59. (a) $x^2 = 4py$ passes through point $\left(16, -\frac{2}{5}\right)$

$256 = 4p\left(-\frac{2}{5}\right) \implies p = -160$

$x^2 = 4(-160)y$

$x^2 = -640y$ or $y = \frac{-1}{640}x^2$

(b) $-0.1 = \frac{-1}{640}x^2 \implies x = 8$ feet

61. (a) Escape velocity: $17{,}500\sqrt{2}$

(b) $x^2 = 4p(y - 4100)$ and $p = -4100$.

$x^2 = -16{,}400(y - 4100)$

63. (a) $y = \dfrac{-16}{v^2}x^2 + s$

$= \dfrac{-16}{32^2}x^2 + 75 = -\dfrac{1}{64}x^2 + 75$

(b) $y = 0 = -\dfrac{1}{64}x^2 + 75 \implies x^2 = (75)(64) \implies x \approx 69.28$ feet

65. False. It is not possible for a parabola to intersect its directrix. If the graph crossed the directrix there would exist points nearer the directrix than the focus.

67. $\pm 4, \pm 2, \pm 1$

69. $\pm 16, \pm 8, \pm 4, \pm 2, \pm 1, \pm\frac{1}{2}$

Section 10.2 Ellipses

- An **ellipse** is the set of all points (x, y) the sum of whose distances from two distinct fixed points **(foci)** is constant.
- The standard equation of an ellipse with center (h, k) and major and minor axes of lengths $2a$ and $2b$ is:
  (a) $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ if the major axis is horizontal.
  (b) $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$ if the major axis is vertical.
- $c^2 = a^2 - b^2$ where c is the distance from the center to a focus.
- The eccentricity of an ellipse is $e = \dfrac{c}{a}$

Solutions to Odd-Numbered Exercises

1. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

Center: $(0, 0)$

$a = 3, b = 2$

Vertical major axis

Matches graph (b).

3. $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$

Center: $(0, 0)$

$a = 5, b = 2$

Vertical major axis

Matches graph (d).

5. $\dfrac{(x-2)^2}{16} + (y+1)^2 = 1$

Center: $(2, -1)$

$a = 4, b = 1$

Horizontal major axis

Matches graph (a).

7. $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

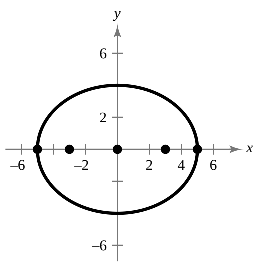
Center: $(0, 0)$

$a = 5, b = 4, c = 3$

Foci: $(\pm 3, 0)$

Vertices: $(\pm 5, 0)$

$e = \dfrac{3}{5}$



9. $\dfrac{x^2}{5} + \dfrac{y^2}{9} = 1$

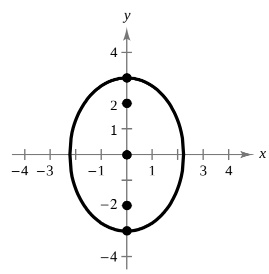
Center: $(0, 0)$

$a = 3, b = \sqrt{5}, c = 2$

Foci: $(0, \pm 2)$

Vertices: $(0, \pm 3)$

$e = \dfrac{2}{3}$



11. $\dfrac{(x + 3)^2}{16} + \dfrac{(y - 5)^2}{25} = 1$

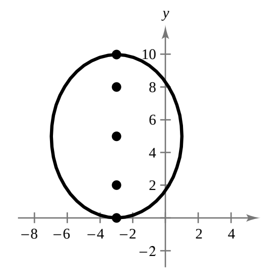
Center: $(-3, 5)$

$a = 5, b = 4, c = 3$

Foci: $(-3, 5 \pm 3) = (-3, 8),$
$(-3, 2)$

Vertices: $(-3, 5 \pm 5) = (-3, 10),$
$(-3, 0)$

$e = \dfrac{3}{5}$



13. $\dfrac{(x + 5)^2}{\frac{9}{4}} + (y - 1)^2 = 1$

Center: $(-5, 1)$

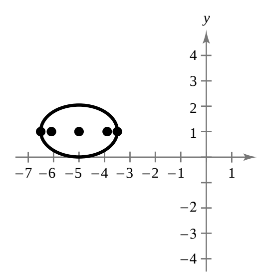
$a = \dfrac{3}{2}, b = 1, c = \sqrt{\dfrac{9}{4} - 1} = \dfrac{\sqrt{5}}{2}$

Foci: $\left(-5 + \dfrac{\sqrt{5}}{2}, 1\right), \left(-5 - \dfrac{\sqrt{5}}{2}, 1\right)$

Vertices: $\left(-5 + \dfrac{3}{2}, 1\right) = \left(-\dfrac{7}{2}, 1\right)$

$\left(-5 - \dfrac{3}{2}, 1\right) = \left(-\dfrac{13}{2}, 1\right)$

$e = \dfrac{\sqrt{5}/2}{3/2} = \sqrt{5}/3$



15. $9x^2 + 4y^2 + 36x - 24y + 36 = 0$

$9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = -36 + 36 + 36$

$$\frac{(x + 2)^2}{4} + \frac{(y - 3)^2}{9} = 1$$

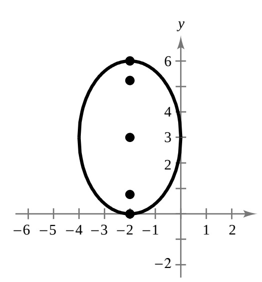
$a = 3, b = 2, c = \sqrt{5}$

Center: $(-2, 3)$

Foci: $\left(-2, 3 \pm \sqrt{5}\right)$

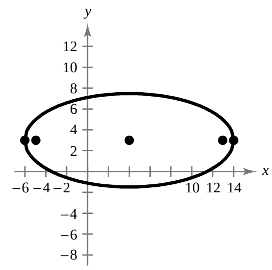
Vertices: $(-2, 6), (-2, 0)$

$e = \dfrac{\sqrt{5}}{3}$



17. $x^2 + 5y^2 - 8x - 30y - 39 = 0$

$$(x^2 - 8x + 16) + 5(y^2 - 6y + 9) = 39 + 16 + 45$$
$$(x - 4)^2 + 5(y - 3)^2 = 100$$
$$\frac{(x - 4)^2}{100} + \frac{(y - 3)^2}{20} = 1$$



Center: $(4, 3)$

$a = 10, b = 2\sqrt{5}, c = \sqrt{80} = 4\sqrt{5}$

Foci: $\left(4 + 4\sqrt{5}, 3\right), \left(4 - 4\sqrt{5}, 3\right)$

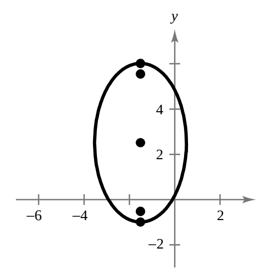
Vertices: $(4 + 10, 3) = (14, 3)$

$(4 - 10, 3) = (-6, 3)$

$$e = \frac{4\sqrt{5}}{10} = \frac{2\sqrt{5}}{5}$$

19. $6x^2 + 2y^2 + 18x - 10y + 2 = 0$

$$6\left(x^2 + 3x + \frac{9}{4}\right) + 2\left(y^2 - 5y + \frac{25}{4}\right) = -2 + \frac{27}{2} + \frac{25}{2}$$
$$6\left(x + \frac{3}{2}\right)^2 + 2\left(y - \frac{5}{2}\right)^2 = 24$$
$$\frac{\left(x + \frac{3}{2}\right)^2}{4} + \frac{\left(y - \frac{5}{2}\right)^2}{12} = 1$$



Center: $\left(-\frac{3}{2}, \frac{5}{2}\right)$

$a = 2\sqrt{3}, b = 2, c = 2\sqrt{2}$

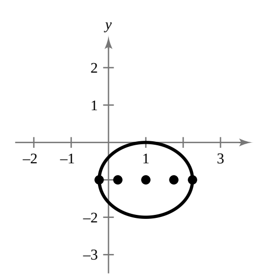
Foci: $\left(-\frac{3}{2}, \frac{5}{2} \pm 2\sqrt{2}\right)$

Vertices: $\left(-\frac{3}{2}, \frac{5}{2} \pm 2\sqrt{3}\right)$

$$e = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

21. $16x^2 + 25y^2 - 32x + 50y + 16 = 0$

$$16(x^2 - 2x + 1) + 25(y^2 + 2y + 1) = -16 + 16 + 25$$
$$\frac{(x - 1)^2}{25/16} + (y + 1)^2 = 1$$



$a = \frac{5}{4}, b = 1, c = \frac{3}{4}$

Center: $(1, -1)$

Foci: $\left(\frac{7}{4}, -1\right), \left(\frac{1}{4}, -1\right)$

Vertices: $\left(\frac{9}{4}, -1\right), \left(-\frac{1}{4}, -1\right)$

$$e = \frac{3}{5}$$

23. $5x^2 + 3y^2 = 15$

$$\frac{x^2}{3} + \frac{y^2}{5} = 1$$

Center: $(0, 0)$

$a = \sqrt{5}, b = \sqrt{3}, c = \sqrt{2}$

Foci: $\left(0, \pm\sqrt{2}\right)$

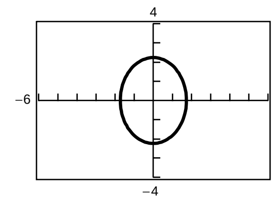
Vertices: $\left(0, \pm\sqrt{5}\right)$

To graph, solve for y.

$$y^2 = \frac{15 - 5x^2}{3}$$

$$y_1 = \sqrt{\frac{15 - 5x^2}{3}}$$

$$y_2 = -\sqrt{\frac{15 - 5x^2}{3}}$$



25. $12x^2 + 20y^2 - 12x + 40y - 37 = 0$

$$12\left(x^2 - x + \frac{1}{4}\right) + 20(y^2 + 2y + 1) = 37 + 3 + 20$$

$$\frac{[x - (1/2)]^2}{5} + \frac{(y + 1)^2}{3} = 1$$

$a = \sqrt{5}, b = \sqrt{3}, c = \sqrt{2}$

Center: $\left(\frac{1}{2}, -1\right)$

Foci: $\left(\frac{1}{2} \pm \sqrt{2}, -1\right)$

Vertices: $\left(\frac{1}{2} \pm \sqrt{5}, -1\right)$

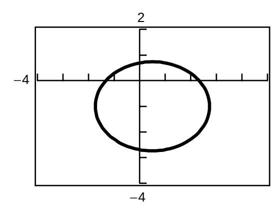
$$e = \frac{\sqrt{10}}{5}$$

To graph, solve for y.

$$(y + 1)^2 = 3\left[1 - \frac{(x - 0.5)^2}{5}\right]$$

$$y_1 = -1 + \sqrt{3\left[1 - \frac{(x - 0.5)^2}{5}\right]}$$

$$y_2 = -1 - \sqrt{3\left[1 - \frac{(x - 0.5)^2}{5}\right]}$$



27. Center: $(0, 0)$, $a = 4$, $b = 2$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

29. Vertices: $(\pm 6, 0)$, Foci: $(\pm 2, 0)$

$a = 6, c = 2, b = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$

Horizontal major axis, center: $(0, 0)$

$$\frac{x^2}{36} + \frac{y^2}{32} = 1$$

31. Foci: $(\pm 5, 0) \Rightarrow c = 5$

Center: $(0, 0)$

Horizontal major axis

Major axis of length 12 $\Rightarrow 2a = 12$

$a = 6$

$6^2 - b^2 = 5^2 \Rightarrow b^2 = 11$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{x^2}{36} + \frac{y^2}{11} = 1$$

33. Vertices: $(0, \pm 5) \Rightarrow a = 5$

Center: $(0, 0)$

Vertical major axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{25} = 1$$

Point: $(4, 2)$

$$\frac{4^2}{b^2} + \frac{2^2}{25} = 1$$

$$\frac{16}{b^2} = 1 - \frac{4}{25} = \frac{21}{25}$$

$$400 = 21b^2$$

$$\frac{400}{21} = b^2$$

$$\frac{x^2}{400/21} + \frac{y^2}{25} = 1$$

$$\frac{21x^2}{400} + \frac{y^2}{25} = 1$$

35. Center: $(2, 3)$

$a = 3, \quad b = 1$

Vertical major axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-2)^2}{1} + \frac{(y-3)^2}{9} = 1$$

37. Center: $(-2, 3)$, $a = 4$, $b = 3$

$$\frac{(x+2)^2}{16} + \frac{(y-3)^2}{9} = 1$$

39. Center: $(2, 4)$, $a = 2$, $b = \frac{2}{2} = 1$

$$\frac{(x-2)^2}{4} + \frac{(y-4)^2}{1} = 1$$

41. Foci: $(0, 0), (0, 8) \Rightarrow c = 4$

Major axis of length 16 $\Rightarrow a = 8$

$b^2 = a^2 - c^2 = 64 - 16 = 48$

Center: $(0, 4) = (h, k)$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{x^2}{48} + \frac{(y-4)^2}{64} = 1$$

43. Vertices: $(3, 1), (3, 9) \Rightarrow a = 4$

Center: $(3, 5)$

Minor axis of length 6 $\Rightarrow b = 3$

Vertical major axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$$

45. Center: $(0, 4)$

Vertices: $(-4, 4), (4, 4) \Rightarrow a = 4$

$a = 2c \Rightarrow 4 = 2c \Rightarrow c = 2$

$2^2 = 4^2 - b^2 \Rightarrow b^2 = 12$

Horizontal major axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{16} + \frac{(y-4)^2}{12} = 1$$

47. Vertices: $(\pm 5, 0) \Rightarrow a = 5$

Eccentricity: $\frac{3}{5} \Rightarrow c = \frac{3}{5}a = 3$

$b^2 = a^2 - c^2 = 25 - 9 = 16$

Center: $(0, 0) = (h, k)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

49. Vertices: $(\pm 3, 0) \quad \Longrightarrow \quad a = 3$

Half of minor axis length: $2 \quad \Longrightarrow \quad b = 2$

$c^2 = a^2 - b^2 = 9 - 4 = 5 \quad \Longrightarrow \quad c = \sqrt{5}$

Place the tacks $\sqrt{5}$ feet from the center: $\left(\pm\sqrt{5}, 0\right)$

Length of string: $2a = 2(3) = 6$ feet

51. Area of ellipse = 2 (area of circle)

$$\pi ab = 2\pi r^2$$

$$\pi a(10) = 2\pi(10)^2$$

$$\pi a(10) = 200$$

$$a = 20$$

Length of major axis: $2a = 2(20) = 40$ units

53. Center: $(0, 0) \implies h = 0, k = 0$

$2a = 0.34 + 4.08 = 4.42$

$a = 2.21$

$c = 2.21 - 0.34 = 1.87$

$b^2 = a^2 - c^2 = 4.8841 - 3.4969 = 1.3872$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4.88} + \frac{y^2}{1.39} = 1$$

55. For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have $c^2 = a^2 - b^2$.

When $x = c$:

$$\frac{c^2}{a^2} + \frac{y^2}{b^2} = 1 \implies y^2 = b^2\left(1 - \frac{a^2 - b^2}{a^2}\right) \implies y^2 = \frac{b^4}{a^2} \implies 2y = \frac{2b^2}{a}.$$

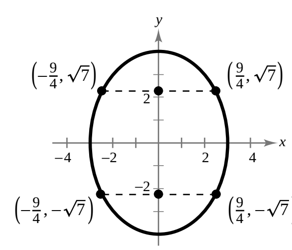
57. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$a = 4, b = 3, c = \sqrt{7}$

Points on the ellipse: $(\pm 3, 0), (0, \pm 4)$

Length of latus recta: $\frac{2b^2}{a} = \frac{2(3)^2}{4} = \frac{9}{2}$

Additional points; $\left(\pm\frac{9}{4}, -\sqrt{7}\right), \left(\pm\frac{9}{4}, \sqrt{7}\right)$



59. $5x^2 + 3y^2 = 15$

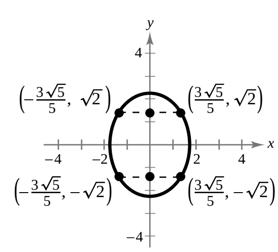
$\frac{x^3}{3} + \frac{y^2}{5} = 1$

$a = \sqrt{5}, b = \sqrt{3}, c = \sqrt{2}$

Points on the ellipse: $\left(\pm\sqrt{3}, 0\right), \left(0, \pm\sqrt{5}\right)$

Length of latus recta: $\frac{2b^2}{a} = \frac{2 \cdot 3}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$

Additional points; $\left(\pm\frac{3\sqrt{5}}{5}, -\sqrt{2}\right), \left(\pm\frac{3\sqrt{5}}{5}, \sqrt{2}\right)$



61. True. If $e \approx 1$ then the ellipse is elongated, not circular.

63. False. $c < a$ always