CHAPTER 6
Additional Topics in Trigonometry

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Section 6.1 Law of Sines

- If ABC is any oblique triangle with sides a, b, and c, then the Law of Sines says

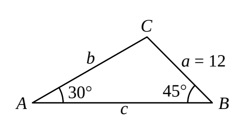
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

- You should be able to use the Law of Sines to solve an oblique triangle for the remaining three parts, given:
  (a) Two angles and any side (AAS or ASA)
  (b) Two sides and an angle opposite one of them (SSA)
    1. If A is acute and $h = b \sin A$:
       (a) $a < h$, no triangle is possible.
       (b) $a = h$ or $a > b$, one triangle is possible.
       (c) $h < a < b$, two triangles are possible.
    2. If A is obtuse and $h = b \sin A$:
       (a) $a \le b$, no triangle is possible.
       (b) $a > b$, one triangle is possible.
- The area of any triangle equals one-half the product of the lengths of two sides times the sine of their included angle.

$$A = \tfrac{1}{2}ab \sin C = \tfrac{1}{2}ac \sin B = \tfrac{1}{2}bc \sin A$$

Solutions to Odd-Numbered Exercises

1. Given: $A = 30°, B = 45°, a = 12$

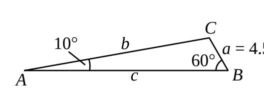


$C = 180° - 30° - 45° = 105°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{12}{\sin 30°}(\sin 45°) = 12\sqrt{2} \approx 16.97$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{12}{\sin 30°}(\sin 105°) \approx 23.18$$

3. Given: $A = 10°, B = 60°, a = 4.5$



$C = 180° - 10° - 60° = 110°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{4.5}{\sin 10°}(\sin 60°) \approx 22.44$$

$$c = \frac{a}{\sin A}(\sin \text{C}) = \frac{4.5}{\sin 10°}(\sin 110°) \approx 24.35$$

5. Given: $A = 36°, a = 10, b = 4$

$$\sin B = \frac{b \sin A}{a} = \frac{4 \sin 36°}{10} \approx 0.2351 \implies B \approx 13.60°$$

$$C = 180° - A - B = 180° - 36° - 13.60° = 130.40°$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{10}{\sin 36°}(\sin 130.40°) \approx 12.96$$

7. Given: $A = 150°,\ C = 20°,\ a = 200$

$$B = 180° - A - C = 180° - 150° - 20° = 10°$$

$$b = \frac{a}{\sin A}(\sin B) = \frac{200}{\sin 150°}(\sin 10°) \approx 69.46$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{200}{\sin 150°}(\sin 20°) \approx 136.81$$

9. Given: $A = 83° 20',\ C = 54.6°,\ c = 18.1$

$$B = 180° - A - C = 180° - 83° 20' - 54° 36' = 42° 4'$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{18.1}{\sin 54.6°}(\sin 83° 20') \approx 22.05$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{18.1}{\sin 54.6°}(\sin 42° 4') \approx 14.88$$

11. Given: $B = 15° 30',\ a = 4.5,\ b = 6.8$

$$\sin A = \frac{a \sin B}{b} = \frac{4.5 \sin 15° 30'}{6.8} \approx 0.17685 \implies A \approx 10° 11'$$

$$C = 180° - A - B \approx 180° - 10° 11' - 15.5° = 154° 19'$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{6.8}{\sin 15° 30'}(\sin 154° 19') \approx 11.03$$

13. Given: $A = 110° 15',\ a = 48,\ b = 16$

$$\sin B = \frac{b \sin A}{a} = \frac{16 \sin 110° 15'}{48} \approx 0.31273 \implies B \approx 18° 13'$$

$$C = 180° - A - B \approx 180° - 110° 15' - 18° 13' = 51° 32'$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{48}{\sin 110° 15'}(\sin 51° 32') \approx 40.05$$

15. Given: $a = 4.5,\ b = 12.8,\ A = 58°$

$$h = 12.8 \sin 58° \approx 10.86$$

Since $a < h$, no triangle is formed.

17. Given: $A = 58°$, $a = 11.4$, $b = 12.8$

$$\sin B = \frac{b \sin A}{a} = \frac{12.8 \sin 58°}{11.4} \approx 0.9522 \implies B \approx 72.21° \text{ or } 107.79°$$

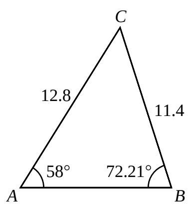
Case 1

$B \approx 72.21°$

$C = 180° - 58° - 72.21° = 49.79°$

$c = \dfrac{a}{\sin A}(\sin C) = \dfrac{11.4}{\sin 58°}(\sin 49.79°)$

≈ 10.27



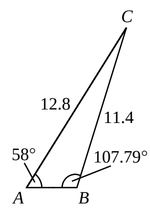
Case 2

$B \approx 107.79$

$C = 180° - 58° - 107.79° = 14.21°$

$c = \dfrac{a}{\sin A}(\sin C) = \dfrac{11.4}{\sin 58°}(\sin 14.21°)$

≈ 3.30



19. Given: A $= 110°$, $a = 125$, $b = 100$

$$\sin B = \frac{b \sin A}{a} = \frac{100 \sin 110°}{125} \approx 0.75175 \implies B \approx 48.74°$$

$C = 180° - A - B = 21.26°$

$$c = \frac{a}{\sin A}(\sin C) = \frac{125 \sin 21.26°}{\sin 110°} \approx 48.23$$

21. Given: $A = 36°$, $a = 5$

(a) One solution if $b \leq 5$ or $b = \dfrac{5}{\sin 36°}$.

(b) Two solutions if $5 < b < \dfrac{5}{\sin 36°}$.

(c) No solution if $b > \dfrac{5}{\sin 36°}$.

23. Area $= \frac{1}{2}ab \sin C$

$= \frac{1}{2}(6)(10) \sin(110°)$

≈ 28.2 square units

25. Area $= \frac{1}{2}bc \sin A$

$= \frac{1}{2}(67)(85) \sin(38° 45')$

≈ 1782.3 square units

27. Area $= \frac{1}{2}ac \sin B$

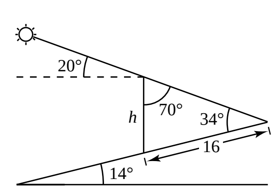
$= \frac{1}{2}(92)(30) \sin 130°$

≈ 1057.1 square units

29. Angle $\measuredangle CAB = 70°$

Angle $B = 20° + 14° = 34°$

(a)

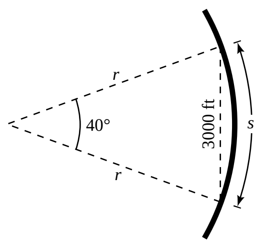


(b) $\dfrac{16}{\sin 70°} = \dfrac{h}{\sin 34°}$ (c) $h = \dfrac{16 \sin 34°}{\sin 70°} \approx 9.52$ meters

31. $\sin A = \dfrac{a \sin B}{b} = \dfrac{500 \sin 46^\circ}{840} \approx 0.4282 \implies A \approx 25^\circ$

The bearing from C to A is S 65° W.

33. (a)



(b) $r = \dfrac{3000 \sin[1/2(180^\circ - 40^\circ)]}{\sin 40^\circ} \approx 4385.71$ feet

(c) $s \approx 40^\circ\left(\dfrac{\pi}{180^\circ}\right)4385.71 \approx 3061.80$ feet

35. $A = 65^\circ - 28^\circ = 37^\circ$

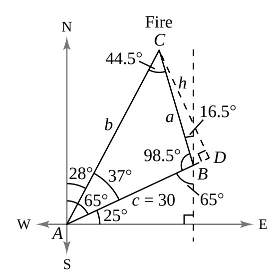
$c = 30$

$B = 180^\circ - 16.5^\circ - 65^\circ = 98.5^\circ$

$C = 180^\circ - 37^\circ - 98.5^\circ = 44.5^\circ$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{30}{\sin 44.5^\circ}(\sin 37^\circ) \approx 25.8$ km to B

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{30}{\sin 44.5^\circ}(\sin 98.5^\circ) \approx 42.3$ km to A



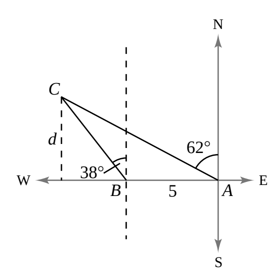
37. $A = 90^\circ - 62^\circ = 28^\circ$,

$B = 90^\circ + 38^\circ = 128^\circ,\ c = 5$

$C = 180^\circ - 128^\circ - 28^\circ = 24^\circ$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{5}{\sin 24^\circ}(\sin 28^\circ) \approx 5.77$

$d = a \sin(90^\circ - 38^\circ) \approx 5.77 \sin 52^\circ \approx 4.55$ miles



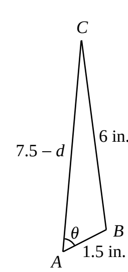
39. (a) $\dfrac{6}{\sin \theta} = \dfrac{1.5}{\sin C}, \sin \theta \neq 0$

$\sin C = \dfrac{1.5 \sin \theta}{6} \implies C = \arcsin \dfrac{1.5 \sin \theta}{6}$

$B = 180^\circ - \theta - \arcsin \dfrac{1.5 \sin \theta}{6}$

$\dfrac{7.5 - d}{\sin B} = \dfrac{6}{\sin \theta}$

$d = 7.5 - \dfrac{6 \sin\left(180^\circ - \theta - \arcsin \dfrac{1.5 \sin \theta}{6}\right)}{\sin \theta}$



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39. —CONTINUED—

For $\theta = 0°, C = 0°, B = 180° \implies 7.5 - d = 1.5 + 6 \implies d = 0.$

θ	0°	45°	90°	135°	180°
d	0	0.5338	1.6905	2.6552	3

For $\theta = 180°, C = 0°, B = 0° \implies 7.5 - d = 6 - 1.5 \implies d = 3.$

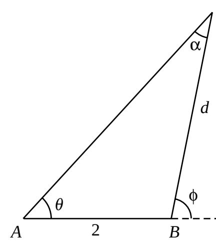
(b) $\theta = 5°$

$$d = 7.5 - \frac{6 \sin\left(180° - 5° - \arcsin \dfrac{1.5 \sin 5°}{6}\right)}{\sin 5°} \approx 0.0071 \text{ inch}$$

41. $\alpha = 180 - (\phi + 180 - \theta) = \theta - \phi$

$$\frac{d}{\sin \phi} = \frac{2}{\sin \alpha}$$

$$d = \frac{2 \sin \phi}{\sin(\phi - \theta)}$$



43. False. If the 3 angles are known, the triangle cannot be solved.

45. False. See page 428.

47. $\tan \theta = \dfrac{\sin \theta}{\cos \theta} = -\dfrac{12}{5}$ $\qquad \sec \theta = \dfrac{13}{5}$

$\cot \theta = -\dfrac{5}{12}$ $\qquad \csc \theta = -\dfrac{13}{12}$

49. $\tan \theta = -\dfrac{1}{11};\ \cos \theta = -\dfrac{11}{\sqrt{122}} = -\dfrac{11\sqrt{122}}{122}$

$\sin \theta = \left(-\dfrac{1}{11}\right)\left(-\dfrac{11}{\sqrt{122}}\right) = \dfrac{1}{\sqrt{122}} = \dfrac{\sqrt{122}}{122};$

$\csc \theta = \sqrt{122}$

51. $\sec^2 x(\csc^2 x - 1) = \sec^2 x(\cot^2 x)$

$$= \frac{1}{\sin^2 x} = \csc^2 x$$

53. $6 \sin 8\theta \cos 3\theta = 6\left(\tfrac{1}{2}\right)[\sin(8\theta + 3\theta) + \sin(8\theta - 3\theta)]$

$$= 3[\sin 11\theta + \sin 5\theta]$$

55. $3 \cos \dfrac{\pi}{6} \sin \dfrac{5\pi}{3} = 3\left(\dfrac{1}{2}\right)\left[\sin\left(\dfrac{\pi}{6} + \dfrac{5\pi}{3}\right) - \sin\left(\dfrac{\pi}{6} - \dfrac{5\pi}{3}\right)\right]$

$$= \frac{3}{2}\left[\sin\left(\frac{11\pi}{6}\right) - \sin\left(-\frac{3\pi}{2}\right)\right]$$

$$= \frac{3}{2}\left[-\frac{1}{2} - 1\right] = -\frac{9}{4}$$