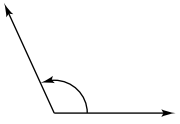
C H A P T E R 4
Trigonometric Functions

C H A P T E R 4
Trigonometric Functions

Section 4.1 Radian and Degree Measure

Solutions to Odd-Numbered Exercises

1.



The angle shown is approximately 2 radians.

3.

The angle shown is approximately -3 radians.

5. (a) Since $0 < \frac{\pi}{5} < \frac{\pi}{2}$, $\frac{\pi}{5}$ lies in Quadrant I.

(b) Since $\pi < \frac{7\pi}{5} < \frac{3\pi}{2}$, $\frac{7\pi}{5}$ lies in Quadrant III.

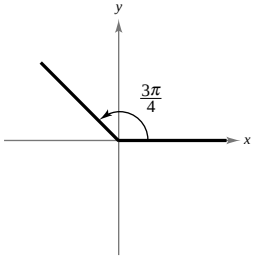
7. (a) Since $-\frac{\pi}{2} < -\frac{\pi}{12} < 0$, $-\frac{\pi}{12}$ lies in Quadrant IV.

(b) Since $-\frac{3\pi}{2} < -\frac{11\pi}{9} < -\pi$, $-\frac{11\pi}{9}$ lies in Quadrant II.

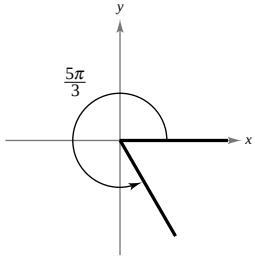
9. (a) Since $\pi < 3.5 < \frac{3\pi}{2}$, 3.5 lies in Quadrant III.

(b) Since $\frac{\pi}{2} < 2.25 < \pi$, 2.25 lies in Quadrant II.

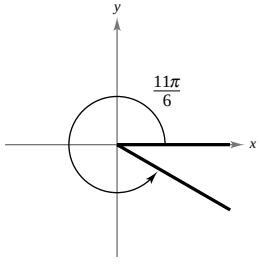
11. (a)



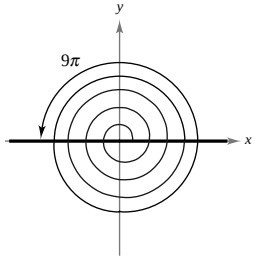
(b)



13. (a)



(b)



15. (a) Coterminal angles for $\frac{\pi}{12}$

$$\frac{\pi}{12} + 2\pi = \frac{25\pi}{12}$$

$$\frac{\pi}{12} - 2\pi = -\frac{23\pi}{12}$$

(b) Coterminal angles for $\frac{2\pi}{3}$

$$\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$$

$$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$$

17. (a) Coterminal angles for $-\frac{11\pi}{4}$

$$-\frac{11\pi}{4} + 4\pi = \frac{5\pi}{4}$$

$$-\frac{11\pi}{4} + 2\pi = -\frac{3\pi}{4}$$

(b) Coterminal angles for $-\frac{2\pi}{15}$

$$-\frac{2\pi}{15} + 2\pi = \frac{28\pi}{15}$$

$$-\frac{2\pi}{15} - 2\pi = -\frac{32\pi}{15}$$

19. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Supplement: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(b) Complement: Not possible; $\frac{3\pi}{4}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$

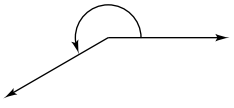
21. (a) Complement: $\frac{\pi}{2} - 1 \approx 0.57$

Supplement: $\pi - 1 \approx 2.14$

(b) Complement: none $\left(2 > \frac{\pi}{2}\right)$

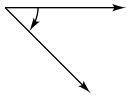
Supplement: $\pi - 2 \approx 1.14$

23.



The angle shown is approximately 210º.

25.



The angle shown is approximately -45º.

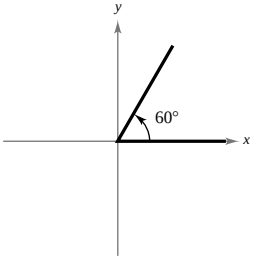
27. (a) Since $90° < 150° < 180°$, $150°$ lies in Quadrant II.

(b) Since $270° < 282° < 360°$, $282°$ lies in Quadrant IV.

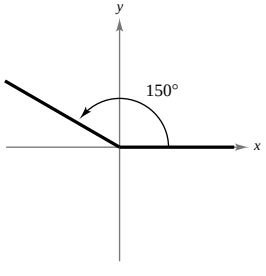
29. (a) Since $-180° < -132°\ 50' < -90°$, $-132°\ 50'$ lies in Quadrant III.

(b) Since $-360° < -336°\ 30' < -270°$, $-336°\ 30'$ lies in Quadrant I.

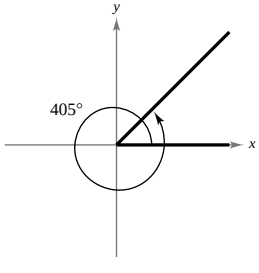
31. (a)



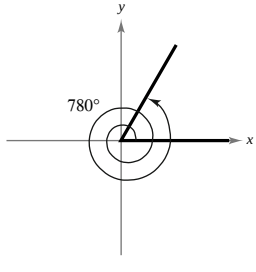
(b)



33. (a)



(b)



35. (a) Coterminal angles for $52°$

$52° + 360° = 412°$

$52° - 360° = -308°$

(b) Coterminal angles for $-36°$

$-36° + 360° = 324°$

$-36° - 360° = -396°$

37. (a) Coterminal angles for $300°$

$300° + 360° = 660°$

$300° - 360° = -60°$

(b) Coterminal angles for $230°$

$230° + 360° = 590°$

$230° - 360° = -130°$

39. (a) Complement of $24°$: $90° - 24° = 66°$

Supplement of $24°$: $180° - 24° = 156°$

(b) Complement of $126°$: Not possible because $126° > 90°$

Supplement of $126°$: $180° - 126° = 54°$

41. (a) Complement: $90° - 79° = 11°$

Supplement: $180° - 79° = 101°$

(b) Complement: does not exist

Supplement: $180° - 150° = 30°$

43. (a) $30° = 30\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$

(b) $150° = 150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6}$

45. (a) $-20° = -20\left(\frac{\pi}{180}\right) = -\frac{\pi}{9}$

(b) $-240° = -240\left(\frac{\pi}{180}\right) = -\frac{4\pi}{3}$

47. $115° = 115\left(\frac{\pi}{180}\right) \approx 2.007$ radians

49. $-216.35° = -216.35\left(\frac{\pi}{180}\right) \approx -3.776$ radians

51. $642° = 642\left(\frac{\pi}{180}\right) \approx 11.205$ radians

53. $-0.78° = -0.78\left(\frac{\pi}{180}\right) \approx -0.014$ radians

55. (a) $\frac{3\pi}{2} = \frac{3\pi}{2}\left(\frac{180}{\pi}\right)^\circ = 270°$

(b) $-\frac{7\pi}{6} = -\frac{7\pi}{6}\left(\frac{180}{\pi}\right)^\circ = -210°$

57. (a) $\frac{7\pi}{3} = \frac{7\pi}{3}\left(\frac{180°}{\pi}\right) = 420°$

(b) $-\frac{13\pi}{60} = -\frac{13\pi}{60}\left(\frac{180°}{\pi}\right) = -39°$

59. $\frac{\pi}{7} = \frac{\pi}{7}\left(\frac{180}{\pi}\right) \approx 25.714°$

61. $\frac{25\pi}{8} = \frac{25\pi}{8}\left(\frac{180}{\pi}\right) = 562.5°$

63. $-4.2\pi = -4.2\pi\left(\frac{180}{\pi}\right) = -756°$

65. $-2 = -2\left(\frac{180}{\pi}\right) \approx -114.592°$

67. (a) $64° 45' = 64° + \left(\frac{45}{60}\right)^\circ = 64.75°$

(b) $-124° 30' = -124° - \left(\frac{30}{60}\right)^\circ = -124.5°$

69. (a) $85° 18' 30'' = 85° + \left(\frac{18}{60}\right)^\circ + \left(\frac{30}{3600}\right)^\circ \approx 85.308°$

(b) $-408° 16' 25'' = -408° - \left(\frac{16}{60}\right)^\circ - \left(\frac{25}{3600}\right)^\circ \approx -408.274°$

71. (a) $280.6^\circ = 280^\circ + 0.6(60)' = 280^\circ\, 36'$

(b) $-115.8^\circ = -115^\circ - 0.8(60)' = -115^\circ\, 48'$

73. (a) $4.5 = 4.5\left(\dfrac{180}{\pi}\right)^\circ \approx 257^\circ\, 49'\, 51.628''$

(b) $-3.58 = -3.58\left(\dfrac{180}{\pi}\right)^\circ \approx -205^\circ\, 7'\, 8.006''$

75. $s = r\theta$

$6 = 5\theta$

$\theta = \frac{6}{5}$ radians

77. $s = r\theta$

$32 = 7\theta$

$\theta = \frac{32}{7} = 4\frac{4}{7}$ radians

79. $s = r\theta$

$8 = 15\theta$

$\theta = \frac{8}{15}$ radians

81. $s = r\theta$

$35 = 14.5\theta$

$\theta = \frac{70}{29} \approx 2.414$ radians

83. $s = r\theta$, θ in radians

$s = 14(180)\left(\dfrac{\pi}{180}\right) = 14\pi \approx 43.982$ inches

85. $s = r\theta$

$s = 6\left(\dfrac{2\pi}{3}\right) = 4\pi \approx 12.57$ meters

87. $\theta = 42^\circ\, 7'\, 15'' - 25^\circ\, 46'\, 37'' = 16^\circ\, 20'\, 38'' \approx 0.2853$ radian

$s = r\theta = 4000(0.2853) \approx 1141.02$ miles

89. $\theta = \dfrac{s}{r} = \dfrac{600}{6378} \approx 0.094$ radian $\approx 5.39^\circ$

91. $\theta = \dfrac{s}{r} = \dfrac{2.5}{6} = \dfrac{25}{60} = \dfrac{5}{12}$ radian $\approx 23.87^\circ$

93. (a) single axel: $1\frac{1}{2}$ revolutions $= 360^\circ + 180^\circ = 540^\circ$

$= 2\pi + \pi = 3\pi$ radians

(b) double axel: $2\frac{1}{2}$ revolutions $= 720^\circ + 180^\circ = 900^\circ$

$= 4\pi + \pi = 5\pi$ radians

(c) triple axel: $3\frac{1}{2}$ revolutions $= 1260^\circ$

$= 7\pi$ radians

95. (a) 40 miles per hour $= 40\dfrac{(5280)}{60} = 3520$ feet per minute

Circumference of tire is $C = 2.5\pi$ feet

Number of revolutions per minute is $r = \dfrac{3520}{2.5\pi} = \dfrac{1408}{\pi} \approx 448.2$ revolutions per minute

(b) The angular speed is $\dfrac{\theta}{t}$:

$\theta = \dfrac{3520}{2.5\pi}(2\pi) = 2816$ radians

Angular speed $= \dfrac{2816 \text{ radians}}{1 \text{ minute}} = 2816$ radians/minute

97. speed $=$ (360 revolutions/minute)$(2\pi\,(1.68)$ inches/revolution)

$= 1209.6\pi$ inches/minute

$= 20.16\pi$ inches/second

99. False, 1 radian $= \left(\frac{180}{\pi}\right)^\circ \approx 57.3^\circ$, so one radian is much larger than one degree.

101. True: $\frac{2\pi}{3} + \frac{\pi}{4} + \frac{\pi}{12} = \frac{8\pi + 3\pi + \pi}{12} = \pi = 180^\circ$

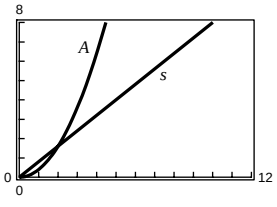
103. Two angles in standard position are coterminal angles if they have the same initial and terminal sides. For example, 30° and 390° are coterminal.

105. $A = \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2 \cdot \frac{\pi}{3} = \frac{50}{3}\pi \text{ m}^2$

107. $A = \frac{1}{2}r^2\theta$, $s = r\theta$

(a) $\theta = 0.8 \implies A = \frac{1}{2}r^2(0.8) = 0.4r^2$ Domain: $r > 0$

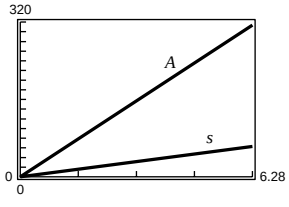
$s = r\theta = r(0.8)$ Domain: $r > 0$



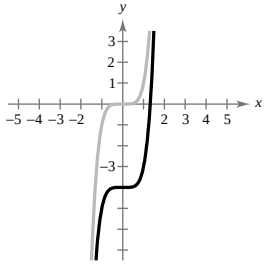
The area function changes more rapidly for $r > 1$ because it is quadratic and the arc length function is linear.

(b) $r = 10 \implies A = \frac{1}{2}(10^2)\theta = 50\theta$ Domain: $0 < \theta < 2\pi$

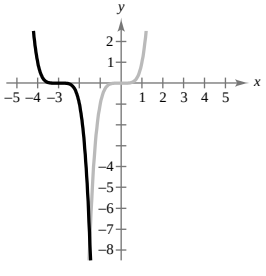
$s = r\theta = 10\theta$ Domain: $0 < \theta < 2\pi$



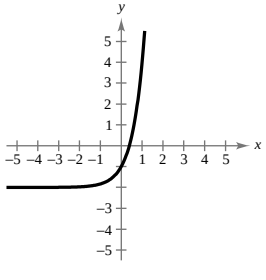
109.



111.



113.



115.