

Section 6.3 Vectors in the Plane

- A vector \mathbf{v} is the collection of all directed line segments that are equivalent to a given directed line segment \overrightarrow{PQ} .
- You should be able to *geometrically* perform the operations of vector addition and scalar multiplication.
- The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$
- The magnitude of $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$.
- You should be able to perform the operations of scalar multiplication and vector addition in component form.
- You should know the following properties of vector addition and scalar multiplication.
 - (a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
 - (b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
 - (c) $\mathbf{u} + \mathbf{0} = \mathbf{u}$
 - (d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
 - (e) $c(d\mathbf{u}) = (cd)\mathbf{u}$
 - (f) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
 - (g) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
 - (h) $1(\mathbf{u}) = \mathbf{u}, 0\mathbf{u} = \mathbf{0}$
 - (i) $\|c\mathbf{v}\| = |c| \|\mathbf{v}\|$
- A unit vector in the direction of \mathbf{v} is given by $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$.
- The standard unit vectors are $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$. $\mathbf{v} = \langle v_1, v_2 \rangle$ can be written as $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$.
- A vector \mathbf{v} with magnitude $\|\mathbf{v}\|$ and direction θ can be written as $\mathbf{v} = a\mathbf{i} + b\mathbf{j} = \|\mathbf{v}\|(\cos \theta)\mathbf{i} + \|\mathbf{v}\|(\sin \theta)\mathbf{j}$ where $\tan \theta = b/a$.

Solutions to Odd-Numbered Exercises

1. $\mathbf{v} = \langle 6 - 2, 5 - 4 \rangle = \langle 4, 1 \rangle = \mathbf{v}$

3. Initial point: (0, 0)

Terminal point: (4, 3)

$$\mathbf{v} = \langle 4 - 0, 3 - 0 \rangle = \langle 4, 3 \rangle$$

$$\|\mathbf{v}\| = \sqrt{4^2 + 3^2} = 5$$

7. Initial point: (3, -2)

Terminal point: (3, 3)

$$\mathbf{v} = \langle 3 - 3, 3 - (-2) \rangle = \langle 0, 5 \rangle$$

$$\|\mathbf{v}\| = 5$$

5. Initial point: (2, 2)

Terminal point: (-1, 4)

$$\mathbf{v} = \langle -1 - 2, 4 - 2 \rangle = \langle -3, 2 \rangle$$

$$\|\mathbf{v}\| = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$$

9. Initial point: $\left(\frac{5}{2}, 1\right)$

Terminal point: $\left(-2, -\frac{3}{2}\right)$

$$\mathbf{v} = \left\langle -2 - \frac{5}{2}, -\frac{3}{2} - 1 \right\rangle = \left\langle -\frac{9}{2}, -\frac{5}{2} \right\rangle$$

$$\|\mathbf{v}\| = \sqrt{\left(-\frac{9}{2}\right)^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{\frac{81 + 25}{4}} = \frac{1}{2}\sqrt{106}$$

11. Initial point: $(-3, -5)$

Terminal point: $(5, 1)$

$$\mathbf{v} = \langle 5 - (-3), 1 - (-5) \rangle = \langle 8, 6 \rangle$$

$$\|\mathbf{v}\| = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

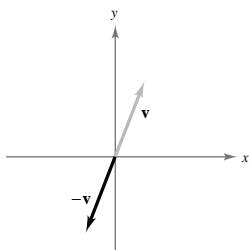
13. Initial point: $(-4.2, 5)$

Terminal point: $(3.7, -12.9)$

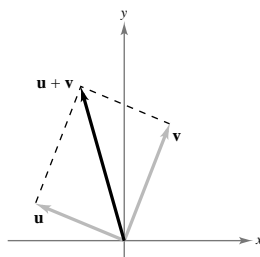
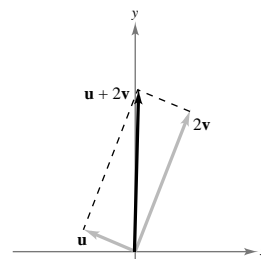
$$\mathbf{v} = \langle 3.7 - (-4.2), -12.9 - 5 \rangle = \langle 7.9, -17.9 \rangle$$

$$\|\mathbf{v}\| = \sqrt{7.9^2 + (-17.9)^2} \approx 19.6$$

15.



17.

19. $\mathbf{u} + 2\mathbf{v}$ 

21. $\mathbf{u} = \langle 4, 2 \rangle, \mathbf{v} = \langle 7, 1 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle 11, 3 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -3, 1 \rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle 8, 4 \rangle - \langle 21, 3 \rangle = \langle -13, 1 \rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle 7, 1 \rangle + \langle 16, 8 \rangle = \langle 23, 9 \rangle$

23. $\mathbf{u} = \langle -5, -2 \rangle, \mathbf{v} = \langle 1, -3 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle -4, -5 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -6, 1 \rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle -10, -4 \rangle - \langle 3, -9 \rangle = \langle -13, 5 \rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle 1, -3 \rangle + \langle -20, -8 \rangle$
 $= \langle -19, -11 \rangle$

25. $\mathbf{u} = \mathbf{i} + \mathbf{j}, \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = -\mathbf{i} + 4\mathbf{j}$

(c) $2\mathbf{u} - 3\mathbf{v} = (2\mathbf{i} + 2\mathbf{j}) - (6\mathbf{i} - 9\mathbf{j})$
 $= -4\mathbf{i} + 11\mathbf{j}$

(d) $\mathbf{v} + 4\mathbf{u} = (2\mathbf{i} - 3\mathbf{j}) + (4\mathbf{i} + 4\mathbf{j})$
 $= 6\mathbf{i} + \mathbf{j}$

27. $\|\langle 6, 0 \rangle\| = 6$

unit vector $= \frac{1}{6}\langle 6, 0 \rangle = \langle 1, 0 \rangle$

29. $\|\mathbf{v}\| = \|\langle -4, 4 \rangle\| = \sqrt{16 + 16} = 4\sqrt{2}$

unit vector $= \frac{1}{4\sqrt{2}}\langle -4, 4 \rangle = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

31. $\|\mathbf{v}\| = \sqrt{(-24)^2 + 7^2} = 25$

unit vector $= \frac{1}{25}\langle -24, -7 \rangle = \left\langle -\frac{24}{25}, -\frac{7}{25} \right\rangle$

33. $\mathbf{u} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$

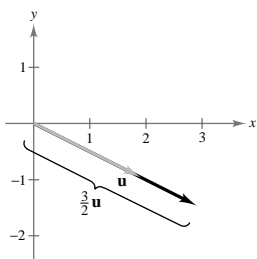
$$= \frac{1}{\sqrt{16 + 9}}(4\mathbf{i} - 3\mathbf{j}) = \frac{1}{5}(4\mathbf{i} - 3\mathbf{j})$$

$$= \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$$

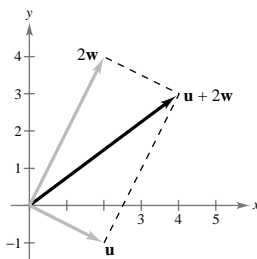
35. $\mathbf{u} = \frac{1}{2}(2\mathbf{j}) = \mathbf{j}$

$$\begin{aligned}
 37. \quad 5\left(\frac{1}{\|\mathbf{v}\|}\mathbf{v}\right) &= 5\left(\frac{1}{\sqrt{3^2+3^2}}\langle 3, 3 \rangle\right) & 39. \quad 7\left(\frac{1}{\|\mathbf{v}\|}\mathbf{v}\right) &= 7\left(\frac{1}{\sqrt{3^2+4^2}}\langle 3, 4 \rangle\right) & 41. \quad 8\left(\frac{1}{\|\mathbf{v}\|}\mathbf{v}\right) &= 8\left(\frac{1}{2}\langle -2, 0 \rangle\right) \\
 &= 5\left(\frac{1}{3\sqrt{2}}\langle 3, 3 \rangle\right) & &= \frac{7}{5}\langle 3, 4 \rangle & &= 4\langle -2, 0 \rangle \\
 &= \left\langle \frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}} \right\rangle & &= \left\langle \frac{21}{5}, \frac{28}{5} \right\rangle & &= \langle -8, 0 \rangle = -8\mathbf{i} \\
 &= \left\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle & & & &
 \end{aligned}$$

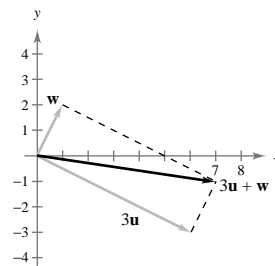
$$\begin{aligned}
 43. \quad \mathbf{v} &= \frac{3}{2}\mathbf{u} \\
 &= \frac{3}{2}(2\mathbf{i} - \mathbf{j}) \\
 &= 3\mathbf{i} - \frac{3}{2}\mathbf{j} = \left\langle 3, -\frac{3}{2} \right\rangle
 \end{aligned}$$



$$\begin{aligned}
 45. \quad \mathbf{v} &= \mathbf{u} + 2\mathbf{w} \\
 &= (2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} + 2\mathbf{j}) \\
 &= 4\mathbf{i} + 3\mathbf{j} = \langle 4, 3 \rangle
 \end{aligned}$$



$$\begin{aligned}
 47. \quad \mathbf{v} &= (3\mathbf{u} + \mathbf{w}) \\
 &= (6\mathbf{i} - 3\mathbf{j} + \mathbf{i} + 2\mathbf{j}) \\
 &= \langle 7, -1 \rangle
 \end{aligned}$$

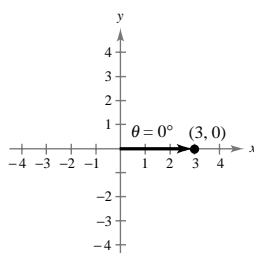


$$\begin{aligned}
 49. \quad \mathbf{v} &= 5(\cos 30^\circ\mathbf{i} + \sin 30^\circ\mathbf{j}) \\
 \|\mathbf{v}\| &= 5, \quad \theta = 30^\circ
 \end{aligned}$$

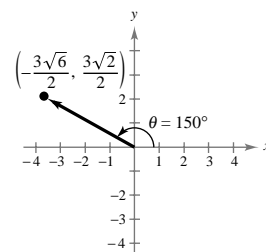
$$\begin{aligned}
 51. \quad \mathbf{v} &= 6\mathbf{i} - 6\mathbf{j} \\
 \|\mathbf{v}\| &= \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2} \\
 \tan \theta &= \frac{-6}{6} = -1 \\
 \text{Since } \mathbf{v} \text{ lies in Quadrant IV, } &\theta = 315^\circ.
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \mathbf{v} &= -2\mathbf{i} + 5\mathbf{j} \\
 \|\mathbf{v}\| &= \sqrt{(-2)^2 + 5^2} = \sqrt{29} \\
 \tan \theta &= -\frac{5}{2} \\
 \text{Since } \mathbf{v} \text{ lies in Quadrant II,} &\theta \approx 111.8^\circ.
 \end{aligned}$$

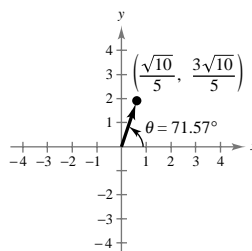
$$\begin{aligned}
 55. \quad \mathbf{v} &= \langle 3 \cos 0^\circ, 3 \sin 0^\circ \rangle \\
 &= \langle 3, 0 \rangle
 \end{aligned}$$



$$\begin{aligned}
 57. \quad \mathbf{v} &= \langle 3\sqrt{2} \cos 150^\circ, \\
 &3\sqrt{2} \sin 150^\circ \rangle \\
 &= \left\langle -\frac{3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2} \right\rangle
 \end{aligned}$$



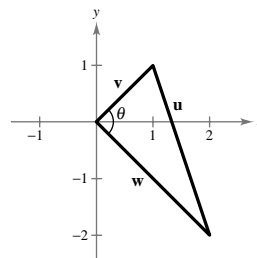
$$\begin{aligned}
 59. \quad \mathbf{v} &= 2\left(\frac{1}{\sqrt{3^2+1^2}}\right)(\mathbf{i} + 3\mathbf{j}) \\
 &= \frac{2}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j}) \\
 &= \frac{\sqrt{10}}{5}\mathbf{i} + \frac{3\sqrt{10}}{5}\mathbf{j} = \left\langle \frac{\sqrt{10}}{5}, \frac{3\sqrt{10}}{5} \right\rangle
 \end{aligned}$$



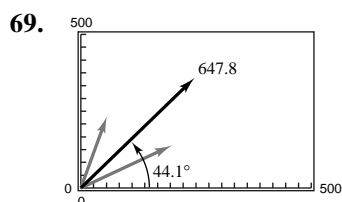
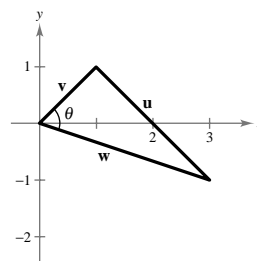
61. $\mathbf{u} = \langle 5 \cos 60^\circ, 5 \sin 60^\circ \rangle = \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle$
 $\mathbf{v} = \langle 5 \cos 90^\circ, 5 \sin 90^\circ \rangle = \langle 0, 5 \rangle$
 $\mathbf{u} + \mathbf{v} = \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle + \langle 0, 5 \rangle = \left\langle \frac{5}{2}, 5 + \frac{5\sqrt{3}}{2} \right\rangle$

63. $\mathbf{u} = \langle 20 \cos 45^\circ, 20 \sin 45^\circ \rangle = \langle 10\sqrt{2}, 10\sqrt{2} \rangle$
 $\mathbf{v} = \langle 50 \cos 150^\circ, 50 \sin 150^\circ \rangle = \langle -25\sqrt{3}, 25 \rangle$
 $\mathbf{u} + \mathbf{v} = \langle 10\sqrt{2} - 25\sqrt{3}, 10\sqrt{2} + 25 \rangle$

65. $\mathbf{v} = \mathbf{i} + \mathbf{j}$
 $\mathbf{w} = 2(\mathbf{i} - \mathbf{j})$
 $\mathbf{u} = \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j}$
 $\|\mathbf{v}\| = \sqrt{2}$
 $\|\mathbf{w}\| = 2\sqrt{2}$
 $\|\mathbf{v} - \mathbf{w}\| = \sqrt{10}$
 $\cos \alpha = \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\| \|\mathbf{w}\|} = \frac{2 + 8 - 10}{2\sqrt{2} \cdot 2\sqrt{2}} = 0$
 $\alpha = 90^\circ$



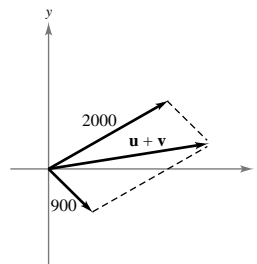
67. $\mathbf{v} = \mathbf{i} + \mathbf{j}$
 $\mathbf{w} = 3\mathbf{i} - \mathbf{j}$
 $\mathbf{u} = \mathbf{v} - \mathbf{w} = -2\mathbf{i} + 2\mathbf{j}$
 $\cos \alpha = \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\| \|\mathbf{w}\|} = \frac{2 + 10 - 8}{2\sqrt{2} \sqrt{10}} \approx 0.4472$
 $\alpha = 63.4^\circ$



$\mathbf{u} = 400 \cos 25^\circ \mathbf{i} + 400 \sin 25^\circ \mathbf{j}$
 $\mathbf{v} = 300 \cos 70^\circ \mathbf{i} + 300 \sin 70^\circ \mathbf{j}$
 $\mathbf{u} + \mathbf{v} \approx 465.13\mathbf{i} + 450.96\mathbf{j}$
 $\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(465.13)^2 + (450.96)^2} \approx 647.8$
 $\alpha = \arctan\left(\frac{450.96}{465.13}\right) \approx 44.1^\circ$

71. Force One: $\mathbf{u} = 45\mathbf{i}$
 Force Two: $\mathbf{v} = 60 \cos \theta \mathbf{i} + 60 \sin \theta \mathbf{j}$
 Resultant Force: $\mathbf{u} + \mathbf{v} = (45 + 60 \cos \theta)\mathbf{i} + 60 \sin \theta \mathbf{j}$
 $\|\mathbf{u} + \mathbf{v}\| = \sqrt{(45 + 60 \cos \theta)^2 + (60 \sin \theta)^2} = 90$
 $2025 + 5400 \cos \theta + 3600 = 8100$
 $5400 \cos \theta = 2475$
 $\cos \theta = \frac{2475}{5400} \approx 0.4583$
 $\theta \approx 62.7^\circ$

73. $\mathbf{u} = (2000 \cos 30^\circ)\mathbf{i} + (2000 \sin 30^\circ)\mathbf{j}$
 $\approx 1732.05\mathbf{i} + 1000\mathbf{j}$
 $\mathbf{v} = (900 \cos(-45^\circ))\mathbf{i} + (900 \sin(-45^\circ))\mathbf{j}$
 $\approx 636.4\mathbf{i} + -636.4\mathbf{j}$
 $\mathbf{u} + \mathbf{v} \approx 2368.4\mathbf{i} + 363.6\mathbf{j}$
 $\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(2368.4)^2 + (363.6)^2} \approx 2396.19$
 $\tan \theta = \frac{363.6}{2368.4} \approx 0.1535 \Rightarrow \theta \approx 8.7^\circ$



75. Horizontal component of velocity: $70 \cos 40^\circ \approx 53.62$ ft/sec
 Vertical component of velocity: $70 \sin 40^\circ \approx 45.0$ ft/sec

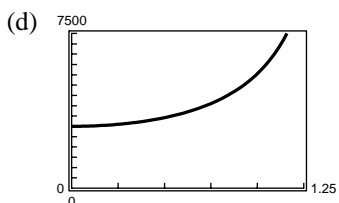
77. Rope \overrightarrow{AC} : $\mathbf{u} = 10\mathbf{i} - 24\mathbf{j}$
 The vector lies in Quadrant IV and its reference angle is $\arctan(\frac{12}{5})$.
 $\mathbf{u} = \|\mathbf{u}\| [\cos(\arctan \frac{12}{5})\mathbf{i} - \sin(\arctan \frac{12}{5})\mathbf{j}]$
 Rope \overrightarrow{BC} : $\mathbf{v} = -20\mathbf{i} - 24\mathbf{j}$
 The vector lies in Quadrant III and its reference angle is $\arctan(\frac{6}{5})$.
 $\mathbf{v} = \|\mathbf{v}\| [-\cos(\arctan \frac{6}{5})\mathbf{i} - \sin(\arctan \frac{6}{5})\mathbf{j}]$
 Resultant: $\mathbf{u} + \mathbf{v} = -5000\mathbf{j}$
 $\|\mathbf{u}\| \cos(\arctan \frac{12}{5}) - \|\mathbf{v}\| \cos(\arctan \frac{6}{5}) = 0$
 $-\|\mathbf{u}\| \sin(\arctan \frac{12}{5}) - \|\mathbf{v}\| \sin(\arctan \frac{6}{5}) = -5000$
 Solving this system of equations yields: $T_{AC} = \|\mathbf{u}\| \approx 3611.1$ pounds
 $T_{BC} = \|\mathbf{v}\| \approx 2169.5$ pounds

79. (a) Tow line 1: $\mathbf{u} = \|\mathbf{u}\| (\cos 20^\circ\mathbf{i} + \sin 20^\circ\mathbf{j})$
 Tow line 2: $\mathbf{v} = \|\mathbf{u}\| (\cos(-20^\circ)\mathbf{i} + \sin(-20^\circ)\mathbf{j})$
 Resultant: $\mathbf{u} + \mathbf{v} = 6000\mathbf{i} = [\|\mathbf{u}\| \cos 20^\circ + \|\mathbf{u}\| \cos(-20^\circ)]\mathbf{i}$
 $\Rightarrow 6000 = 2\|\mathbf{u}\| \cos 20^\circ$
 $\Rightarrow \|\mathbf{u}\| \approx 3192.5$ lb

(b) $\mathbf{u} + \mathbf{v} = 6000\mathbf{i} = 2\|\mathbf{u}\| \cos \theta \Rightarrow T = \|\mathbf{u}\| = 3000 \sec \theta$. Domain: $0^\circ \leq \theta < 90^\circ$

(c)

θ	10°	20°	30°	40°	50°	60°
T	3046.3	3192.5	3464.1	2916.2	4667.2	6000.0



(e) The tension increases because the component in the direction of the motion of the barge decreases.

81. Airspeed: $\mathbf{v} = 860(\cos 302^\circ\mathbf{i} + \sin 302^\circ\mathbf{j})$

Groundspeed: $\mathbf{u} = 800(\cos 310^\circ\mathbf{i} + \sin 310^\circ\mathbf{j})$

$\mathbf{w} + \mathbf{v} = \mathbf{u}$

$\mathbf{w} = \mathbf{u} - \mathbf{v} = 800(\cos 310^\circ\mathbf{i} + \sin 310^\circ\mathbf{j}) - 860(\cos 302^\circ\mathbf{i} + \sin 302^\circ\mathbf{j}) \approx 58.50\mathbf{i} + 116.49\mathbf{j}$

$\|\mathbf{w}\| = \sqrt{58.50^2 + 116.49^2} \approx 130.35 \text{ km/hr}$

$\theta = \arctan\left(\frac{116.49}{58.50}\right) \approx 63.3^\circ$

Direction: N 26.7° E

83. (a) $\mathbf{u} = 220\mathbf{i}$, $\mathbf{v} = 150 \cos 30^\circ\mathbf{i} + 150 \sin 30^\circ\mathbf{j}$

$\mathbf{u} + \mathbf{v} = (220 + 75\sqrt{3})\mathbf{i} + 75\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(220 + 75\sqrt{3})^2 + 75^2} \approx 357.85 \text{ newtons}$

$\tan \theta = \frac{75}{220 + 75\sqrt{3}} \Rightarrow \theta \approx 12.1^\circ$

(b) $\mathbf{u} + \mathbf{v} = 220\mathbf{i} + (150 \cos \theta\mathbf{i} + 150 \sin \theta\mathbf{j})$

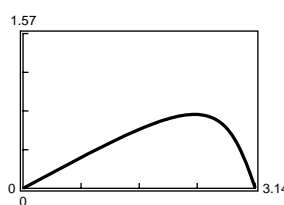
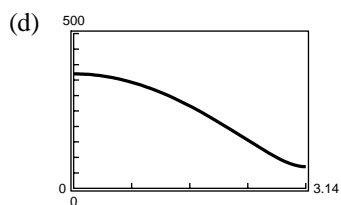
$M = \|\mathbf{u} + \mathbf{v}\| = \sqrt{(220^2 + 150^2(\cos^2 \theta + \sin^2 \theta) + 2(220)(150) \cos \theta)}$

$= \sqrt{70,900 + 66,000 \cos \theta}$

$= 10\sqrt{709 + 660 \cos \theta}$

$\alpha = \arctan\left(\frac{15 \sin \theta}{22 + 15 \cos \theta}\right)$

(c)	θ	0°	30°	60°	90°	120°	150°	180°
	M	370.0	357.9	322.3	266.3	194.7	117.2	70.0
	α	0°	12.1°	23.8°	34.3°	41.9°	39.8°	0°



(e) For increasing θ the two vectors tend to work against each other resulting in a decrease in the magnitude of the resultant.

85. True. See page 444

87. True. In fact, $a = b = 0$.

89. (a) The angle between them is 0° .

(b) The angle between them is 180° .

(c) No. At most it can be equal to the sum when the angle between them is 0° .

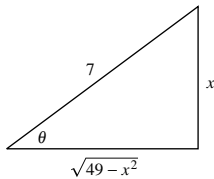
91. Let $\mathbf{v} = (\cos \theta)\mathbf{i} + (\sin \theta)\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{\cos^2 \theta + \sin^2 \theta} = \sqrt{1} = 1$

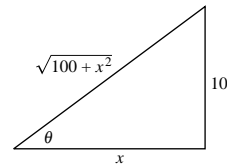
Therefore, \mathbf{v} is a unit vector for any value of θ .

$$\begin{aligned}
 93. \quad \mathbf{u} &= \langle 5 - 1, 2 - 6 \rangle = \langle 4, -4 \rangle \\
 \mathbf{v} &= \langle 9 - 4, 4 - 5 \rangle = \langle 5, -1 \rangle \\
 \mathbf{u} - \mathbf{v} &= \langle -1, -3 \rangle \\
 \mathbf{v} - \mathbf{u} &= \langle 1, 3 \rangle
 \end{aligned}$$

$$95. \sin \theta = \frac{x}{7} \Rightarrow \sqrt{49 - x^2} = 7 \cos \theta$$



$$97. \cot \theta = \frac{x}{10} \Rightarrow \sqrt{x^2 + 100} = 10 \cdot \csc \theta$$

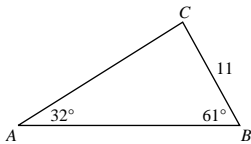


$$99. \text{ Given: } A = 32^\circ, a = 11, B = 61^\circ$$

$$C = 180^\circ - 32^\circ - 61^\circ = 87^\circ$$

$$b = \frac{a}{\sin A} (\sin B) = \frac{11}{\sin 32^\circ} (\sin 61^\circ) \approx 18.16$$

$$c = \frac{a}{\sin A} (\sin C) = \frac{11}{\sin 32^\circ} (\sin 87^\circ) \approx 20.73$$



$$101. \text{ Given: } a = 12, b = 15, c = 24$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{15^2 + 24^2 - 12^2}{2(15)(24)} = 0.9125 \Rightarrow A \approx 24.1^\circ$$

$$\sin B = \frac{b \sin A}{a} \approx 0.5104 \Rightarrow B \approx 30.7^\circ$$

$$C = 180^\circ - 24.1^\circ - 30.7^\circ = 125.2^\circ$$