CHAPTER 10
Topics in Analytic Geometry

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Section 10.1 Introduction to Conics: Parabolas

- A **parabola** is the set of all points (x, y) that are equidistant from a fixed line **(directrix)** and a fixed point **(focus)** not on the line.
- The standard equation of a parabola with vertex (h, k) and:
  (a) Vertical axis $x = h$ and directrix $y = k - p$ is:
  $(x - h)^2 = 4p(y - k),\ p \neq 0$
  (b) Horizontal axis $y = k$ and directrix $x = h - p$ is:
  $(y - k)^2 = 4p(x - h),\ p \neq 0$
- The tangent line to a parabola at a point P makes **equal angles** with:
  (a) the line through P and the focus
  (b) the axis of the parabola

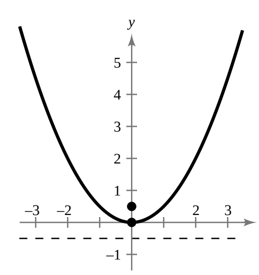
Solutions to Odd-Numbered Exercises

1. $y^2 = -4x$
Vertex: $(0, 0)$
Opens to the left since p is negative.
Matches graph (e).

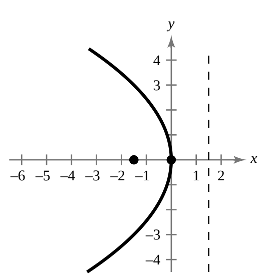
3. $x^2 = -8y$
Vertex: $(0, 0)$
Opens downward since p is negative.
Matches graph (d).

5. $(y - 1)^2 = 4(x - 3)$
Vertex: $(3, 1)$
Opens to the right since p is positive.
Matches graph (a).

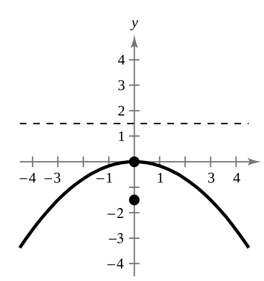
7. $y = \frac{1}{2}x^2$
$x^2 = 2y$
$x^2 = 4\left(\frac{1}{2}\right)y \Rightarrow h = 0, k = 0, p = \frac{1}{2}$
Vertex: $(0, 0)$
Focus: $\left(0, \frac{1}{2}\right)$
Directrix: $y = -\frac{1}{2}$



9. $y^2 = -6x$
$y^2 = 4\left(-\frac{3}{2}\right)x \Rightarrow h = 0, k = 0, p = -\frac{3}{2}$
Vertex: $(0, 0)$
Focus: $\left(-\frac{3}{2}, 0\right)$
Directrix: $x = \frac{3}{2}$



11. $x^2 + 6y = 0$
$x^2 = 4\left(-\frac{3}{2}\right)y \Rightarrow h = 0, k = 0, p = -\frac{3}{2}$
Vertex: $(0, 0)$
Focus: $\left(0, -\frac{3}{2}\right)$
Directrix: $y = \frac{3}{2}$



13. $(x - 1)^2 + 8(y + 2) = 0$

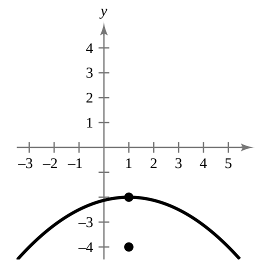
$$(x - 1)^2 = 4(-2)(y + 2)$$

$h = 1, k = -2, p = -2$

Vertex: $(1, -2)$

Focus: $(1, -4)$

Directrix: $y = 0$

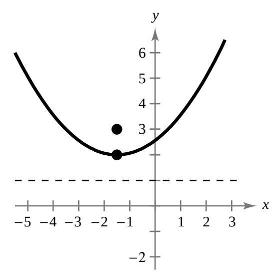


15. $\left(x + \frac{3}{2}\right)^2 = 4(y - 2) \implies h = -\frac{3}{2}, k = 2, p = 1$

Vertex: $\left(-\frac{3}{2}, 2\right)$

Focus: $\left(-\frac{3}{2}, 2 + 1\right) = \left(-\frac{3}{2}, 3\right)$

Directrix: $y = 1$



17. $y = \frac{1}{4}(x^2 - 2x + 5)$

$4y - 4 = (x - 1)^2$

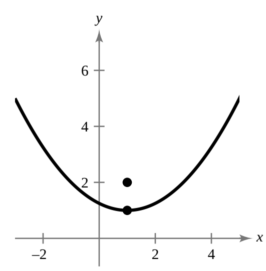
$(x - 1)^2 = 4(1)(y - 1)$

$h = 1, k = 1, p = 1$

Vertex: $(1, 1)$

Focus: $(1, 2)$

Directrix: $y = 0$



19. $y^2 + 6y + 8x + 25 = 0$

$$y^2 + 6y + 9 = -8x - 25 + 9$$

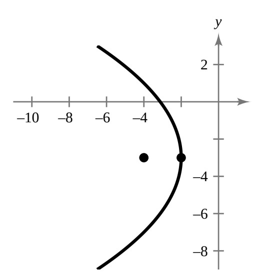
$$(y + 3)^2 = 4(-2)(x + 2)$$

$h = -2, k = -3, p = -2$

Vertex: $(-2, -3)$

Focus: $(-4, -3)$

Directrix: $x = 0$



21. $x^2 + 4x + 6y - 2 = 0$

$$x^2 + 4x + 4 = -6y + 2 + 4 = -6y + 6$$

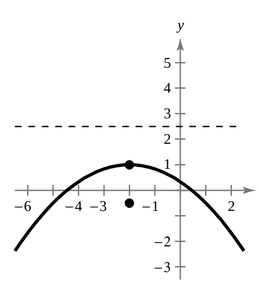
$$(x + 2)^2 = -6(y - 1)$$

$$(x + 2)^2 = 4\left(-\frac{3}{2}\right)(y - 1)$$

Vertex: $(-2, 1)$

Focus: $\left(-2, 1 - \frac{3}{2}\right) = \left(-2, -\frac{1}{2}\right)$

Directrix: $y = \frac{5}{2}$



23. $y^2 + x + y = 0$

$y^2 + y + \frac{1}{4} = -x + \frac{1}{4}$

$\left(y + \frac{1}{2}\right)^2 = 4\left(-\frac{1}{4}\right)\left(x - \frac{1}{4}\right)$

$h = \frac{1}{4}, k = -\frac{1}{2}, p = -\frac{1}{4}$

Vertex: $\left(\frac{1}{4}, -\frac{1}{2}\right)$

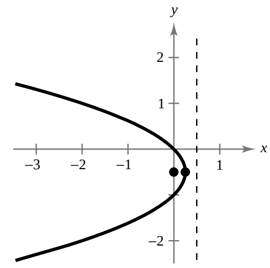
Focus: $\left(0, -\frac{1}{2}\right)$

Directrix: $x = \frac{1}{2}$

To use a graphing calculator, enter:

$y_1 = -\frac{1}{2} + \sqrt{\frac{1}{4} - x}$

$y_2 = -\frac{1}{2} - \sqrt{\frac{1}{4} - x}$



25. $y = -\sqrt{-6x}$

27. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Graph opens upward.

$x^2 = 4py$

Point on graph: $(3, 6)$

$3^2 = 4p(6)$

$9 = 24p$

$\frac{3}{8} = p$

Thus, $x^2 = 4\left(\frac{3}{8}\right)y \Rightarrow y = \frac{2}{3}x^2$.

29. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Focus: $\left(0, -\frac{3}{2}\right) \Rightarrow p = -\frac{3}{2}$

$(x - h)^2 = 4p(y - k)$

$x^2 = 4\left(-\frac{3}{2}\right)y$

$x^2 = -6y$

31. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Focus: $(-2, 0) \Rightarrow p = -2$

$(y - k)^2 = 4p(x - h)$

$y^2 = 4(-2)x$

$y^2 = -8x$

33. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Directrix: $y = -1 \Rightarrow p = 1$

$(x - h)^2 = 4p(y - k)$

$(x - 0)^2 = 4(1)(y - 0)$

$x^2 = 4y$ or $y = \frac{1}{4}x^2$

35. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Directrix: $x = 2 \Rightarrow p = -2$

$y^2 = 4px$

$y^2 = -8x$

37. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Horizontal axis and passes through the point $(4, 6)$

$(y - k)^2 = 4p(x - h)$

$(y - 0)^2 = 4p(x - 0)$

$y^2 = 4px$

$6^2 = 4p(4)$

$36 = 16p \Rightarrow p = \frac{9}{4}$

$y^2 = 4\left(\frac{9}{4}\right)x$

$y^2 = 9x$

39. Vertex: $(3, 1)$ and opens downward. Passes through $(2, 0)$ and $(4, 0)$.

$y = -(x - 2)(x - 4)$

$= -x^2 + 6x - 8$

$= -(x - 3)^2 + 1$

$(x - 3)^2 = -(y - 1)$

41. Vertex: $(-2, 0)$ and opens to the right. Passes through $(0, 2)$.

$(y - 0)^2 = 4p(x + 2)$

$2^2 = 4p(0 + 2)$

$\frac{1}{2} = p$

$y^2 = 4\left(\frac{1}{2}\right)(x + 2)$

$y^2 = 2(x + 2)$

43. Vertex: $(5, 2)$

Focus: $(3, 2)$

Horizontal axis: $p = 3 - 5 = -2$

$(y - 2)^2 = 4(-2)(x - 5)$

$(y - 2)^2 = -8(x - 5)$

45. Vertex: $(0, 4)$

Directrix: $y = 2$

Vertical axis

$p = 4 - 2 = 2$

$(x - 0)^2 = 4(2)(y - 4)$

$x^2 = 8(y - 4)$

47. Focus: $(2, 2)$

Directrix: $x = -2$

Horizontal axis

Vertex: $(0, 2)$

$p = 2 - 0 = 2$

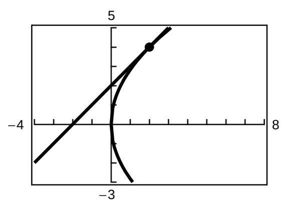
$(y - 2)^2 = 4(2)(x - 0)$

$(y - 2)^2 = 8x$

49. $y^2 - 8x = 0 \Rightarrow y = \pm\sqrt{8x}$

$x - y + 2 = 0 \Rightarrow y = x + 2$

The point of tangency is $(2, 4)$.



51. $x^2 = 2y$, $(4, 8)$, $p = \frac{1}{2}$, focus: $\left(0, \frac{1}{2}\right)$

Following Example 4, we find the y-intercept $(0, 6)$:

$d_1 = \frac{1}{2} - b.$

$d_2 = \sqrt{(4 - 0)^2 + \left(8 - \frac{1}{2}\right)^2} = \frac{17}{2}$

$d_1 = d_2 \quad \Rightarrow \quad \frac{1}{2} - b = \frac{17}{2} \quad \Rightarrow \quad b = -8$

$m = \frac{8 - (-8)}{4 - 0} = 4$

$y = 4x - 8$ Tangent line

Let $y = 0 \quad \Rightarrow \quad x = 2 \quad \Rightarrow \quad x$-intercept $(2, 0)$

53. $y = -2x^2 \quad \Rightarrow \quad x^2 = -\frac{1}{2}y = 4\left(-\frac{1}{8}\right)y \quad \Rightarrow \quad p = -\frac{1}{8}$, focus: $\left(0, -\frac{1}{8}\right)$

Following Example 4, we find the y-intercept $(0, b)$:

$d_1 = \frac{1}{8} + b$

$d_2 = \sqrt{(-1 - 0)^2 + \left(-2 + \frac{1}{8}\right)^2} = \frac{17}{8}$

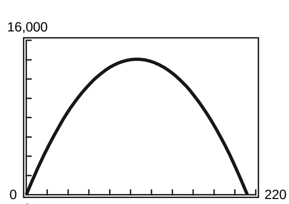
$d_1 = d_2 \quad \Rightarrow \quad \frac{1}{8} + b = \frac{17}{8} \quad \Rightarrow \quad b = 2$

$m = \frac{-2 - 2}{-1 - 0} = 4$

$y = 4x + 2$

Let $y = 0 \quad \Rightarrow \quad x = -\frac{1}{2} \quad \Rightarrow \quad x$-intercept $\left(-\frac{1}{2}, 0\right)$

55.



$R = 265x - \frac{5}{4}x^2$ is a maximum (14045) when $x = 106$ units.

57. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Focus: $(0, 3.5) \Rightarrow p = 3.5$

$(x - h)^2 = 4p(y - k)$

$(x - 0)^2 = 4(3.5)(y - 0)$

$x^2 = 14y$ or $y = \frac{1}{14}x^2$