Section 4.2 Trigonometric Functions: The Unit Circle

Solutions to Odd-Numbered Exercises

1. $\sin t = y = \frac{15}{17}$

$\cos t = x = -\frac{8}{17}$

$\tan t = \frac{y}{x} = -\frac{15}{8}$

$\cot t = \frac{x}{y} = -\frac{8}{15}$

$\sec t = \frac{1}{x} = -\frac{17}{8}$

$\csc t = \frac{1}{y} = \frac{17}{15}$

3. $\sin t = y = -\frac{5}{13}$

$\cos t = x = \frac{12}{13}$

$\tan t = \frac{y}{x} = -\frac{5}{12}$

$\cot t = \frac{x}{y} = -\frac{12}{5}$

$\sec t = \frac{1}{x} = \frac{13}{12}$

$\csc t = \frac{1}{y} = -\frac{13}{5}$

5. $t = \frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

7. $t = \frac{7\pi}{6}$ corresponds to $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9. $t = \frac{4\pi}{3}$ corresponds to $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

11. $t = \frac{3\pi}{2}$ corresponds to $(0, -1)$.

13. $t = \frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = \frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = 1$

15. $t = -\frac{\pi}{6}$ corresponds to $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin t = y = -\frac{1}{2}$

$\cos t = x = \frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = -\frac{\sqrt{3}}{3}$

17. $t = -\frac{7\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = \frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = 1$

19. $t = \frac{11\pi}{6}$ corresponds to $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin t = y = -\frac{1}{2}$

$\cos t = x = \frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = -\frac{\sqrt{3}}{3}$

21. $t = -\frac{3\pi}{2}$ corresponds to $(0, 1)$.

$\sin t = y = 1$

$\cos t = x = 0$

$\tan t = \frac{y}{x}$ is undefined.

23. $t = \frac{3\pi}{4}$ corresponds to $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin t = y = \frac{\sqrt{2}}{2}$ $\quad \csc t = \frac{1}{y} = \sqrt{2}$

$\cos t = x = -\frac{\sqrt{2}}{2}$ $\quad \sec t = \frac{1}{x} = -\sqrt{2}$

$\tan t = \frac{y}{x} = -1$ $\quad \cot t = \frac{x}{y} = -1$

25. $t = \frac{\pi}{2}$ corresponds to $(0, 1)$.

$\sin t = y = 1$ $\quad \csc t = \frac{1}{y} = 1$

$\cos t = x = 0$ $\quad \sec t = \frac{1}{x}$ is undefined.

$\tan t = \frac{y}{x}$ is undefined. $\quad \cot t = \frac{x}{y} = 0$

27. $t = -\frac{\pi}{3}$ corresponds to $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\sin t = y = -\frac{\sqrt{3}}{2}$, $\cos t = x = \frac{1}{2}$, $\tan t = \frac{y}{x} = -\sqrt{3}$

$\csc t = \frac{1}{y} = -\frac{2}{\sqrt{3}}$, $\sec t = \frac{1}{x} = 2$, $\cot t = \frac{x}{y} = -\frac{1}{\sqrt{3}}$

29. $\sin 5\pi = \sin \pi = 0$

31. $\cos \frac{8\pi}{3} = \cos \frac{2\pi}{3} = -\frac{1}{2}$

33. $\cos(-3\pi) = \cos \pi = -1$

35. $\sin\left(-\frac{9\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

37. $\sin t = \frac{1}{3}$

(a) $\sin(-t) = -\sin t = -\frac{1}{3}$

(b) $\csc(-t) = -\csc t = -3$

39. $\cos(-t) = -\frac{1}{5}$

(a) $\cos t = \cos(-t) = -\frac{1}{5}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = -5$

41. $\sin t = \frac{4}{5}$

(a) $\sin(\pi - t) = \sin t = \frac{4}{5}$

(b) $\sin(t + \pi) = -\sin t = -\frac{4}{5}$

43. $\sin \frac{\pi}{4} \approx 0.7071$

45. $\csc 1.3 \approx 1.0378$

47. $\cos(-1.7) \approx -0.1288$

49. $\csc 0.8 = \frac{1}{\sin 0.8} \approx 1.3940$

51. $\sec 22.8 = \frac{1}{\cos 22.8} \approx -1.4486$

53. (a) $\sin 5 \approx -1$

(b) $\cos 2 \approx -0.4$

55. (a) $\sin t = 0.25$

$t \approx 0.25$ or 2.89

(b) $\cos t = -0.25$

$t \approx 1.82$ or 4.46

57. $I = 5e^{-2(0.7)} \sin(0.7) \approx 0.794$

59. $y(t) = \frac{1}{4} \cos 6t$

(a) $y(0) = \frac{1}{4} \cos 0 = 0.2500$ feet

(b) $y\left(\frac{1}{4}\right) = \frac{1}{4} \cos \frac{3}{2} \approx 0.0177$ feet

(c) $y\left(\frac{1}{2}\right) = \frac{1}{4} \cos 3 \approx -0.2475$ feet

61. $\cos 1.5 \approx 0.0707$, $2 \cos 0.75 \approx 1.4634$

Thus, $\cos 2t \neq 2 \cos t$

63. False. $\sin\left(\dfrac{-4\pi}{3}\right) = \dfrac{\sqrt{3}}{2} > 0$

65. (a) The points have y-axis symmetry.

(b) $\sin t_1 = \sin(\pi - t_1)$ since they have the same y-value.

(c) $-\cos t_1 = \cos(\pi - t_1)$ since the x-values have the opposite signs.

67. $\cos \theta = x = \cos(-\theta)$

$\sin \theta = \dfrac{1}{x} = \sec(-\theta)$

$\sin \theta = y$

$\sin(-\theta) = -y = -\sin \theta$

$\sec \theta = \dfrac{1}{y}$

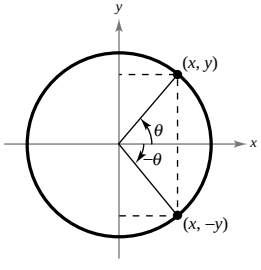
$\sec(-\theta) = -\dfrac{1}{y} = -\sec \theta$

$\tan \theta = \dfrac{y}{x}$

$\tan(-\theta) = \dfrac{-y}{x} = -\tan \theta$

$\cot \theta = \dfrac{x}{y}$

$\cot(-\theta) = \dfrac{x}{-y} = -\cot \theta$



69. $f(t) = \sin t$ and $g(t) = \tan t$

Both f and g are odd functions.

$h(t) = f(t)g(t) = \sin t \tan t$

$h(-t) = \sin(-t) \tan(-t)$

$= (-\sin t)(-\tan t)$

$= \sin t \tan t = h(t)$

The function $h(t) = f(t)g(t)$ is even.

71. $f(x) = \frac{1}{4}x^3 + 1$

$y = \frac{1}{4}x^3 + 1$

$x = \frac{1}{4}y^3 + 1$

$x - 1 = \frac{1}{4}y^3$

$4(x - 1) = y^3$

$y = \sqrt[3]{4(x - 1)}$

$f^{-1}(x) = \sqrt[3]{4(x - 1)}$