Section 4.8 Applications and Models

- You should be able to solve right triangles.
- You should be able to solve right triangle applications.
- You should be able to solve applications of simple harmonic motion: $d = a \sin wt$ or $d = a \cos wt$

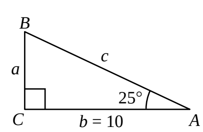
Solutions to Odd-Numbered Exercises

1. Given: $A = 25°$, $b = 10$

$$\tan A = \frac{a}{b} \Longrightarrow a = b \tan A = 10 \tan 25° \approx 4.66$$

$$\cos A = \frac{b}{c} \Longrightarrow c = \frac{b}{\cos A} = \frac{10}{\cos(25°)} \approx 11.03$$

$$B = 90° - 25° = 65°$$

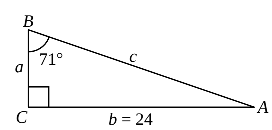


3. Given: $B = 71°$, $b = 24$

$$\tan B = \frac{b}{a} \Longrightarrow a = \frac{b}{\tan B} = \frac{24}{\tan 71°} \approx 8.26$$

$$\sin B = \frac{b}{c} \Longrightarrow c = \frac{b}{\sin B} = \frac{24}{\sin 71°} \approx 25.38$$

$$A = 90° - 71° = 19°$$

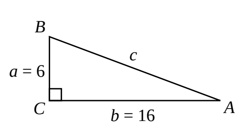


5. Given: $a = 6$, $b = 16$

$$c^2 = a^2 + b^2 \Longrightarrow c = \sqrt{292} \approx 17.09$$

$$\tan A = \frac{a}{b} = \frac{6}{16} \Longrightarrow A = \arctan\left(\frac{3}{8}\right) \approx 20.56°$$

$$B = 90° - 20.56° = 69.44°$$

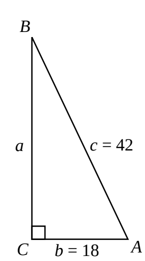


7. Given: $b = 18$, $c = 42$

$$a = \sqrt{c^2 - b^2} = \sqrt{1440} \approx 37.95$$

$$\cos A = \frac{18}{42} \Longrightarrow A = \arccos\left(\frac{3}{7}\right) \approx 64.62°$$

$$B = 90° - 64.62° = 25.38°$$



9. $A = 12° \, 15', \; c = 430.5$

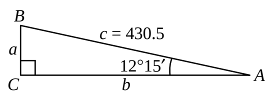
$B = 90° - 12° \, 15' = 77° \, 45'$

$\sin 12° \, 15' = \dfrac{a}{430.5}$

$a = 430.5 \sin 12° \, 15' \approx 91.34$

$\cos 12° \, 15' = \dfrac{b}{430.5}$

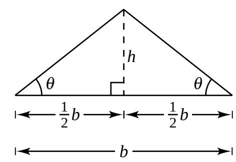
$b = 430.5 \cos 12° \, 15' \approx 420.70$



11. $\tan \theta = \dfrac{h}{\frac{1}{2}b}$

$h = \dfrac{1}{2} b \tan \theta$

$h = \dfrac{1}{2}(4) \tan 52° \approx 2.56$ in.

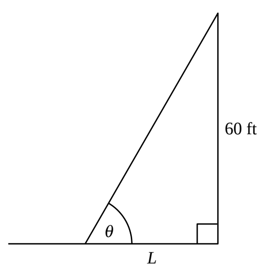


13. $\tan \theta = \dfrac{h}{\frac{1}{2}b} \Longrightarrow h = \dfrac{1}{2} b \tan \theta = \dfrac{1}{2}(14.2) \tan (41.6°) \approx 6.30$ feet

15. (a) $\tan \theta = \dfrac{60}{L}$

$L = \dfrac{60}{\tan \theta}$

$= 60 \cot \theta$



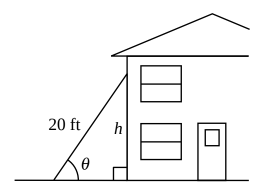
(b)

θ	10°	20°	30°	40°	50°
L	340	165	104	72	50

(c) No, the shadow lengths do not increase in equal increments. The cotangent function is not linear.

17. (a) $\sin \theta = \dfrac{h}{20}$

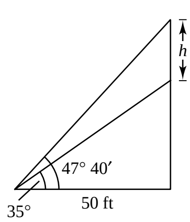
$h = 20 \sin \theta$



(b)

θ	60°	65°	70°	75°	80°
h	17.3	18.1	18.8	19.3	19.7

19. (a)



(b) Let the height of the church $= x$ and the height of the church and steeple $= y$. Then:

$\tan 35° = \dfrac{x}{50}$ and $\tan 47° \, 40' = \dfrac{y}{50}$

$x = 50 \tan 35°$ and $y = 50 \tan 47° \, 40'$

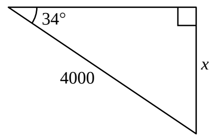
$h = y - x = 50(\tan 47° \, 40' - \tan 35°)$

(c) $h \approx 19.9$ feet

21. $\sin 31.5^\circ = \dfrac{x}{4000}$

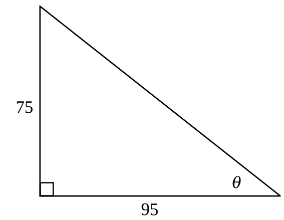
$x = 4000 \sin 31.5^\circ$

≈ 2089.99 feet



23. $\tan\theta = \dfrac{75}{95}$

$\theta = \arctan\left(\dfrac{15}{19}\right) \approx 38.29^\circ$

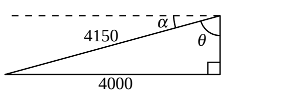


25. $\sin\theta = \dfrac{4000}{4150}$

$\theta = \arcsin\left(\dfrac{4000}{4150}\right)$

$\theta \approx 74.5^\circ$

$\alpha = 90^\circ - 74.5^\circ = 15.5^\circ$



27. Since the airplane speed is

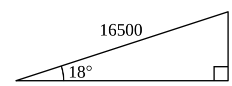
$\left(275 \dfrac{\text{ft}}{\text{sec}}\right)\left(60 \dfrac{\text{sec}}{\text{min}}\right) = 16{,}500 \dfrac{\text{ft}}{\text{min}},$

after one minute its distance travelled is 16,500 feet.

$\sin 18^\circ = \dfrac{a}{16{,}500}$

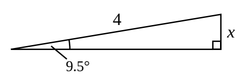
$a = 16{,}500 \sin 18^\circ$

≈ 5099 ft



29. $\sin 9.5^\circ = \dfrac{x}{4}$

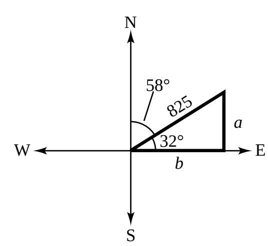
$x = 4 \sin 9.5^\circ \approx 0.66$ mile



31. The plane has traveled $550(1.5) = 825$ miles

$\sin 32^\circ = \dfrac{a}{825} \Longrightarrow a \approx 437.2$ miles north

$\cos 32^\circ = \dfrac{b}{825} \Longrightarrow b \approx 699.6$ miles east



33. $\theta = 32^\circ,\ \phi = 68^\circ$. Note: *ABC* form a right triangle.

(a) $\alpha = 90^\circ - 32^\circ = 58^\circ$

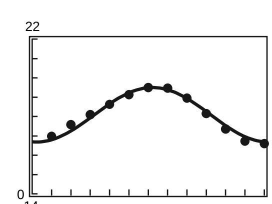
Bearing from *A* to *C*: N 58° E

(b) $\beta = \theta = 32^\circ$

$\gamma = 90^\circ - \phi = 22^\circ$

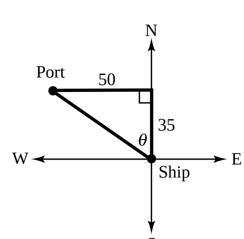
$C = \beta + \gamma = 54^\circ$

$\tan C = \dfrac{d}{50} \implies \tan 54^\circ = \dfrac{d}{50} \implies d \approx 68.82$ yd

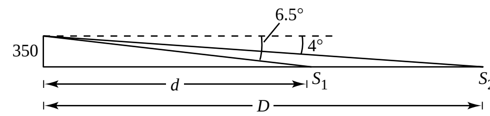


35. $\tan\theta = \dfrac{50}{35} \Longrightarrow \theta \approx 55.0^\circ$

Bearing: N 55.0° W



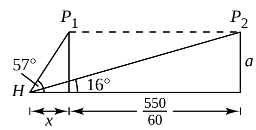
37. $\tan 6.5° = \dfrac{350}{d} \implies d \approx 3071.91$ ft



$\tan 4° = \dfrac{350}{D} \implies D \approx 5005.23$ ft

Distance between ships: $D - d \approx 1933.3$ ft

39. $\tan 57° = \dfrac{a}{x} \implies x = a \cot 57°$



$$\tan 16° = \frac{a}{x + (55/6)}$$

$$\tan 16° = \frac{a}{a \cot 57° + (55/6)}$$

$$\cot 16° = \frac{a \cot 57° + (55/6)}{a}$$

$$a \cot 16° - a \cot 57° = \frac{55}{6} \implies a \approx 3.23 \text{ miles}$$

$$\approx 17{,}054 \text{ ft}$$

41. L_1: $3x - 2y = 5 \implies y = \dfrac{3}{2}x - \dfrac{5}{2} \implies m_1 = \dfrac{3}{2}$

L_2: $x + y = 1 \implies y = -x + 1 \implies m_2 = -1$

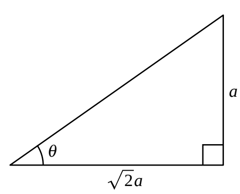
$$\tan \alpha = \left|\frac{-1 - (3/2)}{1 + (-1)(3/2)}\right| = \left|\frac{-5/2}{-1/2}\right| = 5$$

$$\alpha = \arctan 5 \approx 78.7°$$

43. The diagonal of the base has a length of $\sqrt{a^2 + a^2} = \sqrt{2}a$.

Now, we have:

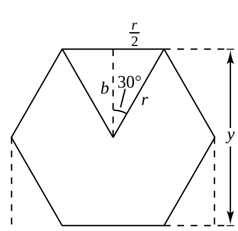
$$\tan \theta = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$



$$\theta = \arctan \frac{1}{\sqrt{2}}$$

$$\theta \approx 35.3°$$

45. $\cos 30° = \dfrac{b}{r}$



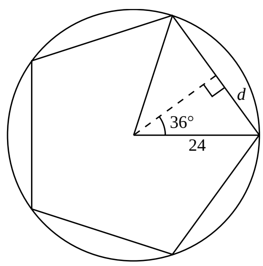
$$b = \cos 30° r$$

$$b = \frac{\sqrt{3}r}{2}$$

$$y = 2b = 2\left(\frac{\sqrt{3}r}{2}\right) = \sqrt{3}r$$

47. $\sin 36^\circ = \dfrac{d}{24} \implies d \approx 14.1068$

Length of side: $2d \approx 28.2$ inches

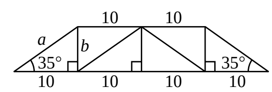


49. $\tan 35^\circ = \dfrac{b}{10}$

$b = 10 \tan 35^\circ \approx 7$

$\cos 35 = \dfrac{10}{a}$

$a = \dfrac{10}{\cos 35^\circ} \approx 12.2$



51. $d = 4 \cos 8\pi t$

(a) Maximum displacement = amplitude = 4

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{8\pi}{2\pi}$

$= 4$ cycles per unit of time

(c) $8\pi t = \dfrac{\pi}{2} \implies t = \dfrac{1}{16}$

53. $d = \dfrac{1}{16} \sin 140\pi t$

(a) Maximum displacement = amplitude $= \dfrac{1}{16}$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{140\pi}{2\pi}$

$= 70$ cycles per unit of time

(c) $140\pi t = \pi \implies t = \dfrac{1}{140}$

55. $d = 0$ when $t = 0$, $a = 8$, period $= 2$

Use $d = a \sin wt$ since $d = 0$ when $t = 0$

$\dfrac{2\pi}{w} = 2 \implies w = \pi$

Thus, $d = 8 \sin \pi t$

57. $d = 3$ when $t = 0$, $a = 3$, Period $= 1.8$

Use $d = a \cos wt$ since $d = 3$ when $t = 0$

$\dfrac{2\pi}{w} = 1.8 \implies w = \dfrac{10}{9}\pi$

Thus, $d = 3 \cos\left(\dfrac{10}{9}\pi t\right)$

59. $d = a \sin \omega t$

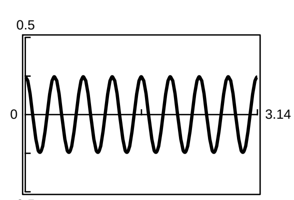
Period $= \dfrac{2\pi}{\omega} = \dfrac{1}{\text{frequency}}$

$\dfrac{2\pi}{\omega} = \dfrac{1}{264}$

$\omega = 2\pi(264) = 528\pi$

61. $y = \dfrac{1}{4} \cos 16t, \ t > 0$

(a)

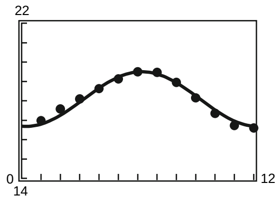


(b) Period: $\dfrac{2\pi}{16} = \dfrac{\pi}{8}$ seconds

(c) $\dfrac{1}{4} \cos 16t = 0$ when $16t = \dfrac{\pi}{2} \implies t = \dfrac{\pi}{32}$ seconds.

63. $S(t) = 18.09 + 1.41 \sin\left(\frac{\pi t}{6} + 4.60\right)$

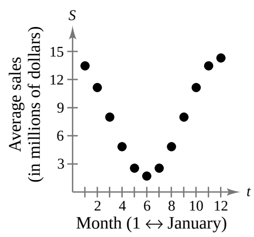
(a)



(b) The period is 12 months, which corresponds to 1 year.

(c) The amplitude is 1.41. This gives the maximum change in time from the average time (18.09) of sunset.

65. (a)



(b) $a = \frac{1}{2}(14.30 - 1.70) = 6.3$

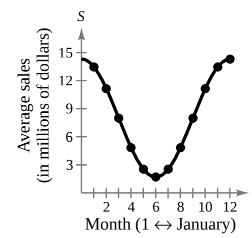
$\frac{2\pi}{b} = 12 \implies b = \frac{\pi}{6}$

Shift: $d = 14.3 - 6.3 = 8$

$S = d + a \cos bt$

$S = 8 + 6.3 \cos\left(\frac{\pi t}{6}\right)$

The model is a good fit.



(c) Period: $\frac{2\pi}{\pi/6} = 12$

This corresponds to the 12 months in a year. Since the sales of outerwear is seasonal, this is reasonable.

(d) The amplitude represents the maximum displacement from the average sale of 8 million dollars. Sales are greatest in December (cold weather + holidays) and least in June.

67. False. The other acute angle is $90° - 48.1° = 41.9°$. Then $\tan(41.9°) = \frac{\text{opp}}{\text{adj}} = \frac{a}{22.56} \implies a = 22.56 \cdot \tan(41.9°)$

69. $\frac{x^2}{x + 6} - \frac{1}{2x + 1} = \frac{2x^3 + x^2 - x - 6}{(x + 6)(2x + 1)}$

71. $\frac{3x^2 - 13x + 4}{x^2 + 8x + 12} \cdot \frac{x^3 + 3x^2 - 18x}{12x^2 - 4x} = \frac{(x - 4)(3x - 1)}{(x + 2)(x + 6)} \cdot \frac{x(x - 3)(x + 6)}{4x(3x - 1)}$

$= \frac{(x - 4)(x - 3)}{4(x + 2)}, x \neq 0, -6, \frac{1}{3}$

73. $e^{2x} = 54$

$2x = \ln 54$

$x = \frac{1}{2}\ln 54 \approx 1.994$

75. $\ln(x^2 + 1) = 3.2$

$x^2 + 1 = e^{3.2}$

$x = \pm\sqrt{e^{3.2} - 1} \approx \pm 4.851$

77. arc cos $0.13 \approx 1.44$ or $82.53°$

79. $\arcsin(-0.11) \approx -0.11$ or $-6.32°$