87. Not one-to-one

89. One-to-one.

$y = \sqrt[3]{x - 5}$

$x = \sqrt[3]{y - 5}$

$x^3 = y - 5$

$y = x^3 + 5$

$f^{-1}(x) = x^3 + 5$

91. hyp $= \sqrt{9^2 + 14^2} = \sqrt{277}$

$\sin\theta = \dfrac{9}{\sqrt{277}}$ $\quad\cos\theta = \dfrac{14}{\sqrt{277}}$

$\tan\theta = \dfrac{9}{14}$ $\quad\cot\theta = \dfrac{14}{9}$

$\csc\theta = \dfrac{\sqrt{277}}{9}$ $\quad\sec\theta = \dfrac{\sqrt{277}}{14}$

Section 4.7 Inverse Trigonometric Functions

- You should know the definitions, domains, and ranges of $y = \arcsin x$, $y = \arccos x$, and $y = \arctan x$.

Function	Domain	Range
$y = \arcsin x \implies x = \sin y$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x \implies x = \cos y$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x \implies x = \tan y$	$-\infty < x < \infty$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$

- You should know the inverse properties of the inverse trigonometric functions.

  $\sin(\arcsin x) = x,\ -1 \le x \le 1$ and $\arcsin(\sin y) = y,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

  $\cos(\arccos x) = x,\ -1 \le x \le 1$ and $\arccos(\cos y) = y,\ 0 \le y \le \pi$

  $\tan(\arctan x) = x$ and $\arctan(\tan y) = y,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$

- You should be able to use the triangle technique to convert trigonometric functions of inverse trigonometric functions into algebraic expressions.

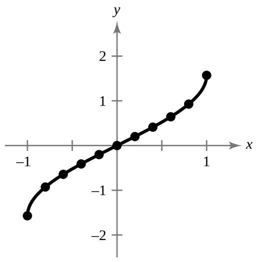
Solutions to Odd-Numbered Exercises

1. (a)

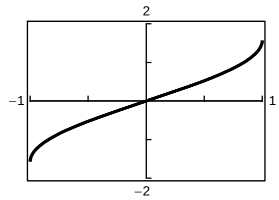
x	-1.0	-0.8	-0.6	-0.4	-0.2
y	-1.5708	-0.9273	-0.6435	-0.4115	-0.2014

x	0	0.2	0.4	0.6	0.8	1
y	0	0.2014	0.4115	0.6435	0.9273	1.5708

(b)



(c)



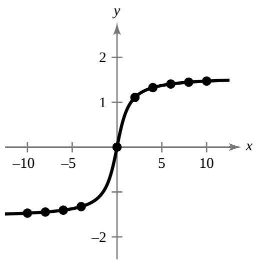
(d) (0, 0), Symmetric to the origin

3. (a)

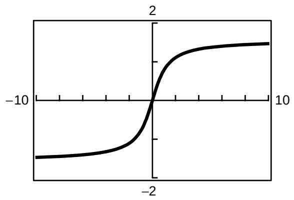
x	-10	-8	-6	-4	-2
y	-1.4711	-1.4464	-1.4056	-1.3258	-1.1071

x	0	2	4	6	8	10
y	0	1.1071	1.3258	1.4056	1.4464	1.4711

(b)



(c)



(d) Horizontal asymptotes: $y = \pm\frac{\pi}{2}$

5. $\tan\left(-\frac{\pi}{4}\right) = -1 \Longrightarrow \arctan(-1) = -\frac{\pi}{4}$

7. $\arcsin(-1) = -\frac{\pi}{2} \Longrightarrow \sin\left(-\frac{\pi}{2}\right) = -1$

9. (a) $y = \arccos\frac{1}{2} \Longrightarrow \cos y = \frac{1}{2}$ for $0 \le y \le \pi \Longrightarrow y = \frac{\pi}{3} \approx 1.047$

(b) $y = \arccos 0 \Longrightarrow \cos y = 0$ for $0 \le y \le \pi \Longrightarrow y = \frac{\pi}{2} \approx 1.571$

11. (a) $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4} \approx 2.356$

(b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4} \approx -0.785$

13. (a) $y = \arccos\left(-\frac{1}{2}\right) \Longrightarrow \cos y = -\frac{1}{2}$ for $0 \le y \le \pi \Longrightarrow y = \frac{2\pi}{3} \approx 2.094$

(b) $y = \arcsin\frac{\sqrt{2}}{2} \Longrightarrow \sin y = \frac{\sqrt{2}}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Longrightarrow y = \frac{\pi}{4} \approx 0.785$

15. (a) $\arcsin(-1) = -\dfrac{\pi}{2} \approx -1.571$

(b) $\arccos(1) = 0$

17. $y = \arccos x \quad (-1, \pi), \left(-\dfrac{1}{2}, \dfrac{2\pi}{3}\right), \left(\dfrac{\sqrt{3}}{2}, \dfrac{\pi}{6}\right)$

$x = \cos y$

19. (a) $\arcsin(-0.75) \approx -.85$

(b) $\arccos(-0.7) \approx 2.35$

21. (a) $\arcsin 0.41 \approx 0.42$

(b) $\arccos 0.36 \approx 1.20$

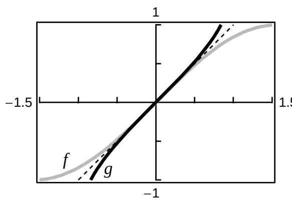
23. (a) $\arctan 0.98 \approx 0.78$

(b) $\arctan 4.7 \approx 1.36$

25. $f(x) = \sin x$

$g(x) = \arcsin x$

$y = x$



27. $\cos\theta = \dfrac{4}{x}$

$\theta = \arccos\dfrac{4}{x}$

29. $\tan\theta = \dfrac{x+1}{10}$

$\theta = \arctan\left(\dfrac{x+1}{10}\right)$

31. $\tan(\arctan 35) = 35$

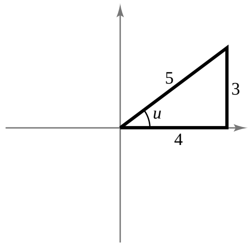
33. $\sin(\arcsin(-0.1)) = -0.1$

35. $\arccos\left(\cos\dfrac{7\pi}{2}\right) = \arccos(0) = \dfrac{\pi}{2}$

37. $\arcsin\left(\sin\dfrac{7\pi}{4}\right) = \arcsin\left(-\dfrac{\sqrt{2}}{2}\right) = -\dfrac{\pi}{4}$

39. Let $u = \arcsin\dfrac{3}{5}$,

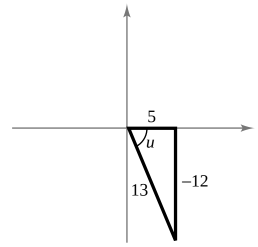
$\sin u = \dfrac{3}{5}, 0 < u < \dfrac{\pi}{2}.$



$\sec\left(\arcsin\dfrac{3}{5}\right) = \sec u = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{5}{4}$

41. Let $u = \arctan\left(-\dfrac{12}{5}\right)$

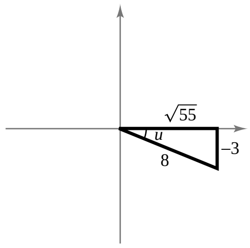
$\tan u = -\dfrac{12}{5}, -\dfrac{\pi}{2} < u < 0.$



$\csc\left[\arctan\left(-\dfrac{12}{5}\right)\right] = \csc u = \dfrac{\text{hyp}}{\text{opp}} = -\dfrac{13}{12}$

43. Let $u = \arcsin\left(-\frac{3}{8}\right)$,

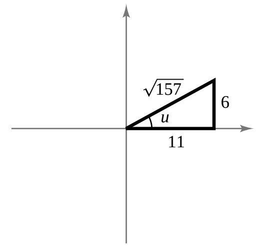
$\sin u = -\frac{3}{8}, -\frac{\pi}{2} < u < 0.$



$$\tan\left[\arcsin\left(-\frac{3}{8}\right)\right] = \tan u = -\frac{3}{\sqrt{55}} = -\frac{3\sqrt{55}}{55}$$

45. Let $u = \arctan \frac{6}{11}$,

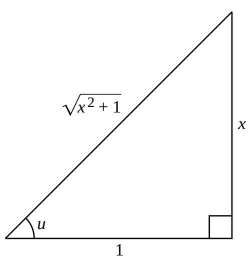
$\tan u = \frac{6}{11}, 0 < u < \frac{\pi}{2}.$



$$\cot\left(\arctan \frac{6}{11}\right) = \cot u = \frac{\text{adj}}{\text{opp}} = \frac{11}{6}$$

47. Let $u = \arctan x$,

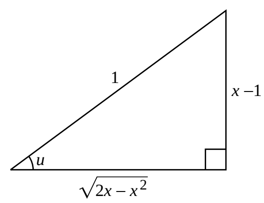
$\tan u = x = \frac{x}{1}.$



$$\sin(\arctan x) = \sin u = \frac{\text{opp}}{\text{hyp}} = \frac{x}{\sqrt{x^2 + 1}}$$

49. Let $u = \arcsin(x - 1)$,

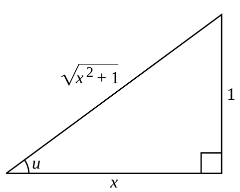
$\sin u = x - 1 = \frac{x - 1}{1}.$



$$\sec[\arcsin (x - 1)] = \sec u = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\sqrt{2x - x^2}}$$

51. Let $u = \arctan \frac{1}{x}$,

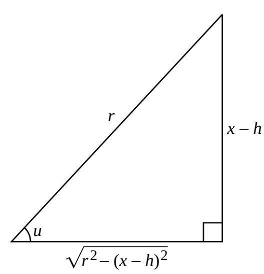
$\tan u = \frac{1}{x}.$



$$\cot\left(\arctan \frac{1}{x}\right) = \cot u = \frac{\text{adj}}{\text{opp}} = x$$

53. Let $u = \arcsin \frac{x - h}{r}$,

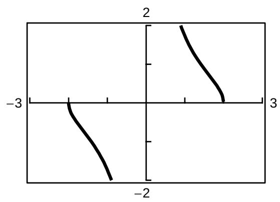
$\sin u = \frac{x - h}{r}.$



$$\cos\left(\arcsin \frac{x - h}{r}\right) = \cos u = \frac{\sqrt{r^2 - (x - h)^2}}{r}$$

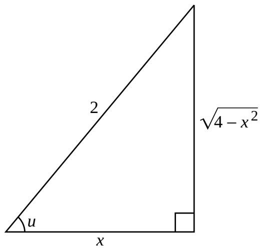
55. $f(x) = \tan\left(\arccos \dfrac{x}{2}\right)$

$g(x) = \dfrac{\sqrt{4 - x^2}}{x}$



Asymptote: $x = 0$

These are equal because:

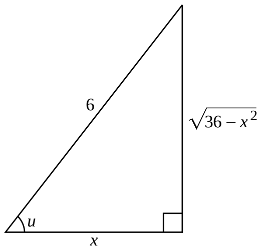


Let $u = \arccos \dfrac{x}{2}$.

$\tan\left(\arccos \dfrac{x}{2}\right) = \tan u = \dfrac{\sqrt{4 - x^2}}{x}$

57. If $\arcsin \dfrac{\sqrt{36 - x^2}}{6} = u$,

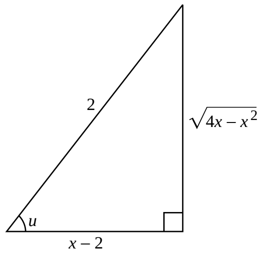
then $\sin u = \dfrac{\sqrt{36 - x^2}}{6}$.



$\arcsin \dfrac{\sqrt{36 - x^2}}{6} = \arccos \dfrac{x}{6}$

59. If $\arccos \dfrac{x - 2}{2} = u$,

then $\cos u = \dfrac{x - 2}{2}$.

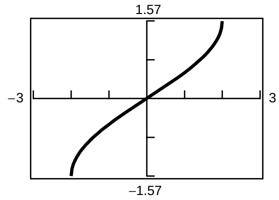


$\arccos \dfrac{x - 2}{2} = \arctan \dfrac{\sqrt{4x - x^2}}{x - 2}$

61. $y = \arcsin \dfrac{x}{2}$

Domain: $-2 \le x \le 2$

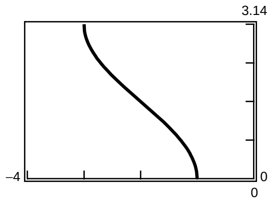
Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$



63. $g(t) = \arccos(t + 2)$

Domain: $-3 \le t \le -1$

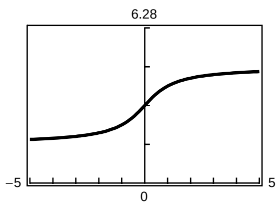
This is the graph of $y = \arccos t$ shifted two units to the left.



65. $f(x) = \pi + \arctan x$

Domain: $(-\infty, \infty)$

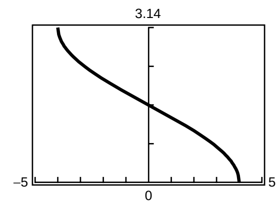
Range: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



67. $f(x) = \arccos \frac{x}{4}$

Domain: $[-4, 4]$

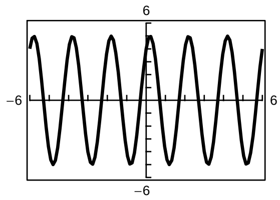
Range: $[0, \pi]$



69. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

$$= \sqrt{4^2 + 3^2} \sin\left(\pi t + \arctan \frac{4}{3}\right)$$

$$= 5 \sin\left(\pi t + \arctan \frac{4}{3}\right)$$



The graph suggests that $A \cos \omega t + B \sin \omega t = \sqrt{A^2 + b^2} \sin\left(\omega t + \arctan \frac{A}{B}\right)$ is true.

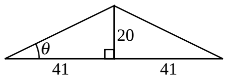
71. (a) $\tan \theta = \frac{s}{750}$

$$\theta = \arctan\left(\frac{s}{750}\right)$$

(b) When $s = 400$, $\theta = \arctan\left(\frac{400}{750}\right) \approx 0.4900\ (\approx 28.07°)$

When $s = 1600$, $\theta = \arctan\left(\frac{1600}{750}\right) \approx 1.1325\ (\approx 64.89°)$

73. (a) $\theta = \arctan\left(\frac{20}{41}\right) \approx 26.0°$ (.45 rad)



(b) $\tan(26.0°) = \frac{h}{50}$

$$h = 50 \tan(26.0°) \approx 24.4 \text{ feet}$$

75. Area $=$ arctan b $-$ arctan a

(a) $a = 0, b = 1$

Area $= \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

(b) $a = -1, b = 1$

Area $= \arctan 1 - \arctan(-1)$

$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

(c) $a = 0, b = 3$

Area $= \arctan 3 - \arctan 0$

$\approx 1.25 - 0 = 1.25$

$= 1.25$

(d) $a = -1, b = 3$

Area $= \arctan 3 - \arctan(-1)$

$\approx 1.25 - \left(-\frac{\pi}{4}\right) \approx 2.03$

77. (a) $\tan\theta = \frac{x}{20}$

$\theta = \arctan\frac{x}{20}$

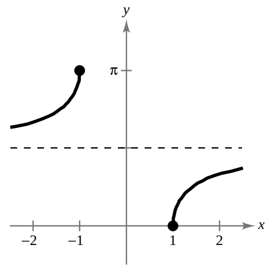
(b) When $x = 5$,

$\theta = \arctan\frac{5}{20} \approx 14.0°$, (0.24 rad).

When $x = 12$, $\theta = \arctan\frac{12}{20} \approx 31.0°. = (0.54$ rad.).

79. False. $\tan x = \frac{\sin x}{\cos x}$.

81. $y = \operatorname{arcsec} x$ if and only if $\sec y = x$ where $x \le -1 \cup x \ge 1$ and $0 \le y < \pi/2$ and $\pi/2 < y \le \pi$. The domain of y, arcsec x is $(-\infty, -1] \cup [1, \infty)$ and the range is $[0, \pi/2) \cup (\pi/2, \pi]$.



83. (a) $y = \operatorname{arcsec}\sqrt{2} \Longrightarrow \sec y = \sqrt{2}$ and $0 \le y < \frac{\pi}{2} \cup \frac{\pi}{2} < y \le \pi \Longrightarrow y = \frac{\pi}{4}$

(b) $y = \operatorname{arcsec} 1 \Longrightarrow \sec y = 1$ and $0 \le y < \frac{\pi}{2} \cup \frac{\pi}{2} < y \le \pi \Longrightarrow y = 0$

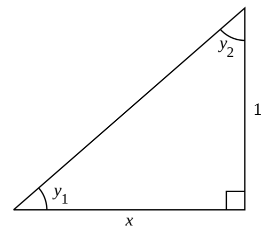
(c) $y = \operatorname{arccot}\left(-\sqrt{3}\right) \Longrightarrow \cot y = -\sqrt{3}$ and $0 < y < \pi \Longrightarrow y = \frac{5\pi}{6}$

(d) $y = \operatorname{arccsc} 2 \Longrightarrow \csc y = 2$ and $-\frac{\pi}{2} \le y < 0 \cup 0 < y \le \frac{\pi}{2} \Longrightarrow y = \frac{\pi}{6}$

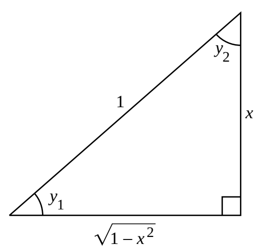
85.
$$y = \arctan(-x)$$
$$\tan y = -x,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$$
$$-\tan y = x$$
$$\tan(-y) = x,\ -\frac{\pi}{2} < -y < \frac{\pi}{2}$$
$$\arctan(\tan(-y)) = \arctan x$$
$$-y = \arctan x$$
$$y = -\arctan x$$

87. $y_2 = \dfrac{\pi}{2} - y_1$

$$\arctan x + \arctan\frac{1}{x} = y_1 + y_2$$
$$= y_1 + \left(\frac{\pi}{2} - y_1\right) = \frac{\pi}{2}$$



89. $\arcsin x = \arcsin\dfrac{x}{1} = \arctan\dfrac{x}{\sqrt{1 - x^2}}$



91. $-585°$ coterminal with $720° - 585° = 135°$.
Quadrant II

Reference angle $\theta' = 45°$

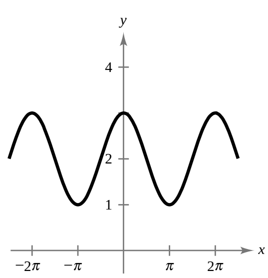
$$\sin(-585°) = \frac{\sqrt{2}}{2}$$
$$\cos(-585°) = -\frac{\sqrt{2}}{2}$$
$$\tan(-585°) = -1$$

93. $-\dfrac{19\pi}{4}$ coterminal with $6\pi - \dfrac{19\pi}{4} = \dfrac{5\pi}{4}$.
Quadrant III

Reference angle $\theta' = \dfrac{\pi}{4}$

$$\sin\left(-\frac{19\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$
$$\cos\left(-\frac{19\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$
$$\tan\left(-\frac{19\pi}{4}\right) = 1$$

95. $y = 2 - \cos(x + \pi)$



97. $y = 4\cot\left(\dfrac{1}{2}x\right) = \dfrac{4}{\tan\left(\dfrac{1}{2}x\right)}$.

Asymptotes: $x = 0, x = 2\pi, x = -2\pi, \ldots$