CHAPTER 9
Sequences, Series, and Probability

Section 9.1 Sequences and Series **428**

Section 9.2 Arithmetic Sequences and Partial Sums **434**

Section 9.3 Geometric Sequences and Series **439**

Section 9.4 Mathematical Induction **446**

Section 9.5 The Binomial Theorem **453**

Section 9.6 Counting Principles **459**

Section 9.7 Probability **462**

Review Exercises **465**

Practice Test **472**

CHAPTER 9
Sequences, Series, and Probability

Section 9.1 Sequences and Series

- Given the general nth term in a sequence, you should be able to find, or list, some of the terms.
- You should be able to find an expression for the nth term of a sequence.
- You should be able to use and evaluate factorials.
- You should be able to use sigma notation for a sum.

Solutions to Odd-Numbered Exercises

1. $a_n = 2n + 5$

$a_1 = 2(1) + 5 = 7$

$a_2 = 2(2) + 5 = 9$

$a_3 = 2(3) + 5 = 11$

$a_4 = 2(4) + 5 = 13$

$a_5 = 2(5) + 5 = 15$

3. $a_n = 2^n$

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

5. $a_n = (-2)^n$

$a_1 = (-2)^1 = -2$

$a_3 = (-2)^2 = 4$

$a_3 = (-2)^3 = -8$

$a_4 = (-2)^4 = 16$

$a_5 = (-2)^5 = -32$

7. $a_n = \frac{n+1}{n}$

$a_1 = \frac{1+1}{1} = 2$

$a_2 = \frac{3}{2}$

$a_3 = \frac{4}{3}$

$a_4 = \frac{5}{4}$

$a_5 = \frac{6}{5}$

9. $a_n = \frac{6n}{3n^2 - 1}$

$a_1 = \frac{6(1)}{3(1)^2 - 1} = 3$

$a_2 = \frac{6(2)}{3(2)^2 - 1} = \frac{12}{11}$

$a_3 = \frac{6(3)}{3(3)^2 - 1} = \frac{9}{13}$

$a_4 = \frac{6(4)}{3(4)^2 - 1} = \frac{24}{47}$

$a_5 = \frac{6(5)}{3(5)^2 - 1} = \frac{15}{37}$

11. $a_n = \frac{1 + (-1)^n}{n}$

$a_1 = 0$

$a_2 = \frac{2}{2} = 1$

$a_3 = 0$

$a_4 = \frac{2}{4} = \frac{1}{2}$

$a_5 = 0$

13. $a_n = 3 - \frac{1}{2^n}$

$a_1 = 3 - \frac{1}{2} = \frac{5}{2}$

$a_2 = 3 - \frac{1}{4} = \frac{11}{4}$

$a_3 = 3 - \frac{1}{8} = \frac{23}{8}$

$a_4 = 3 - \frac{1}{16} = \frac{47}{16}$

$a_5 = 3 - \frac{1}{32} = \frac{95}{32}$

15. $a_n = \frac{1}{n^{3/2}}$

$a_1 = \frac{1}{1} = 1$

$a_2 = \frac{1}{2^{3/2}}$

$a_3 = \frac{1}{3^{3/2}}$

$a_4 = \frac{1}{4^{3/2}} = \frac{1}{8}$

$a_5 = \frac{1}{5^{3/2}}$

17. $a_n = \frac{3^n}{n!}$

$a_1 = \frac{3^1}{1!} = \frac{3}{1} = 3$

$a_2 = \frac{3^2}{2!} = \frac{9}{2}$

$a_3 = \frac{27}{6} = \frac{9}{2}$

$a_4 = \frac{81}{24} = \frac{27}{8}$

$a_5 = \frac{243}{120} = \frac{81}{40}$

19. $a_n = \frac{(-1)^n}{n^2}$

$a_1 = \frac{-1}{1} = -1$

$a_2 = \frac{1}{4}$

$a_3 = \frac{-1}{9}$

$a_4 = \frac{1}{16}$

$a_5 = \frac{-1}{25}$

21. $a_n = (2n - 1)(2n + 1)$

$a_1 = (1)(3) = 3$

$a_2 = (3)(5) = 15$

$a_3 = (5)(7) = 35$

$a_4 = (7)(9) = 63$

$a_5 = (9)(11) = 99$

23. $a_{25} = (-1)^{25}[3(25) - 2] = -73$

25. $a_n = \frac{2^n}{n!}$

$a_{10} = \frac{2^{10}}{10!} = \frac{1024}{3,628,800} = \frac{4}{14,175}$

27. $a_n = \frac{4n}{2n^2 - 3}$

$a_{12} = \frac{4(12)}{2(12)^2 - 3} = \frac{48}{285} = \frac{16}{95}$

29. $a_1 = 28$ and $a_{k+1} = a_k - 4$

$a_1 = 28$

$a_2 = a_1 - 4 = 28 - 4 = 24$

$a_3 = a_2 - 4 = 24 - 4 = 20$

$a_4 = a_3 - 4 = 20 - 4 = 16$

$a_5 = a_4 - 4 = 16 - 4 = 12$

31. $a_1 = 3$ and $a_{k+1} = 2(a_k - 1)$

$a_1 = 3$

$a_2 = 2(a_1 - 1) = 2(3 - 1) = 4$

$a_3 = 2(a_2 - 1) = 2(4 - 1) = 6$

$a_4 = 2(a_3 - 1) = 2(6 - 1) = 10$

$a_5 = 2(a_4 - 1) = 2(10 - 1) = 18$

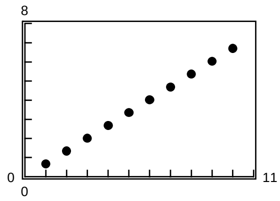
33. $a_1 = 2, a_2 = 6, a_{k+2} = a_{k+1} + 2a_k$

$a_3 = a_2 + 2a_1 = 6 + 2(2) = 10$

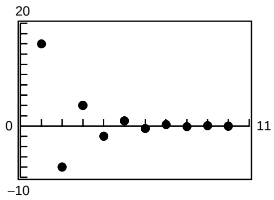
$a_4 = a_3 + 2a_2 = 10 + 2(6) = 22$

$a_5 = a_4 + 2a_3 = 22 + 2(10) = 42$

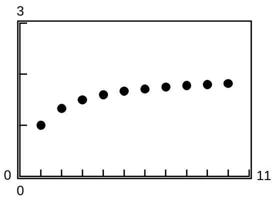
35. $a_n = \frac{2}{3}n$



37. $a_n = 16(-0.5)^{n-1}$



39. $a_n = \frac{2n}{n+1}$



41. $a_n = 2(3n - 1) + 5$

n	1	2	3	4	5	6	7	8	9	10
a_n	9	15	21	27	33	39	45	51	57	63

43. $a_n = \frac{6^n}{n!}$

n	1	2	3	4	5	6	7	8	9	10
a_n	6	18	36	54	64.8	64.8	55.543	41.657	22.771	16.663

45. $a_n = 1 + \frac{n+1}{n}$

n	1	2	3	4	5	6	7	8	9	10
a_n	3	2.5	2.33	2.25	2.2	2.17	2.14	2.13	2.11	2.1

47. $a_n = \frac{8}{n+1}$

$a_n \longrightarrow 0$ as $n \longrightarrow \infty$

$a_1 = 4, \; a_{10} = \frac{8}{11}$

Matches graph (c).

49. $a_n = 4(0.5)^{n-1}$

$a_n \longrightarrow 0$ as $n \longrightarrow \infty$

$a_1 = 4, \; a_{10} \approx 0.008$

Matches graph (d).

51. $1, 4, 7, 10, 13, \ldots$

$a_n = 1 + (n - 1)3 = 3n - 2$

53. $0, 3, 8, 15, 24, \ldots$

$a_n = n^2 - 1$

55. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$

$a_n = \frac{n+1}{n+2}$

57. $\frac{1}{2}, \frac{-1}{4}, \frac{1}{8}, \frac{-1}{16}, \ldots$

$a_n = \frac{(-1)^{n+1}}{2^n}$

59. $1 + \frac{1}{1}, 1 + \frac{1}{2}, 1 + \frac{1}{3}, 1 + \frac{1}{4}, 1 + \frac{1}{5}, \ldots$

$a_n = 1 + \frac{1}{n}$

61. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

$a_n = \frac{1}{n!}$

63. $1, 3, 1, 3, 1, 3, \ldots$

$a_n = 2 + (-1)^n$

65. $a_1 = 6$ and $a_{k+1} = a_k + 2$

$a_1 = 6$

$a_2 = a_1 + 2 = 6 + 2 = 8$

$a_3 = a_2 + 2 = 8 + 2 = 10$

$a_4 = a_3 + 2 = 10 + 2 = 12$

$a_5 = a_4 + 2 = 12 + 2 = 14$

In general, $a_n = 2n + 4$.

67. $a_1 = 81$ and $a_{k+1} = \frac{1}{3}a_k$

$a_1 = 81$

$a_2 = \frac{1}{3}a_1 = \frac{1}{3}(81) = 27$

$a_3 = \frac{1}{3}a_2 = \frac{1}{3}(27) = 9$

$a_4 = \frac{1}{3}a_3 = \frac{1}{3}(9) = 3$

$a_5 = \frac{1}{3}a_4 = \frac{1}{3}(3) = 1$

In general, $a_n = 81\left(\frac{1}{3}\right)^{n-1} = 81(3)\left(\frac{1}{3}\right)^n = \frac{243}{3^n}$.

69. $\frac{3!}{6!} = \frac{3!}{6 \cdot 5 \cdot 4 \cdot 3!} = \frac{1}{6 \cdot 5 \cdot 4} = \frac{1}{120}$

71. $\frac{10!}{8!} = \frac{10 \cdot 9 \cdot 8!}{8!} = 90$

73. $\frac{12!}{4!8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{4!8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2} = 495$

75. $\frac{(n+1)!}{n!} = \frac{(n+1)n!}{n!} = n + 1$

77. $\frac{(2n-1)!}{(2n+1)!} = \frac{(2n-1)!}{(2n+1)(2n)(2n-1)!}$

$= \frac{1}{2n(2n+1)}$

79. $\sum_{i=1}^{5}(2i + 1) = (2 + 1) + (4 + 1) + (6 + 1) + (8 + 1) + (10 + 1) = 35$

81. $\sum_{k=1}^{4}10 = 10 + 10 + 10 + 10 = 40$

83. $\sum_{i=0}^{4} i^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 = 30$

85. $\sum_{k=0}^{3} \frac{1}{k^2 + 1} = \frac{1}{1} + \frac{1}{1+1} + \frac{1}{1+4} + \frac{1}{9+1} = \frac{9}{5}$

87. $\sum_{i=1}^{4}[(i-1)^2 + (i+1)^3] = [(0)^2 + (2)^3] + [(1)^2 + (3)^3] + [(2)^2 + (4)^3] + [(3)^2 + (5)^3] = 238$

89. $\sum_{i=1}^{4}2^i = 2^1 + 2^2 + 2^3 + 2^4 = 30$

91. $\sum_{j=1}^{6}(24 - 3j) = 81$

93. $\sum_{k=0}^{4} \frac{(-1)^k}{k+1} = \frac{47}{60}$

95. $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \cdots + \frac{1}{3(9)} = \sum_{i=1}^{9} \frac{1}{3i} \approx 0.94299$

97. $\left[2\left(\frac{1}{8}\right) + 3\right] + \left[2\left(\frac{2}{8}\right) + 3\right] + \left[2\left(\frac{3}{8}\right) + 3\right] + \cdots + \left[2\left(\frac{8}{8}\right) + 3\right] = \sum_{i=1}^{8}\left[2\left(\frac{i}{8}\right) + 3\right] = 33$

99. $3 - 9 + 27 - 81 + 243 - 729 = \sum_{i=1}^{6} (-1)^{i+1}3^i = -546$

101. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots + -\frac{1}{20^2} = \sum_{i=1}^{20} \frac{(-1)^{i+1}}{i^2} \approx 0.82128$

103. $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64} = \sum_{i=1}^{5} \frac{2^i - 1}{2^{i+1}} = \frac{129}{64} \approx 2.0156$

105. $\sum_{i=1}^{4} 5\left(\frac{1}{2}\right)^i = 4.6875 = \frac{75}{16}$

107. $\sum_{n=1}^{3} 4\left(-\frac{1}{2}\right)^n = -1.5 = -\frac{3}{2}$

109. $\sum_{i=1}^{\infty} 6\left(\frac{1}{10}\right)^i = 6[0.1 + 0.01 + 0.001 + \cdots]$

$= 6[0.111\ldots]$

$= 0.666\ldots$

$= \frac{2}{3}$

111. $\sum_{k=1}^{\infty} \left(\frac{1}{10}\right)^k = 0.1 + 0.11 + 0.111 + \cdots$

$= 0.11111$

$= \frac{1}{9}$

113. $A_n = 5000\left(1 + \frac{0.08}{4}\right)^n, n = 1, 2, 3, \ldots$

(a) $A_1 = \$5100.00$

$A_2 = \$5202.00$

$A_3 = \$5306.04$

$A_4 = \$5412.16$

$A_5 = \$5520.40$

$A_6 = \$5630.81$

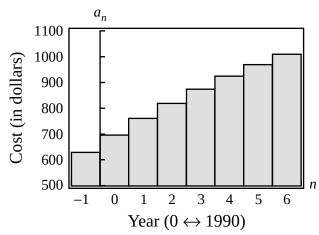
$A_7 = \$5743.43$

$A_8 = \$5858.30$

(b) $A_{40} = \$11,040.20$

115. $a_n = 696.39 + 66.44n - 2.37n^2,\ n = -1, 0, . . 6$

n	-1	0	1	2	3	4	5	6
a_n	627.58	696.39	760.46	819.79	874.38	924.23	969.34	1009.71



According to the graph, hospital costs are increasing.

117. $\sum_{n=0}^{8}[1215.16 + 608.19n - 114.83n^2 + 11n^3] = \$23,661.96$ million

119. True

121. $a_1 = 1, a_2 = 1,\ a_{k+2} = a_{k+1} + a_k$

$a_1 = 1$ $\quad b_1 = \frac{1}{1} = 1$

$a_2 = 1$ $\quad b_2 = \frac{2}{1} = 2$

$a_3 = 1 + 1 = 2$ $\quad b_3 = \frac{3}{2}$

$a_4 = 2 + 1 = 3$ $\quad b_4 = \frac{5}{3}$

$a_5 = 3 + 2 = 5$ $\quad b_5 = \frac{8}{5}$

$a_6 = 5 + 3 = 8$ $\quad b_6 = \frac{13}{8}$

$a_7 = 8 + 5 = 13$ $\quad b_7 = \frac{21}{13}$

$a_8 = 13 + 8 = 21$ $\quad b_8 = \frac{34}{21}$

$a_9 = 21 + 13 = 34$ $\quad b_9 = \frac{55}{34}$

$a_{10} = 34 + 21 = 55$ $\quad b_{10} = \frac{89}{55}$

$a_{11} = 55 + 34 = 89$

$a_{12} = 89 + 55 = 144$

123. $a_n = n^2 - n + 11$

$a_1 = 11$

$a_2 = 13$

$a_3 = 17$

$a_4 = 23$

$a_5 = 31$

The terms seem to be prime numbers. However, $a_{11} = 121$ is not prime.

125. $a_n = \dfrac{(-1)^n x^{2n+1}}{2n + 1}$

$a_1 = \dfrac{-x^3}{3}$

$a_2 = \dfrac{x^5}{5}$

$a_3 = -\dfrac{x^7}{7}$

$a_4 = \dfrac{x^9}{9}$

$a_5 = \dfrac{-x^{11}}{11}$

127. $a_n = \dfrac{(-1)x^{2n+1}}{(2n + 1)!}$

$a_1 = \dfrac{-x^3}{3!}$

$a_2 = \dfrac{x^5}{5!}$

$a_3 = -\dfrac{x^7}{7!}$

$a_4 = \dfrac{x^9}{9!}$

$a_5 = \dfrac{-x^{11}}{11!}$