129. $\begin{bmatrix} 2 & 1 & 3 & \vdots & -3 \\ -1 & 5 & 0 & \vdots & 14 \\ -3 & -6 & -7 & \vdots & -7 \end{bmatrix}$

131. (a) $A - B = \begin{bmatrix} 10 & 19 \\ -12 & -5 \end{bmatrix}$

(b) $2B - 3A = \begin{bmatrix} -30 & -45 \\ 28 & 4 \end{bmatrix}$

(c) $AB = \begin{bmatrix} 56 & -43 \\ 48 & 114 \end{bmatrix}$

(d) $BA = \begin{bmatrix} 48 & -72 \\ 36 & 122 \end{bmatrix}$

133. (a) $A - B = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$

(b) $2B - 3A = \begin{bmatrix} 3 & -4 & 0 \\ -9 & -1 & -10 \\ -2 & 3 & -5 \end{bmatrix}$

(c) $AB = \begin{bmatrix} 12 & 0 & -8 \\ 1 & 21 & 2 \\ -6 & -1 & 8 \end{bmatrix}$

(d) $BA = \begin{bmatrix} 20 & 4 & 8 \\ 2 & 15 & -4 \\ 1 & -6 & 6 \end{bmatrix}$

135. $\det \begin{bmatrix} -4 & 11 \\ 13 & 20 \end{bmatrix} = (-4)(20) - 11(13) = -223$

137. $\det(A) = 664$

Section 9.2 Arithmetic Sequences and Partial Sums

- You should be able to recognize an arithmetic sequence, find its common difference, and find its nth term.
- You should be able to find the nth partial sum of an arithmetic sequence with common difference d using the formula

$$S_n = \frac{n}{2}(a_1 + a_n).$$

Solutions to Odd-Numbered Exercises

1. 10, 8, 6, 4, 2, . . .

Arithmetic sequence, $d = -2$

3. $3, \frac{5}{2}, 2, \frac{3}{2}, 1, \ldots$

Arithmetic sequence, $d = -\frac{1}{2}$

5. −24, −16, −8, 0, 8

Arithmetic sequence, $d = 8$

7. 3.7, 4.3, 4.9, 5.5, 6.1, . . .

Arithmetic sequence, $d = 0.6$

9. $a_n = 8 + 13n$

21, 34, 47, 60, 73

Arithmetic sequence, $d = 13$

11. $a_n = \dfrac{1}{n + 1}$

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$

Not an arithmetic sequence

13. $a_n = 150 - 7n$

143, 136, 129, 122, 115

Arithmetic sequence, $d = -7$

15. $a_n = 3 + \frac{(-1)^n 2}{n}$

$1, \ 4, \ \frac{7}{3}, \frac{7}{2}, \frac{13}{5}$

Not an arithmetic sequence

17. $a_1 = 15, a_{k+1} = a_k + 9$

$a_2 = 15 + 9 = 24$

$a_3 = 24 + 9 = 33$

$a_4 = 33 + 9 = 42$

$a_5 = 42 + 9 = 51$

$d = 9, a_n = 6 + 9n$

19. $a_1 = \frac{7}{2}, a_{k+1} = a_k - \frac{1}{4}$

$a_2 = \frac{7}{2} - \frac{1}{4} = \frac{13}{4}$

$a_3 = \frac{13}{4} - \frac{1}{4} = \frac{12}{4} = 3$

$a_4 = \frac{12}{4} - \frac{1}{4} = \frac{11}{4}$

$a_5 = \frac{11}{4} - \frac{1}{4} = \frac{10}{4} = \frac{5}{2}$

$d = -\frac{1}{4}, a_n = \frac{15}{4} - \frac{1}{4}n$

21. $a_1 = 5, \ d = 6$

$a_1 = 5$

$a_2 = 5 + 6 = 11$

$a_3 = 11 + 6 = 17$

$a_4 = 17 + 6 = 23$

$a_5 = 23 + 6 = 29$

23. $a_1 = -2.6, \ d = -0.4$

$a_1 = -2.6$

$a_2 = -2.6 + (-0.4) = -3.0$

$a_3 = -3.0 + (-0.4) = -3.4$

$a_4 = -3.4 + (-0.4) = -3.8$

$a_5 = -3.8 + (-0.4) = -4.2$

25. $a_8 = 26, \ a_{12} = 42$

$26 = a_8 = a_1 + (n - 1)d = a_1 + 7d$

$42 = a_{12} = a_1 + (n - 1)d = a_1 + 11d$

Answer: $d = 4, \ a_1 = -2$

$a_1 = -2$

$a_2 = -2 + 4 = 2$

$a_3 = 2 + 4 = 6$

$a_4 = 6 + 4 = 10$

$a_5 = 10 + 4 = 14$

27. $a_3 = 19, a_{15} = -1.7$

$a_{15} = a_3 + 12d$

$-1.7 = 19 + 12d \Longrightarrow d = -1.725$

$a_3 = a_1 + 2d \Longrightarrow 19 = a_1 + 2(-1.725) \Longrightarrow a_1 = 22.45$

$a_2 = a_1 - 1.725 = 20.725$

$a_3 = 19$

$a_4 = 19 - 1.725 = 17.275$

$a_5 = 17.275 - 1.725 = 15.55$

29. $a_1 = 5, a_2 = 11 \Longrightarrow d = 6$

$a_{10} = a_1 + 9d = 5 + 9(6) = 59$

31. $a_1 = 2, a_2 = -2 \Longrightarrow d = -4$

$a_{14} = a_1 + 13d = 2 + 13(-4) = -50$

33. $a_1 = 4.2, a_2 = 6.6 \Longrightarrow d = 2.4$

$a_7 = a_1 + 6d = 4.2 + 6(2.4) = 18.6$

35. $a_1 = 1,\ d = 3$

$a_n = a_1 + (n-1)d = 1 + (n-1)(3) = 3n - 2$

37. $a_1 = 100,\ d = -8$

$a_n = a_1 + (n-1)d = 100 + (n-1)(-8) = 108 - 8n$

39. $4, \frac{3}{2}, -1, -\frac{7}{2}, \ldots$

$d = -\frac{5}{2}$

$a_n = a_1 + (n-1)d = 4 + (n-1)\left(-\frac{5}{2}\right) = \frac{13}{2} - \frac{5}{2}n$

41. $a_1 = 5,\ a_4 = 15$

$a_4 = a_1 + 3d \implies 15 = 5 + 3d \implies d = \frac{10}{3}$

$a_n = a_1 + (n-1)d = 5 + (n-1)\left(\frac{10}{3}\right) = \frac{10}{3}n + \frac{5}{3}$

43. $a_3 = 94,\ a_6 = 85$

$a_6 = a_3 + 3d \implies 85 = 94 + 3d \implies d = -3$

$a_1 = a_3 - 2d \implies a_1 = 94 - 2(-3) = 100$

$a_n = a_1 + (n-1)d = 100 + (n-1)(-3) = 103 - 3n$

45. $a_n = -\frac{2}{3}n + 6$

$d = -\frac{2}{3}$ so the sequence is decreasing, and $a_1 = 5\frac{1}{3}$.

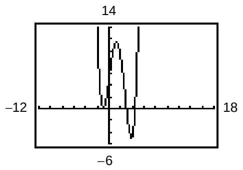
Matches (b).

47. $a_n = 2 + \frac{3}{4}n$

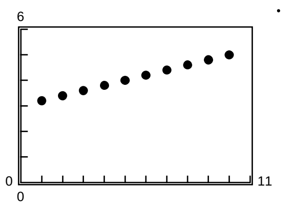
$d = \frac{3}{4}$ so the sequence is increasing, and $a_1 = 2\frac{3}{4}$.

Matches (c).

49. $a_n = 15 - \frac{3}{2}n$



51. $a_n = 0.2n + 3$



53. $a_n = 4n - 5$

n	1	2	3	4	5	6	7	8	9	10
a_n	−1	3	7	11	15	19	23	27	31	35

55. $a_n = 20 - \frac{3}{4}n$

n	1	2	3	4	5	6	7	8	9	10
a_n	19.25	18.5	17.75	17	16.25	15.5	14.75	14	13.25	12.5

57. $a_n = 1.5 + 0.005n$

n	1	2	3	4	5	6	7	8	9	10
a_n	1.505	1.51	1.515	1.52	1.525	1.53	1.535	1.54	1.545	1.55

59. $a_1 = 8, a_2 = 26 \Longrightarrow d = 18$

$a_{10} = a_1 + 9d = 8 + 9(18) = 170$

$S_{10} = \frac{10}{2}(a_1 + a_{10}) = 5(8 + 170) = 890$

61. $a_1 = 0.5, a_2 = 1.3 \Longrightarrow d = 0.8$

$a_{10} = a_1 + 9d = 0.5 + 9(0.8) = 7.7$

$S_{10} = \frac{10}{2}(a_1 + a_{10}) = 5(0.5 + 7.7) = 41$

63. $a_1 = 100, a_{25} = 220$

$a_{25} = a_1 + 24d \Longrightarrow d = 5$

$S_{25} = \frac{25}{2}(a_1 + a_{25}) = 12.5(100 + 220) = 4000$

65. $a_1 = 1,\ a_{50} = 50,\ n = 50$

$\sum_{n=1}^{50} n = \frac{50}{2}(1 + 50) = 1275$

67. $a_1 = 5,\ a_{100} = 500, n = 100$

$\sum_{n=1}^{100} 5n = \frac{100}{2}(5 + 500) = 25{,}250$

69. $\sum_{n=11}^{30} n - \sum_{n=1}^{10} n = \frac{20}{2}(11 + 30) - \frac{10}{2}(1 + 10) = 355$

71. $a_1 = 4,\ a_{500} = 503, n = 500$

$\sum_{n=1}^{500} (n + 3) = \frac{500}{2}(4 + 503) = 126{,}750$

73. $a_1 = 7, a_{20} = 45,\ n = 20$

$\sum_{n=1}^{20} (2n + 5) = \frac{20}{2}(7 + 45) = 520$

75. $a_0 = 1000,\ a_{50} = 750,\ n = 51$

$\sum_{n=0}^{50} (1000 - 5n) = \frac{51}{2}(1000 + 750) = 44{,}625$

77. $a_1 = \frac{742}{3},\ a_{60} = 90,\ n = 60$

$\sum_{i=1}^{60} \left(250 - \frac{8}{3}i\right) = \frac{60}{2}\left(\frac{742}{3} + 90\right) = 10{,}120$

79. $a_1 = 1,\ a_{100} = 199,\ n = 100$

$\sum_{n=1}^{100} (2n - 1) = \frac{100}{2}(1 + 199) = 10{,}000$

81. (a) $a_1 = 32{,}500,\ d = 1500$

$a_6 = a_1 + 5d = 32{,}500 + 5(1500) = \$40{,}000$

(b) $S_6 = \frac{6}{2}[32{,}500 + 40{,}000] = \$217{,}500$

(c) first year: \$32,500; second year: \$34,000; third year: \$35,500; fourth year: \$37,000; fifth year: \$38,500; sixth year: \$40,000

\$32,500 + \$34,000 + \$35,500 + \$37,000 + \$38,500 + \$40,000 = \$217,500

83. $a_1 = 20,\ d = 4,\ n = 30$

$a_{30} = 20 + 29(4) = 136$

$S_{30} = \frac{30}{2}(20 + 36) = 2340$ seats

85. $a_1 = 14, a_{18} = 31$

$S_{18} = \frac{18}{2}(14 + 31) = 405$ bricks

87. $a_1 = 25,\ a_2 = 25 + 2 = 27$, etc. $\Longrightarrow$ $d = 2$ and $n = 15$.

$a_{15} = 2(15) + 23 = 53$

$S_{15} = \frac{15}{2}(25 + 53) = \frac{15}{2} \cdot 78 = 585$ seats

89. $(1 + 2 + \cdots + 12) + (1 + 2 + \cdots + 12) = \frac{12}{2}(1 + 12) \times 2 = 12 \cdot 13 = 156$ times

91. True. Given a_1 and a_2, you know $d = a_2 - a_1$. Thus, $a_n = a_1 + (n - 1)d$.

93. $a_1 = x$ $\quad a_6 = 11x$

$a_2 = x + 2x = 3x$ $\quad a_7 = 13x$

$a_3 = 3x + 2x = 5x$ $\quad a_8 = 15x$

$a_4 = 7x$ $\quad a_9 = 17x$

$a_5 = 9x$ $\quad a_{10} = 19x$

95. (a) $1 + 3 = 4$

$1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 = 16$

$1 + 3 + 5 + 7 + 9 = 25$

$1 + 3 + 5 + 7 + 9 + 11 = 36$

(b) $S_n = n^2$

$S_7 = 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7^2$

(c) $S_n = \frac{n}{2}[1 + (2n - 1)] = \frac{n}{2}(2n) = n^2$

97. Let $S_n = \frac{n}{2}(a_1 + a_n)$ be the sum of the first n terms of the original sequence. If the first term is increased by 5, then the new sum is

$$\begin{aligned} S' = \frac{n}{2}(a_1 + 5 + a_n + 5) &= \frac{n}{2}(a_1 + a_n + 10) \\ &= \frac{n}{2}(a_1 + a_n) + \frac{n}{2}(10) \\ &= \frac{n}{2}(a_1 + a_n) + 5n \\ &= S_n + 5n \end{aligned}$$

99. $\begin{bmatrix} -1 & 4 & 10 & \vdots & 4 \\ 5 & -3 & 1 & \vdots & 31 \\ 8 & 2 & -3 & \vdots & -5 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$

Answer: $(2, -6, 3)$

101. $\begin{vmatrix} -1 & 2 & 1 \\ 5 & 1 & 1 \\ 3 & 8 & 1 \end{vmatrix} = 40$

Area $= \frac{1}{2}(40) = 20$ square units

103. $\dfrac{6!8!}{14!} = \dfrac{6!8!}{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!} = \dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9} = \dfrac{1}{3003}$