good sets and 3 defective sets.

(a) $_{22}C_4 = 7315$ ways

(b) $({}_{22}C_2)({}_3C_2) = (231)(3) = 693$ ways

(c) $_{22}C_4 + ({}_{22}C_3)({}_3C_1) + ({}_{22}C_2)({}_3C_2) = 7315 + (1540)(3) + 693 = 12{,}628$ ways

60. Select type of card for three of a kind: $_{13}C_1$

Select three of four cards for three of a kind: $_4C_3$

Select type of card for pair: $_{12}C_1$

Select two of four cards for pair: $_4C_2$

$_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_1 \cdot {}_4C_2 = 13 \cdot 4 \cdot 12 \cdot 6 = 3744$ ways to get a full house

62. (a) $_3C_2 = \dfrac{3!}{2!1!} = 3$ relationships

(b) $_8C_2 = \dfrac{8!}{2!6!} = \dfrac{8 \cdot 7}{2} = 28$ relationships

(c) $_{12}C_2 = \dfrac{12!}{2!10!} = \dfrac{12 \cdot 11}{2} = 66$ relationships

(d) $_{20}C_2 = \dfrac{20!}{2!18!} = \dfrac{20 \cdot 19}{2} = 190$ relationships

64. ${}_6C_2 - 6 = 15 - 6 = 9$ diagonals

66. ${}_{10}C_2 - 10 = 45 - 10 = 35$ diagonals

68. True

70. ${}_{100}P_{80} \approx 3.836 \times 10^{139}$.

This number is too large for some calculators to evaluate.

72. ${}_nC_r = {}_nC_{n-r} = \dfrac{n!}{r!(n-r)!)}$

74. ${}_nP_{n-1} = \dfrac{n!}{(n-(n-1))!} = \dfrac{n!}{1!} = \dfrac{n!}{0!} = {}_nP_n$

76. ${}_nC_{n-1} = \dfrac{n!}{[n-(n-1)]!(n-1)!} = \dfrac{n!}{(1)!(n-1)!} = \dfrac{n!}{(n-1)!1!} = {}_nC_1$

78. From the graph of $y = \sqrt{x-3} - x + 6$, you see that there is one zero, $x \approx 8.303$. Analytically,

$$\begin{aligned} \sqrt{x-3} &= x - 6 \\ x - 3 &= x^2 - 12x + 36 \\ 0 &= x^2 - 13x + 39. \end{aligned}$$

By the Quadratic Formula, $x = \dfrac{13 \pm \sqrt{(-13)^2 - 4(39)}}{2} = \dfrac{13 \pm \sqrt{13}}{2}$.

Selecting the larger solution, $x = \dfrac{13 + \sqrt{13}}{2} \approx 8.303$.

80.
$$\begin{aligned} \log_2(x-3) &= 5 \\ 2^5 &= x - 3 \\ 2^5 + 3 &= x \\ x &= 35 \end{aligned}$$

82. $x = \dfrac{\begin{vmatrix} -14 & 3 \\ 2 & -2 \end{vmatrix}}{\begin{vmatrix} -5 & 3 \\ 7 & -2 \end{vmatrix}} = \dfrac{22}{-11} = -2$

$y = \dfrac{\begin{vmatrix} -5 & -14 \\ 7 & 2 \end{vmatrix}}{\begin{vmatrix} -5 & 3 \\ 7 & -2 \end{vmatrix}} = \dfrac{88}{-11} = -8$

Answer: $(-2, -8)$

84. $x = \dfrac{\begin{vmatrix} -1 & -4 \\ -4 & 5 \end{vmatrix}}{\begin{vmatrix} -3 & -4 \\ 9 & 5 \end{vmatrix}} = \dfrac{-21}{21} = -1$

$y = \dfrac{\begin{vmatrix} -3 & -1 \\ 9 & -4 \end{vmatrix}}{\begin{vmatrix} -3 & -4 \\ 9 & 5 \end{vmatrix}} = \dfrac{21}{21} = 1$

Answer: $(-1, 1)$

86. $(x-4)^3 = x^3 - 12x^2 + 48x - 64$

88. $(x^2+4)^5 = x^{10} + 20x^8 + 160x^6 + 640x^4 + 1280x^2 + 1024$