Section 4.5 Graphs of Sine and Cosine Functions

- You should be able to graph $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$.
- Amplitude: $|a|$
- Period: $\frac{2\pi}{|b|}$
- Shift: Solve $bx - c = 0$ and $bx - c = 2\pi$.
- Key increments: $\frac{1}{4}$ (period)

Solutions to Odd-Numbered Exercises

1. $y = 3\sin 2x$

Period: $\frac{2\pi}{2} = \pi$

Amplitude: $|3| = 3$

Xmin = -2π
Xmax = 2π
Xscl = $\pi/2$
Ymin = -4
Ymax = 4
Yscl = 1

3. $y = \frac{5}{2}\cos\frac{x}{2}$

Period: $\frac{2\pi}{1/2} = 4\pi$

Amplitude $\left|\frac{5}{2}\right| = \frac{5}{2}$

Xmin = -4π
Xmax = 4π
Xscl = π
Ymin = -3
Ymax = 3
Yscl = 1

5. $y = \frac{2}{3}\sin \pi x$

Period: $\frac{2\pi}{\pi} = 2$

Amplitude: $\left|\frac{2}{3}\right| = \frac{2}{3}$

Xmin = -π
Xmax = π
Xscl = $\pi/2$
Ymin = -1
Ymax = 1
Yscl = .5

7. $y = -2\sin x$

Period: $\frac{2\pi}{1} = 2\pi$

Amplitude: $|-2| = 2$

9. $y = 3\sin 6x$

Period: $\frac{2\pi}{6} = \frac{\pi}{3}$

Amplitude: $|3| = 3$

11. $y = \frac{1}{4}\cos\frac{2x}{3}$

Period: $\frac{2\pi}{2/3} = 3\pi$

Amplitude: $\left|\frac{1}{4}\right| = \frac{1}{4}$

13. $y = 3\sin 4\pi x$

Period: $\frac{2\pi}{4\pi} = \frac{1}{2}$

Amplitude: $|3| = 3$

15. $f(x) = \sin x$

$g(x) = \sin(x - \pi)$

The graph of g is a horizontal shift to the right π units of the graph of f (a phase shift).

17. $f(x) = \cos 2x$

$g(x) = -\cos 2x$

The graph of g is a reflection in the x-axis of the graph of f.

19. $f(x) = \cos x$

$g(x) = -5\cos x$

The graph of g is five times the amplitude of f, and reflected in the x-axis.

21. $f(x) = \sin x$

$f(x) = 4 + \sin x$

The graph of g is a vertical shift upward of 4 units of the graph of f.

23. The graph of g has twice the amplitude as the graph of f. The period is the same.

25. The graph of g is a horizontal shift π units to the right of the graph of f.

27. $f(x) = -2 \sin x$

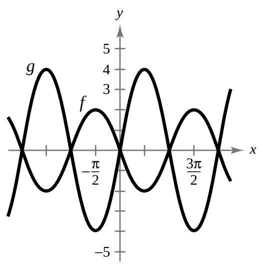
Period: 2π

Amplitude: 2

$g(x) = 4 \sin x$

Period: 2π

Amplitude: 4.

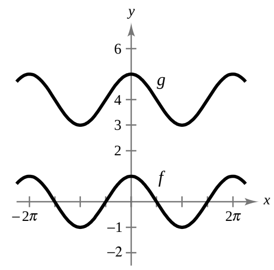


29. $f(x) = \cos x$

Period: 2π

Amplitude: 1

$g(x) = 4 + \cos x$ is a vertical shift of the graph of $f(x)$ four units upward.

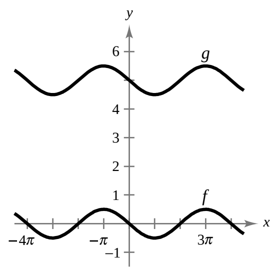


31. $f(x) = -\frac{1}{2} \sin \frac{x}{2}$

Period: 4π

Amplitude: $\frac{1}{2}$

$g(x) = 5 - \frac{1}{2} \sin \frac{x}{2}$ is the graph of $f(x)$ shifted vertically five units upward.

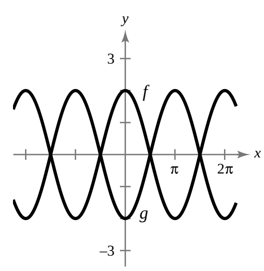


33. $f(x) = 2 \cos x$

Period: 2π

Amplitude: 2

$g(x) = 2 \cos(x + \pi)$ is the graph of $f(x)$ shifted π units to the left.

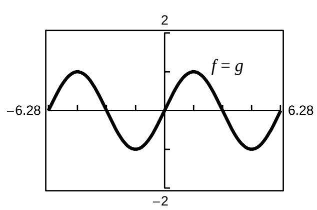


35. Since sine and cosine are cofunctions and x and $x - (\pi/2)$ are complementary, we have

$\sin x = \cos\left(x - \frac{\pi}{2}\right)$.

Period: 2π

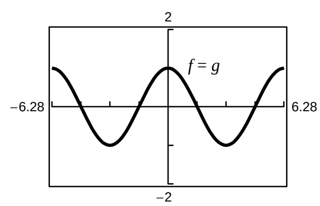
Amplitude: 1



37. $f(x) = \cos x$

$g(x) = -\sin\left(x - \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - x\right) = \cos x$

Thus, $f(x) = g(x)$.

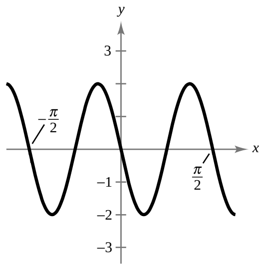


39. $y = -2 \sin 4x.$

Period: $\frac{2\pi}{4} = \frac{\pi}{2}.$

Amplitude: 2

Key points: $(0, 0), \left(\frac{\pi}{8}, -2\right), \left(\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{8}, 2\right), \left(\frac{\pi}{2}, 0\right)$

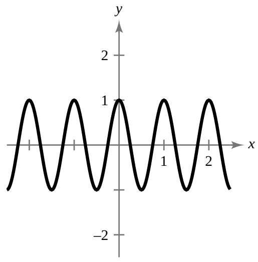


41. $y = \cos 2\pi x$

Period: $\frac{2\pi}{2\pi} = 1$

Amplitude: 1

Key points: $(0, 1), \left(\frac{1}{4}, 0\right), \left(\frac{1}{2}, 0\right), \left(\frac{3}{4}, 0\right), (1, 1)$

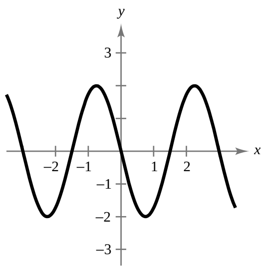


43. $y = -2 \sin \frac{2\pi x}{3}$

Period: $\frac{2\pi}{\frac{2\pi}{3}} = 3$

Amplitude: 2

Key points: $(0, 0), \left(\frac{3}{4}, -2\right), \left(\frac{3}{2}, 0\right), \left(\frac{9}{4}, 2\right), (3, 0)$



45. $y = \sin\left(x - \frac{\pi}{4}\right);\ a = 1,\ b = 1,\ c = \frac{\pi}{4}$

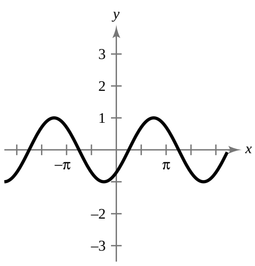
Period: 2π

Amplitude: 1

Shift: Set $x - \frac{\pi}{4} = 0$ and $x - \frac{\pi}{4} = 2\pi$

$x = \frac{\pi}{4}$ $\qquad x = \frac{9\pi}{4}$

Key points: $\left(\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{4}, 1\right), \left(\frac{5\pi}{4}, 0\right), \left(\frac{7\pi}{4}, -1\right), \left(\frac{9\pi}{4}, 0\right)$

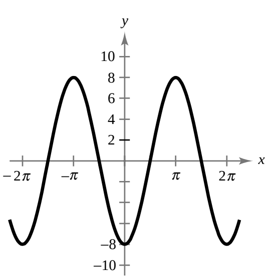


47. $y = 8 \cos (x + \pi)$

Period: 2π

Amplitude: 8

Key points: $(-\pi, 8), \left(-\frac{\pi}{2}, 0\right), (0, -8), \left(\frac{\pi}{2}, 0\right), (\pi, 8)$

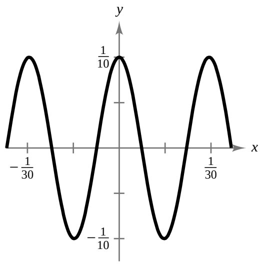


49. $y = \frac{1}{10}\cos(60\pi x);\ a = \frac{1}{10},\ b = 60\pi,\ c = 0$

Period: $\frac{2\pi}{60\pi} = \frac{1}{30}$

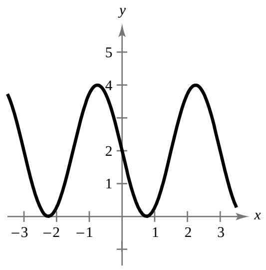
Amplitude: $\frac{1}{10}$

Key points: $\left(0, \frac{1}{10}\right), \left(\frac{1}{120}, 0\right), \left(\frac{1}{60}, -\frac{1}{10}\right), \left(\frac{1}{40}, 0\right), \left(\frac{1}{30}, \frac{1}{10}\right)$



51. $y = 2 - 2\sin\frac{2\pi x}{3}$

Vertical shift 2 units upward of the graph in Exercise #43.



53. $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right);\ a = \frac{2}{3},\ b = \frac{1}{2}, c = \frac{\pi}{4}$

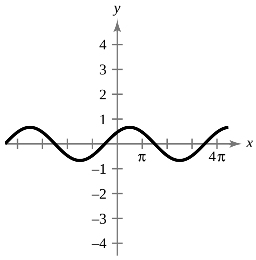
Period: 4π

Amplitude: $\frac{2}{3}$

Shift: Set $\frac{x}{2} - \frac{\pi}{4} = 0$ and $\frac{x}{2} - \frac{\pi}{4} = 2\pi$

$x = \frac{\pi}{2}$ $\qquad$ $x = \frac{9\pi}{2}$

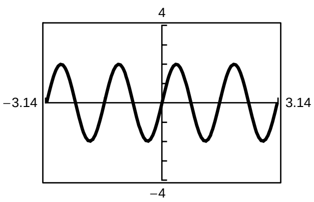
Key points: $\left(\frac{\pi}{2}, \frac{2}{3}\right), \left(\frac{3\pi}{2}, 0\right), \left(\frac{5\pi}{2}, \frac{-2}{3}\right), \left(\frac{7\pi}{2}, 0\right), \left(\frac{9\pi}{2}, \frac{2}{3}\right)$



55. $y = -2\sin(4x + \pi)$

Amplitude: 2

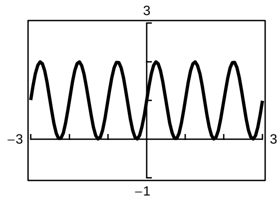
period: $\frac{\pi}{2}$



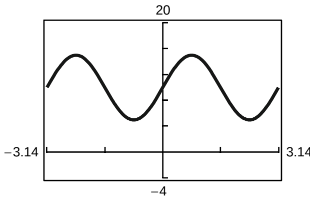
57. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$

Amplitude: 1

period: 1



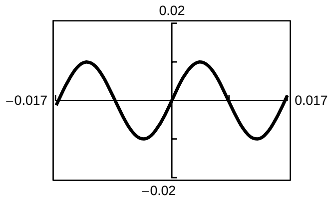
59. $y = 5\sin(\pi - 2x) + 10$



Amplitude: 5

Period: π

61. $y = \dfrac{1}{100}\sin 120\,\pi t$



Amplitude: $\frac{1}{100}$

Period: $\frac{1}{60}$

63. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[8 - 0] = 4$

Since $f(x)$ is the graph of $g(x) = 4\cos x$ reflected about the x-axis and shifted vertically 4 units upward, we have $a = -4$ and $d = 4$. Thus, $f(x) = -4\cos x + 4 = 4 - 4\cos x$.

65. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[7 - (-5)] = 6$

Graph of f is the graph of $g(x) = 6\cos x$ reflected about the x-axis and shifted vertically 1 unit upward. Thus $f(x) = -6\cos x + 1$.

67. $y = a\sin(bx - c)$

Amplitude: $|a| = |3|$.

Since the graph is reflected about the x-axis, we have $a = -3$.

Period: $\dfrac{2\pi}{b} = \pi \implies b = 2$

Phase shift: $c = 0$

Thus, $y = -3\sin 2x$.

69. $y = a\sin(bx + c)$

Amplitude: $a = 1$

Period: $2\pi \implies b = 1$

Phase shift: $bx + c = 0$ when $x = \dfrac{\pi}{4}$

$$(1)\frac{\pi}{4} + c = 0 \implies c = -\frac{\pi}{4}$$

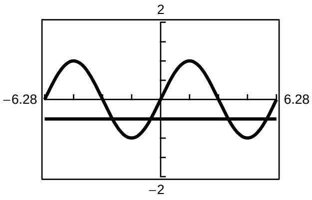
Thus, $y = \sin\left(x - \dfrac{\pi}{4}\right)$.

71. $y_1 = \sin x$

$y_2 = -\dfrac{1}{2}$

In the interval $[-2\pi, 2\pi]$, $\sin x = -\dfrac{1}{2}$ when

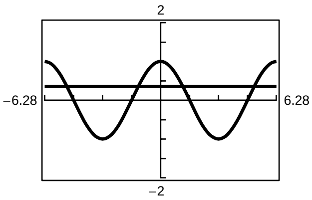
$x = -\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$.



73. $y_1 = \cos x$

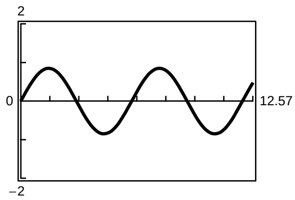
$y_2 = \dfrac{\sqrt{2}}{2}$

In the interval $[-2\pi, 2\pi]$, $\cos x = \dfrac{\sqrt{2}}{2}$ when $x = \pm\dfrac{\pi}{4}, \pm\dfrac{7\pi}{4}$.



75. $y = 0.85 \sin \dfrac{\pi t}{3}$

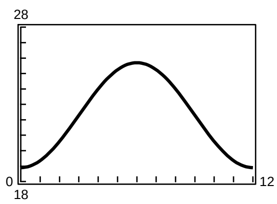
(a)



(b) Time for one cycle = one period = $\dfrac{2\pi}{\pi/3} = 6$ sec

(c) Cycles per min = $\dfrac{60}{6} = 10$ cycles per min

77. $S = 22.3 - 3.4 \cos \dfrac{\pi t}{6}, \; 1 \le t \le 12$

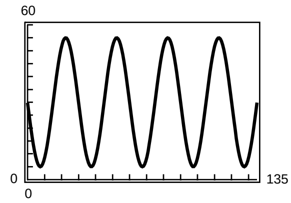


Maximum sales: June

Minimum sales: December

79. $h = 25 \sin \dfrac{\pi}{15}(t - 75) + 30$

(a)



(b) Minimum: $30 - 25 = 5$ feet

Maximum: $30 + 25 = 55$ feet

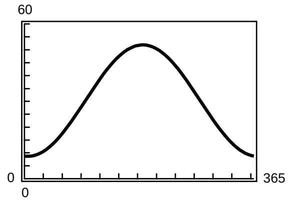
81. $C = 30.3 + 21.6 \sin\left(\dfrac{2\pi t}{365} + 10.9\right)$

(a) period $= \dfrac{2\pi}{b} = \dfrac{2\pi}{(2\pi/365)} = 365$ days

This is to be expected: 365 days = 1 year

(b) The constant 30.3 gallons is the average daily fuel consumption.

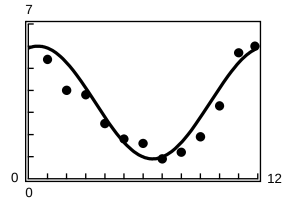
(c)



Consumption exceeds 40 gallons/day when $124 \le x \le 252$. (Graph C together with $y = 40$).

83. (a) $p = 2.55 \cos\left(\dfrac{\pi t}{6} - \dfrac{\pi}{12}\right) + 3.45$

(b)



The graph fits the data fairly well.

85. True. The period is $\dfrac{2\pi}{\left(\dfrac{3}{10}\right)} = \dfrac{20\pi}{3}$

87. The amplitude changes from $\dfrac{1}{2}$ to $\dfrac{3}{2}$ to 3. The graph of $y = -3 \sin x$ is a reflection in the x-axis of the graph of $g(x) = 3 \sin x$