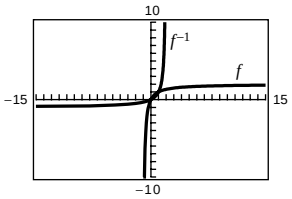
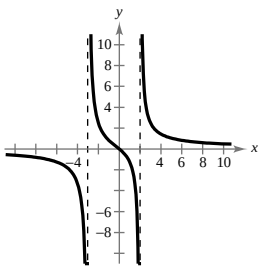
73. $f(x) = \dfrac{2x}{x+1}, x > -1$

$$y = \frac{2x}{x+1}, x > -1$$
$$x = \frac{2y}{y+1}$$
$$xy + x = 2y$$
$$x = 2y - xy$$
$$x = y(2 - x)$$
$$\frac{x}{2-x} = y, x < 2$$
$$f^{-1}(x) = \frac{x}{2-x}, x < 2$$



75. $f(x) = \dfrac{5x}{x^2 - x - 6} = \dfrac{5x}{(x+3)(x-2)}$

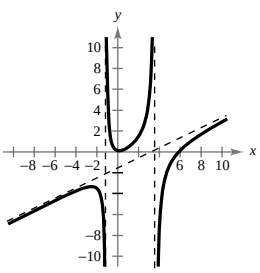
Asymptotes: $x = -3, x = 2, y = 0$



77. $f(x) = \dfrac{x^3 - 6x^2 + x - 1}{2x^2 - 5x - 8} = \dfrac{x}{2} - \dfrac{7}{4} - \dfrac{15(x+4)}{4(2x^2 - 5x - 8)}$

Slant asymptote: $y = \dfrac{x}{2} - \dfrac{7}{4}$

Vertical asymptotes: $x \approx 3.608, x \approx -1.108$



79. $C(10) \approx 69.95(1.045)^{10} \approx \$108.63$

81. (a) $p(0) = \dfrac{1200}{1 + 3e^0} = \dfrac{1200}{4} = 300$

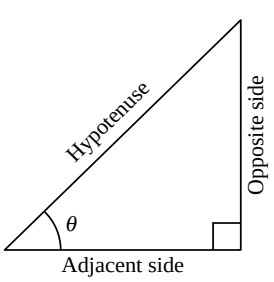
(b) $p(5) \approx 570$ (c) $p = 800$ when $t \approx 8.96$ years

Section 4.3 Right Triangle Trigonometry

- You should know the right triangle definition of trigonometric functions.

(a) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ (b) $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ (c) $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

(d) $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ (e) $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ (f) $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$



- You should know the following identities.

(a) $\sin\theta = \dfrac{1}{\csc\theta}$ (b) $\csc\theta = \dfrac{1}{\sin\theta}$ (c) $\cos\theta = \dfrac{1}{\sec\theta}$

(d) $\sec\theta = \dfrac{1}{\cos\theta}$ (e) $\tan\theta = \dfrac{1}{\cot\theta}$ (f) $\cot\theta = \dfrac{1}{\tan\theta}$

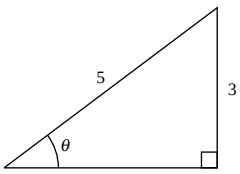
(g) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (h) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ (i) $\sin^2\theta + \cos^2\theta = 1$

(j) $1 + \tan^2\theta = \sec^2\theta$ (k) $1 + \cot^2\theta = \csc^2\theta$

- You should know that two acute angles α and β are complementary if $\alpha + \beta = 90°$, and cofunctions of complementary angles are equal.
- You should know the trigonometric function values of 30°, 45°, and 60°, or be able to construct triangles from which you can determine them.

Solutions to Odd-Numbered Exercises

1.



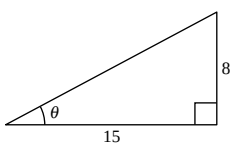
$$\text{adj} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

3.



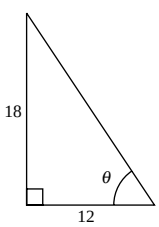
$$\text{hyp} = \sqrt{8^2 + 15^2} = 17$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$$

5.



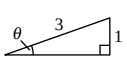
$$\text{hyp} = \sqrt{18^2 + 12^2} = \sqrt{468} = 6\sqrt{13}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{18}{6\sqrt{13}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{13}}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{6\sqrt{13}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{13}}{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{18}{12} = \frac{3}{2} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{3}$$

7.

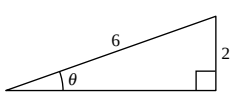


$$\text{adj} = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{3} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = 3$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{2}}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{2}$$



$$\text{adj} = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$$

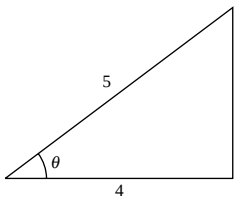
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{6} = \frac{1}{3} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6}{2} = 3$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{6}{4\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{4\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

9.



$\text{opp} = \sqrt{10^2 - 8^2} = 6$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{6}{10} = \dfrac{3}{5}$ $\qquad$ $\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{10}{6} = \dfrac{5}{3}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{8}{10} = \dfrac{4}{5}$ $\qquad$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{10}{8} = \dfrac{5}{4}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{6}{8} = \dfrac{3}{4}$ $\qquad$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{8}{6} = \dfrac{4}{3}$

$\text{opp} = \sqrt{2.5^2 - 2^2} = 1.5$

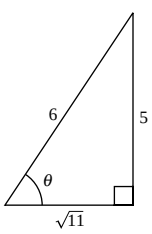
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1.5}{2.5} = \dfrac{3}{5}$ $\qquad$ $\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{2.5}{1.5} = \dfrac{5}{3}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{2}{2.5} = \dfrac{4}{5}$ $\qquad$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{2.5}{2} = \dfrac{5}{4}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1.5}{2} = \dfrac{3}{4}$ $\qquad$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{2}{1.5} = \dfrac{4}{3}$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

11. Given: $\sin\theta = \dfrac{5}{6} = \dfrac{\text{opp}}{\text{hyp}}$



$5^2 + (\text{adj})^2 = 6^2$

$\text{adj} = \sqrt{11}$

$\cos\theta = \dfrac{\sqrt{11}}{6}$

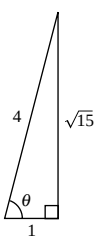
$\tan\theta = \dfrac{5}{\sqrt{11}} = \dfrac{5\sqrt{11}}{11}$

$\cot\theta = \dfrac{\sqrt{11}}{5}$

$\sec\theta = \dfrac{6}{\sqrt{11}} = \dfrac{6\sqrt{11}}{11}$

$\csc\theta = \dfrac{6}{5}$

13. Given: $\sec\theta = 4 = \dfrac{4}{1} = \dfrac{\text{hyp}}{\text{adj}}$



$(\text{opp})^2 + 1^2 = 4^2$

$\text{opp} = \sqrt{15}$

$\sin\theta = \dfrac{\sqrt{15}}{4}$

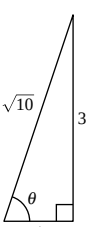
$\cos\theta = \dfrac{1}{4}$

$\tan\theta = \sqrt{15}$

$\cot\theta = \dfrac{1}{\sqrt{15}} = \dfrac{\sqrt{15}}{15}$

$\csc\theta = \dfrac{4}{\sqrt{15}} = \dfrac{4\sqrt{15}}{15}$

15. Given: $\tan\theta = 3 = \dfrac{3}{1} = \dfrac{\text{opp}}{\text{adj}}$



$3^2 + 1^2 = (\text{hyp})^2$

$\text{hyp} = \sqrt{10}$

$\sin\theta = \dfrac{3\sqrt{10}}{10}$

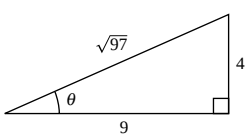
$\cos\theta = \dfrac{\sqrt{10}}{10}$

$\cot\theta = \dfrac{1}{3}$

$\sec\theta = \sqrt{10}$

$\csc\theta = \dfrac{\sqrt{10}}{3}$

17. Given: $\cot\theta = \dfrac{9}{4} = \dfrac{\text{adj}}{\text{hyp}}$



$4^2 + 9^2 = (\text{hyp})^2$

$\text{hyp} = \sqrt{97}$

$\sin\theta = \dfrac{4}{\sqrt{97}} = \dfrac{4\sqrt{97}}{97}$

$\cos\theta = \dfrac{9}{\sqrt{97}} = \dfrac{9\sqrt{97}}{97}$

$\tan\theta = \dfrac{4}{9}$

$\sec\theta = \dfrac{\sqrt{97}}{9}$

$\csc\theta = \dfrac{\sqrt{97}}{4}$

19. $\sin 60^\circ = \dfrac{\sqrt{3}}{2},\ \cos 60^\circ = \dfrac{1}{2}$

(a) $\tan 60^\circ = \dfrac{\sin 60^\circ}{\cos 60^\circ} = \sqrt{3}$

(b) $\sin 30^\circ = \cos 60^\circ = \dfrac{1}{2}$

(c) $\cos 30^\circ = \sin 60^\circ = \dfrac{\sqrt{3}}{2}$

(d) $\cot 60^\circ = \dfrac{\cos 60^\circ}{\sin 60^\circ} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

21. $\csc\theta = 3,\ \sec\theta = \dfrac{3\sqrt{2}}{4}$

(a) $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{3}$

(b) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{2\sqrt{2}}{3}$

(c) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1/3}{(2\sqrt{2})/3} = \dfrac{\sqrt{2}}{4}$

(d) $\sec(90^\circ - \theta) = \csc\theta = 3$

23. $\cos\alpha = \dfrac{1}{4}$

(a) $\sec\alpha = \dfrac{1}{\cos\alpha} = 4$

(b) $\sin^2\alpha + \cos^2\alpha = 1$

$$\sin^2\alpha + \left(\frac{1}{4}\right)^2 = 1$$

$$\sin^2\alpha = \frac{15}{16}$$

$$\sin\alpha = \pm\frac{\sqrt{15}}{4}$$

(c) $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha} = \pm\dfrac{1/4}{\sqrt{15}/4} = \pm\dfrac{1}{\sqrt{15}} = \pm\dfrac{\sqrt{15}}{15}$

(d) $\sin(90^\circ - \alpha) = \cos\alpha = \dfrac{1}{4}$

25. $\tan\theta\cot\theta = \tan\theta\left(\dfrac{1}{\tan\theta}\right) = 1$

27. $\tan\alpha\cos\alpha = \left(\dfrac{\sin\alpha}{\cos\alpha}\right)\cos\alpha = \sin\alpha$

29. $(1 + \cos\theta)(1 - \cos\theta) = 1 - \cos^2\theta$

$$= (\sin^2\theta + \cos^2\theta) - \cos^2\theta$$

$$= \sin^2\theta$$

31. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

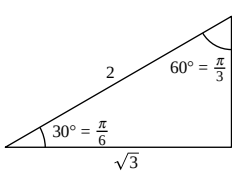
$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$$

$$= \csc\theta\sec\theta$$

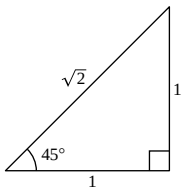
33. (a) $\cos 60^\circ = \dfrac{1}{2}$

(b) $\tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$



35. (a) $\cot\dfrac{\pi}{4} = \cot 45^\circ = 1$

(b) $\cos 45^\circ = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$



37. (a) $\cos\dfrac{\pi}{6} = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$

(b) $\sec 60^\circ = \dfrac{1}{\cos 60^\circ} = 2$

39. (a) $\sin 25^\circ \approx 0.4226$

(b) $\cos 65^\circ \approx 0.4226$

Note: $\sin 25^\circ = \cos(90^\circ - 25^\circ) = \cos 65^\circ$

41. (a) $\sec 42^\circ\, 12' = \sec 42.2^\circ = \dfrac{1}{\cos 42.2^\circ} \approx 1.3499$

(b) $\csc 48^\circ\, 7' = \dfrac{1}{\sin\left(48 + \frac{7}{60}\right)^\circ} \approx 1.3432$

43. Make sure that your calculator is in radian mode.

(a) $\cot\dfrac{\pi}{16} = \dfrac{1}{\tan(\pi/16)} \approx 5.0273$ (b) $\tan\dfrac{\pi}{16} \approx 0.1989$

45. Make sure that your calculator is in radian mode.

(a) $\csc 1 = \dfrac{1}{\sin 1} \approx 1.1884$ (b) $\tan\dfrac{1}{2} \approx 0.5463$

47. (a) $\sin\theta = \dfrac{1}{2} \implies \theta = 30^\circ = \dfrac{\pi}{6}$

(b) $\csc\theta = 2 \implies \theta = 30^\circ = \dfrac{\pi}{6}$

49. (a) $\sec\theta = 2 \implies \theta = 60^\circ = \dfrac{\pi}{3}$

(b) $\cot\theta = 1 \implies \theta = 45^\circ = \dfrac{\pi}{4}$

51. (a) $\csc\theta = \dfrac{2\sqrt{3}}{3} \implies \theta = 60^\circ = \dfrac{\pi}{3}$

(b) $\sin\theta = \dfrac{\sqrt{2}}{2} \implies \theta = 45^\circ = \dfrac{\pi}{4}$

53. (a) $\sin\theta = 0.8191 \implies \theta \approx 55^\circ \approx 0.960$ radian

(b) $\cos\theta = 0.0175 \implies \theta \approx 89^\circ \approx 1.553$ radians

55. (a) $\tan\theta = 1.1920 \implies \theta \approx 50^\circ \approx 0.873$ radian

(b) $\tan\theta = 0.4663 \implies \theta \approx 25^\circ \approx 0.436$ radian

57. $\tan 30^\circ = \dfrac{y}{105}$

$y = 105 \cdot \tan 30^\circ = 105\dfrac{\sqrt{3}}{3} = 35\sqrt{3} \approx 60.6218$

59. $\cot 60^\circ = \dfrac{x}{38}$

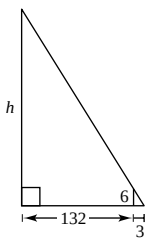
$\dfrac{\sqrt{3}}{3} = \dfrac{x}{38}$

$\dfrac{38\sqrt{3}}{3} = x$

61. $\sin 50^\circ = \dfrac{y}{15}$

$y = 15 \cdot \sin 50^\circ \approx 11.4907 \approx 11.5$

63. (a)



Not drawn to scale

(b) $\tan\theta = \dfrac{6}{3}$ and $\tan\theta = \dfrac{h}{135}$

Thus, $\dfrac{6}{3} = \dfrac{h}{135}$.

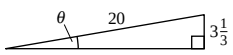
(c) $\dfrac{135 \cdot 6}{3} = h = 270$ feet

65. $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 58^\circ = \dfrac{w}{100}$

$w = 100 \tan 58^\circ \approx 160.0$ feet

67. (a)



(b) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

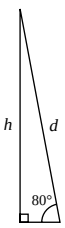
$\sin\theta = \dfrac{10/3}{20} = \dfrac{1}{6}$

(c) $\sin\theta = \dfrac{1}{6} \Longrightarrow \theta = 9.59^\circ$

69. $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 80^\circ = \dfrac{h}{75}$

$h = 75 \tan 80^\circ \approx 425.3$ meters



$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\cos 80^\circ = \dfrac{75}{d}$

$d = 75\,\dfrac{1}{\cos 80^\circ} \approx 431.9$ meters

71. $\tan 3^\circ = \dfrac{x}{15}$

$x = 15 \tan 3^\circ$

$d = 5 + 2x$

$= 5 + 2(15 \tan 3^\circ)$

≈ 6.57 centimeters

73. $x \approx 2.588, y \approx 9.659$

$\sin\theta = \dfrac{y}{10} \approx 0.97$ $\qquad$ $\csc\theta = \dfrac{10}{y} \approx 1.04$

$\cos\theta = \dfrac{x}{10} \approx 0.26$ $\qquad$ $\sec\theta = \dfrac{10}{x} \approx 3.86$

$\tan\theta = \dfrac{y}{x} \approx 3.73$ $\qquad$ $\cot\theta = \dfrac{x}{y} \approx 0.27$