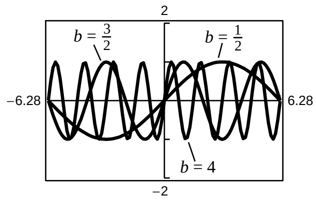
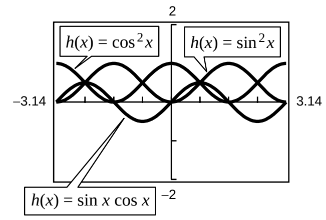
89. The period of the sign function changes from 4π to $\dfrac{4\pi}{3}$ to $\dfrac{\pi}{2}$.



91. (a) $h(x) = \cos^2 x$ is even.

(b) $g(x) = \sin^2 x$ is even.

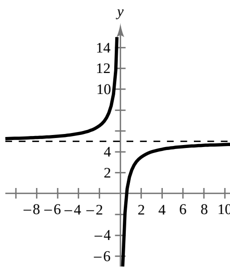
(c) $h(x) = \sin x \cos x$ is odd.



93. $f(x) = 1 - \dfrac{1}{2}x^2$ is the parabola opening downward. $g(x) = \cos x$ is periodic.

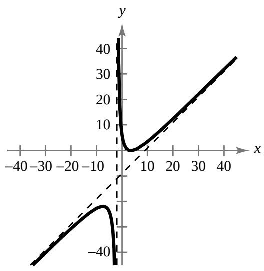
95. $f(x) = \dfrac{5x - 3}{x} = 5 - \dfrac{3}{x}$

Asymptotes: $x = 0, y = 5$



97. $f(x) = \dfrac{x^2 - 7x + 12}{x + 2} = x - 9 + \dfrac{30}{x + 2}$

Asymptotes: $x = -2, y = x - 9$



99. $-\dfrac{\pi}{9} = -\dfrac{\pi}{9}\left(\dfrac{180}{\pi}\right) = -20°$

101. $-0.48 = -0.48\left(\dfrac{180}{\pi}\right) \approx -27.502°$

Section 4.6 Graphs of Other Trigonometric Functions

- You should be able to graph:

  $y = a\tan(bx - c)$ $y = a\cot(bx - c)$

  $y = a\sec(bx - c)$ $y = a\csc(bx - c)$
- When graphing $y = a\sec(bx - c)$ or $y = a\csc(bx - c)$ you should know to first graph $y = a\cos(bx - c)$ or $y = a\sin(bx - c)$ since
  (a) The intercepts of sine and cosine are vertical asymptotes of cosecant and secant.
  (b) The maximums of sine and cosine are local minimums of cosecant and secant.
  (c) The minimums of sine and cosine are local maximums of cosecant and secant.
- You should be able to graph using a damping factor.

Solutions to Odd-Numbered Exercises

1. $y = \sec \frac{x}{2}$

Period: $\frac{2\pi}{1/2} = 4\pi$

Matches graph (g).

3. $y = \tan 2x$

Period: $\frac{\pi}{2}$

Matches graph (f).

5. $y = \cot \frac{\pi x}{2}$

Period: $\frac{\pi}{\pi/2} = 2$

Matches graph (b).

7. $y = -\csc x$

Period: 2π

Matches graph (e).

9. $y = \frac{1}{3} \tan x$

Period: π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	$-\frac{1}{3}$	0	$\frac{1}{3}$

11. $y = -2 \tan 2x$

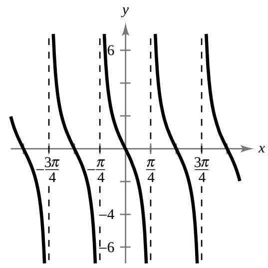
Period: $\frac{\pi}{2}$

Two consecutive asymptotes:

$2x = -\frac{\pi}{2} \Rightarrow x = -\frac{\pi}{4}$

$2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$

x	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$
y	2	0	-2

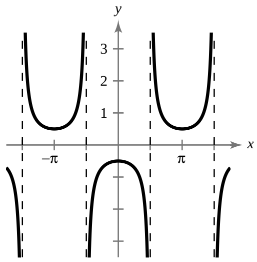


13. $y = -\frac{1}{2} \sec x$

Graph $y = -\frac{1}{2} \cos x$ first.

Period: 2π

One cycle: 0 to 2π

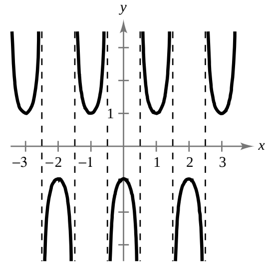


15. $y = -\sec \pi x$

Graph $y = -\cos \pi x$ first.

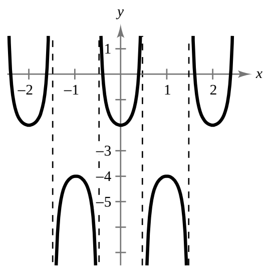
Period: $\frac{2\pi}{\pi} = 2$

One cycle: 0 to 2



17. $y = \sec \pi x - 3$

Reflect the graph in Exercise #15 about the x-axis and then shift it vertically down three units.

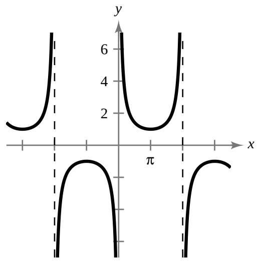


19. $y = \csc \dfrac{x}{2}$

Graph $y = \sin \dfrac{x}{2}$ first.

Period: $\dfrac{2\pi}{1/2} = 4\pi$

One cycle: 0 to 4π



21. $y = \dfrac{1}{2} \cot \dfrac{x}{2}$

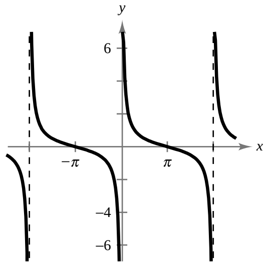
Period: $\dfrac{\pi}{1/2} = 2\pi$

Two consecutive asymptotes:

$\dfrac{x}{2} = 0 \implies x = 0$

$\dfrac{x}{2} = \pi \implies x = 2\pi$

x	$\dfrac{\pi}{2}$	π	$\dfrac{3\pi}{2}$
y	$\dfrac{1}{2}$	0	$-\dfrac{1}{2}$

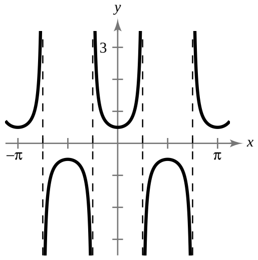


23. $y = \dfrac{1}{2} \sec 2x$

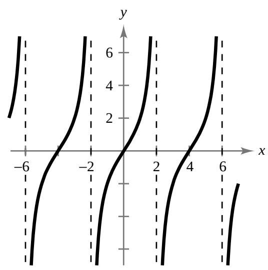
Graph $y = \dfrac{1}{2} \cos 2x$ first.

Period: $\dfrac{2\pi}{2} = \pi$

One cycle: 0 to π



25. $y = 2 \tan \dfrac{\pi x}{4}$



Period: $\dfrac{\pi}{\pi/4} = 4$

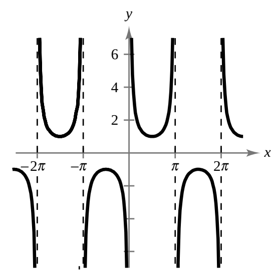
Two consecutive asymptotes:

$\dfrac{\pi x}{4} = -\dfrac{\pi}{2} \Longrightarrow x = -2$

$\dfrac{\pi x}{4} = \dfrac{\pi}{2} \Longrightarrow x = 2$

x	-1	0	1
y	-2	0	2

27. $y = \csc(\pi - x)$



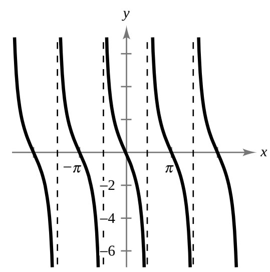
Graph $y = \sin(\pi - x)$ first.

Period: 2π

Shift: Set $\pi - x = 0$ and $\pi - x = 2\pi$

$x = \pi \qquad x = -\pi$

29. $y = 2 \cot\left(x - \dfrac{\pi}{2}\right)$.



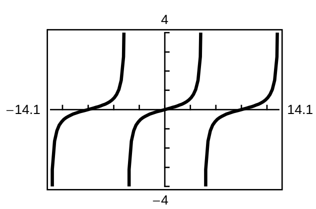
Period: π

Two consecutive asymptotes: $x - \dfrac{\pi}{2} = 0 \Longrightarrow x = \dfrac{\pi}{2}$

$x - \dfrac{\pi}{2} = \pi \Longrightarrow x = \dfrac{3\pi}{2}$

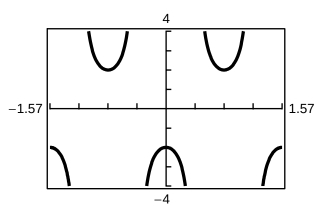
x	$\dfrac{3\pi}{4}$	π	$\dfrac{5\pi}{4}$
y	2	0	-2

31. $y = \dfrac{1}{3} \tan \dfrac{x}{3}$

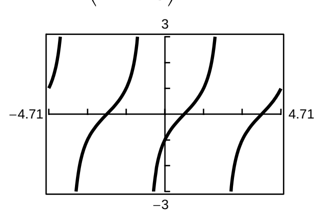


33. $y = -2 \sec 4x$

$= \dfrac{-2}{\cos 4x}$

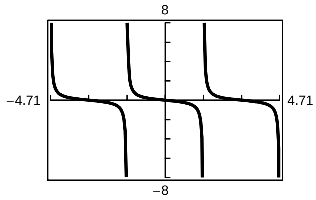


35. $y = \tan\left(x - \dfrac{\pi}{4}\right)$



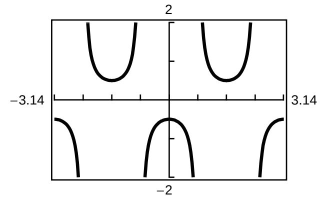
37. $y = \frac{1}{4}\cot\left(x + \frac{\pi}{2}\right)$

$= \frac{1}{4\tan\left(x + \frac{\pi}{2}\right)}$



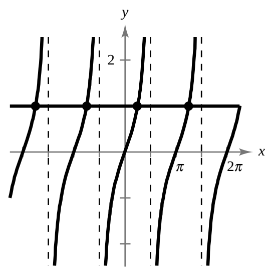
39. $y = \frac{1}{2}\sec(2x - \pi)$

$y = \frac{1}{2\cos(2x - \pi)}$



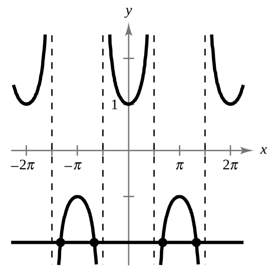
41. $\tan x = 1$

$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$



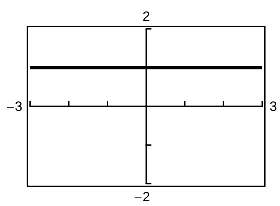
43. $\sec x = -2$

$x = \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}$



45. The graph of $f(x) = \sec x$ has y-axis symmetry. Thus, the function is even.

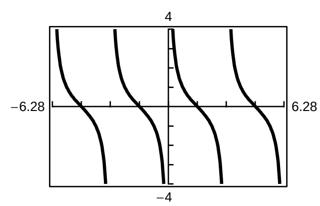
47. $y_1 = \sin x \csc x$ and $y_2 = 1$



Not equivalent because y_1, is not defined at 0

$\sin x \csc x = \sin x\left(\frac{1}{\sin x}\right) = 1, \sin x \neq 0$

49. $y_1 = \frac{\cos x}{\sin x}$ and $y_2 = \cot x = \frac{1}{\tan x}$



Equivalent

$\cot x = \frac{\cos x}{\sin x}$

51. $f(x) = x \cos x$

As $x \longrightarrow 0, f(x) \longrightarrow 0.$

Matches graph (d).

53. $g(x) = |x| \sin x$

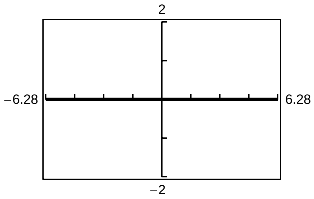
As $x \longrightarrow 0, g(x) \longrightarrow 0.$

Matches graph (b).

55. $f(x) = \sin x + \cos\left(x + \frac{\pi}{2}\right), g(x) = 0$

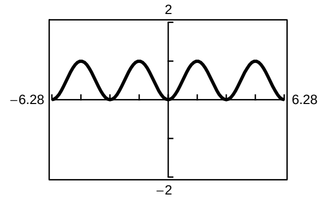
$f(x) = g(x)$

The graph is the line $y = 0$.



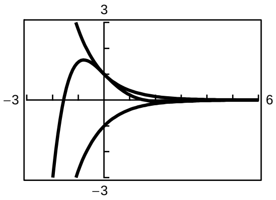
57. $f(x) = \sin^2 x, g(x) = \frac{1}{2}(1 - \cos 2x)$

$f(x) = g(x)$



59. $f(x) = e^{-x} \cos x$

Damping factor: e^{-x}

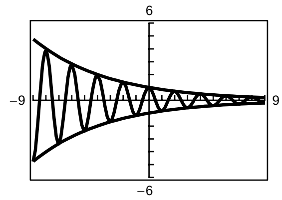


As $x \longrightarrow \infty, f(x) \longrightarrow 0$.

61. $f(x) = 2^{-x/4} \cos \pi x$

$-2^{-x/4} \leq f(x) \leq 2^{-2x/4}$

The damping factor is $y = 2^{-x/4}$.

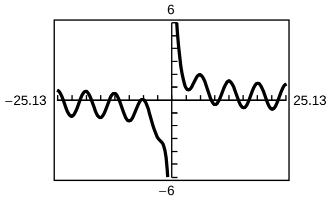


As $x \longrightarrow \infty, f \longrightarrow 0$

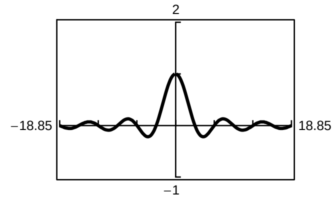
63. $y = \frac{6}{x} + \cos x$

As $x \longrightarrow 0$, from the right, $y \rightarrow \infty$.

As $x \rightarrow 0$. from the left, $y \rightarrow -\infty$.



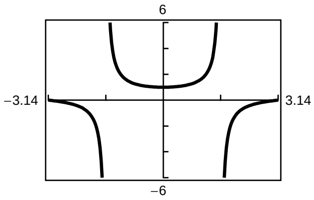
65.



As x tends to 0, $\frac{\sin x}{x}$ approaches 1.

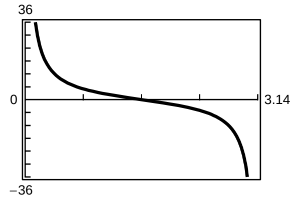
67. $f(x) = \frac{\tan x}{x}$

As $x \rightarrow 0, f(x) \rightarrow 1$



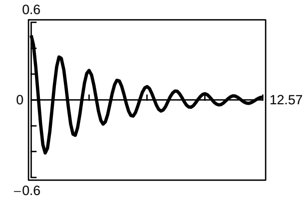
69. $\tan x = \frac{5}{d}$

$d = \frac{5}{\tan x} = 5 \cot x$



71. As the predator population increases, the number of prey decrease. When the number of prey is small, the number of predators decreases.

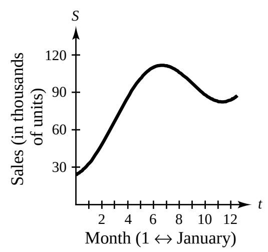
73. (a)



(b) The displacement function is approximately periodic, but damped. It approaches 0 as t increases.

75. $S = 52 + 5t - 28\cos\dfrac{\pi t}{6}$

(a)



(b) least sales: January
$(t = 1, S \approx 32.75$ thousand units$)$

greatest sales: June
$(t \approx 6.66, S \approx 111.64$ thousand units$)$

77. (a) If a spring of less stiffness is used, then c will be less than 8.2.

(b) If the effect of friction is decreased, then b will be greater than 0.22: $0.22 < b < 1$.

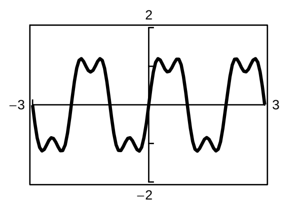
79. True. $-\dfrac{3\pi}{2} + \pi = -\dfrac{\pi}{2}$ and $x = -\dfrac{\pi}{2}$ is a vertical asymptote for the tangent function.

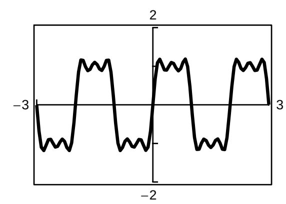
81. True. $2^x \sin x \to 0$ as $x \to -\infty$.

83. For $f(x) = \csc x$, as x approaches π from the left, f approaches ∞. As x approaches π from the right, f approaches $-\infty$.

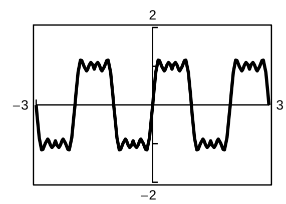
85. (a) $y_1 = \dfrac{4}{\pi}\left(\sin(\pi x) + \dfrac{1}{3}\sin(3\pi x)\right)$

$y_2 = \dfrac{4}{\pi}\left(\sin(\pi x) + \dfrac{1}{3}\sin(3\pi x) + \dfrac{1}{5}\sin(5\pi x)\right)$





(b) $y_3 = \dfrac{4}{\pi}\left(\sin(\pi x) + \dfrac{1}{3}\sin(3\pi x) + \dfrac{1}{5}\sin(5\pi x) + \dfrac{1}{7}\sin(7\pi x)\right)$



(c) $y_4 = \dfrac{4}{\pi}\left(\sin(\pi x) + \dfrac{1}{3}\sin(3\pi x) + \dfrac{1}{5}\sin(5\pi x) + \dfrac{1}{7}\sin(7\pi x) + \dfrac{1}{9}\sin(9\pi x)\right)$