65. $\begin{bmatrix} -3 & 1 & 5 & : & 25 \\ 1 & -2 & 3 & : & 7 \\ 2 & 3 & -1 & : & 0 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 0 & : & -1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$

Answer: $(-1, 2, 4)$

67. $\begin{vmatrix} 7 & 6 \\ -4 & 2 \end{vmatrix} = 14 - (-24) = 38$

69. $(2x^2 - 1)^2 = 4x^4 - 4x^2 + 1$

71. $(5 - 4x)^3 = -64x^3 + 240x^2 - 300x + 125$

Section 9.5 The Binomial Theorem

- You should be able to use the Binomial Theorem

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + {}_nC_r x^{n-r}y^r + \cdots + y^n$$

where ${}_nC_r = \dfrac{n!}{(n-r)!r!}$, to expand$(x + y)^n$.

- You should be able to use Pascal's Triangle.

Solutions to Odd-Numbered Exercises

1. ${}_7C_5 = \dfrac{7!}{2!5!} = \dfrac{7 \cdot 6 \cdot 5!}{2 \cdot 5!} = \dfrac{42}{2} = 21$

3. $\dbinom{12}{0} = {}_{12}C_0 = \dfrac{12!}{0!12!} = 1$

5. ${}_{20}C_{15} = \dfrac{20!}{15!5!} = \dfrac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 15{,}504$

7. ${}_{14}C_1 = \dfrac{14!}{13!1!} = \dfrac{14!13!}{13!} = 14$

9. $\dbinom{100}{98} = {}_{100}C_{98} = \dfrac{100!}{98!2!} = \dfrac{100 \cdot 99}{2 \cdot 1} = 4950$

11. ${}_{100}C_2 = \dfrac{100!}{2!98!} = \dfrac{100 \cdot 99}{2 \cdot 1} = 4950$

13. ${}_{32}C_{28} = 35{,}960$

15. ${}_{22}C_9 = 497{,}420$

17. ${}_{41}C_{36} = 749{,}398$

19.

```
              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5  10  10   5   1
  1   6  15  20  15   6   1
1   7 (21) 35  35  21   7   1
```

${}_7C_2 = 21$, the 3rd entry in the 7th row.

21.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5  10  10   5   1
    1   6  15  20  15   6   1
  1   7  21  35  35  21   7   1
1   8  28  56  70 (56) 28   8   1
```

${}_8C_5 = 56$, the 6th entry in the 8th row.

23. $(x + 1)^4 = {}_4C_0x^4 + {}_4C_1x^3(1) + {}_4C_2x^2(1)^2 + {}_4C_3x(1)^3 + {}_4C_4(1)^4$
$= x^4 + 4x^3 + 6x^2 + 4x + 1$

25. $(a + 3)^3 = {}_3C_0a^3 + {}_3C_1a^2(3) + {}_3C_2a(3)^2 + {}_3C_3(3)^3$
$= a^3 + 3a^2(3) + 3a(3)^2 + (3)^3$
$= a^3 + 9a^2 + 27a + 27$

27. $(y - 2)^4 = {}_4C_0y^4 - {}_4C_1y^3(2) + {}_4C_2y^2(2)^2 - {}_4C_3y(2)^3 + {}_4C_4(2)^4$
$= y^4 - 4y^3(2) + 6y^2(4) - 4y(8) + 16$
$= y^4 - 8y^3 + 24y^2 - 32y + 16$

29. $(x + y)^5 = {}_5C_0x^5 + {}_5C_1x^4y + {}_5C_2x^3y^2 + {}_5C_3x^2y^3 + {}_5C_4xy^4 + {}_5C_5y^5$
$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

31. $(r + 2s)^6 = {}_6C_0r^6 + {}_6C_1r^5(2s) + {}_6C_2r^4(2s)^2 + {}_6C_3r^3(2s)^3 + {}_6C_4r^2(2s)^4$
$+ {}_6C_5r(2s)^5 + {}_6C_6(2s)^6$
$= r^6 + 12r^5s + 60r^4s^2 + 160r^3s^3 + 240r^2s^4 + 192rs^5 + 64s^6$

33. $(x - y)^5 = {}_5C_0x^5 - {}_5C_1x^4y + {}_5C_2x^3y^2 - {}_5C_3x^2y^3 - {}_5C_4xy^4 - {}_5C_5y^5$
$= x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

35. $(1 - 4x)^3 = {}_3C_01^3 - {}_3C_11^2(4x) + {}_3C_21(4x)^2 - {}_3C_3(4x)^3$
$= 1 - 3(4x) + 3(4x)^2 - (4x)^3$
$= 1 - 12x + 48x^2 - 64x^3$

37. $(x^2 + 5)^4 = {}_4C_0(x^2)^4 + {}_4C_1(x^2)^3(5) + {}_4C_2(x^2)^2(5)^2 + {}_4C_3(x^2)(5)^3 + {}_4C_4(5)^4$
$= x^8 + 4x^6(5) + 6x^4(25) + 4x^2(125) + 625$
$= x^8 + 20x^6 + 150x^4 + 500x^2 + 625$

39. $\left(\frac{1}{x} + y\right)^5 = {}_5C_0\left(\frac{1}{x}\right)^5 + {}_5C_1\left(\frac{1}{x}\right)^4y + {}_5C_2\left(\frac{1}{x}\right)^3y^2 + {}_5C_3\left(\frac{1}{x}\right)^2y^3 + {}_5C_4\left(\frac{1}{x}\right)y^4 + {}_5C_5y^5$
$= \frac{1}{x^5} + \frac{5y}{x^4} + \frac{10y^2}{x^3} + \frac{10y^3}{x^2} + \frac{5y^4}{x} + y^5$

41. $2(x - 3)^4 + 5(x - 3)^2 = 2[x^4 - 4(x^3)(3) + 6(x^2)(3^2) - 4(x)(3^3) + 3^4] + 5[x^2 - 2(x)(3) + 3^2]$
$= 2(x^4 - 12x^3 + 54x^2 - 108x + 81) + 5(x^2 - 6x + 9)$
$= 2x^4 - 24x^3 + 113x^2 - 246x + 207$

43. $-3(x-2)^3 - 4(x+1)^6$

$= [-3x^3 + 18x^2 - 36x + 24] - [4x^6 + 24x^5 + 60x^4 + 80x^3 + 60x^2 + 24x + 4]$

$= -4x^6 - 24x^5 - 60x^4 - 83x^3 - 42x^2 - 60x + 20$

45. 5^{th} Row of Pascal's Triangle: 1 5 10 10 5 1

$(3t-s)^5 = 1(3t)^5 + 5(3t)^4(-s) + 10(3t)^3(-s)^2 + 10(3t)^2(-s)^3 + 5(3t)(-s)^4 + 1(-5)^5$

$= 243t^5 - 405t^4s + 270t^3s^2 - 90t^2s^3 + 15ts^4 - s^5$

47. 4^{th} Row of Pascal's Triangle: 1 4 6 4 1

$(3-2z)^4 = 3^4 - 4(3)^3(2z) + 6(3)^2(2z)^2 - 4(3)(2z)^3 + (2z)^4$

$= 81 - 216z + 216z^2 - 96z^3 + 16z^4$

49. The term involving x^4 in the expansion of $(x+3)^{12}$ is ${}_{12}C_8x^4(3)^8 = 495x^4(3)^8 = 3{,}247{,}695x^4$

51. The term involving x^8y^2 in the expansion of $(x-2y)^{10}$ is

${}_{10}C_2x^8(-2y)^2 = \dfrac{10!}{2!8!} \cdot 4x^8y^2 = 180x^8y^2$. The coefficient is 180.

53. The term involving x^6y^3 in $(3x-2y)^9$ is ${}_9C_3(3x)^6(-2y)^3 = 84(3)^6(-2)^3x^6y^3 = -489{,}888$

55. The coefficient of $x^8y^6 = (x^2)^4y^6$ in the expansion of $(x^2+y)^{10}$ is ${}_{10}C_6 = 210$.

57. $(\sqrt{x}+5)^4 = (\sqrt{x})^4 + 4(\sqrt{x})^3(5) + 6(\sqrt{x})^2(5)^2 + 4(\sqrt{x})(5^3) + 5^4$

$= x^2 + 20x\sqrt{x} + 150x + 500\sqrt{x} + 625$

$= x^2 + 20x^{3/2} + 150x + 500x^{1/2} + 625$

59. $(x^{2/3} - y^{1/3})^3 = (x^{2/3})^3 - 3(x^{2/3})^2(y^{1/3}) + 3(x^{2/3})(y^{1/3})^2 - (y^{1/3})^3$

$= x^2 - 3x^{4/3}y^{1/3} + 3x^{2/3}y^{2/3} - y$

61. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^3 - x^3}{h}$

$= \dfrac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$

$= \dfrac{h(3x^2 + 3xh + h^2)}{h}$

$= 3x^2 + 3xh + h^2,\ h \neq 0$

63. $$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}$$
$$= \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$
$$= \frac{(x+h)-x}{h\left[\sqrt{x+h}+\sqrt{x}\right]}$$
$$= \frac{1}{\sqrt{x+h}+\sqrt{x}},\ h \neq 0$$

65. $$(1+i)^4 = {}_4C_0 1^4 + {}_4C_1(1)^3 i + {}_4C_2(1)^2 i^2 + {}_4C_3 1 \cdot i^3 + {}_4C_4 i^4$$
$$= 1 + 4i - 6 - 4i + 1$$
$$= -4$$

67. $$(2-3i)^6 = {}_6C_0 2^6 - {}_6C_1 2^5(3i) + {}_6C_2 2^4(3i)^2 - {}_6C_3 2^3(3i)^3 + {}_6C_4 2^2(3i)^4 - {}_6C_5 2(3i)^5 + {}_6C_6(3i)^6$$
$$= 64 - 576i - 2160 + 4320i + 4860 - 2916i - 729$$
$$= 2035 + 828i$$

69. $$\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3 = \frac{1}{8}\left(-1+\sqrt{3}i\right)^3$$
$$= \frac{1}{8}\left[(-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3\right]$$
$$= \frac{1}{8}\left[-1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i\right]$$
$$= 1$$

71. $$(1.02)^8 = (1+0.02)^8 = 1 + 8(0.02) + 28(0.02)^2 + 56(0.02)^3 + 70(0.02)^4 + 56(0.02)^5$$
$$+ 28(0.02)^6 + 8(0.02)^7 + (0.02)^8$$
$$= 1 + 0.16 + 0.0112 + 0.000448 + \cdots \approx 1.172$$

73. $$(2.99)^{12} = (3-0.01)^{12}$$
$$= 3^{12} - 12(3)^{11}(0.01) + 66(3)^{10}(0.01)^2 - 220(3)^9(0.01)^3 + 495(3)^8(0.01)^4$$
$$- 792(3)^7(0.01)^5 + 924(3)^6(0.01)^6 - 792(3)^5(0.01)^7 + 495(3)^4(0.01)^8$$
$$- 220(3)^3(0.01)^9 + 66(3)^2(0.01)^{10} - 12(3)(0.01)^{11} + (0.01)^{12}$$
$$\approx 510{,}568.785$$

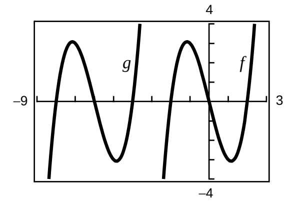
75. $f(x) = x^3 - 4x$

$g(x) = f(x + 6)$

$= (x + 6)^3 - 4(x + 6)$

$= x^3 + 3x^2(6) + 3x(6)^2 + 6^3 - 4x - 24$

$= x^3 + 18x^2 + 104x + 192$



The graph of g is the same as the graph of f shifted 6 units to the left.

77. $f(x) = -x^2 + 3x + 2$

$g(x) = f(x - 2)$

$= -(x - 2)^2 + 3(x - 2) + 2$

$= -x^2 + 4x - 4 + 3x - 6 + 2$

$= -x^2 + 7x - 8$

The graph of g is the same as the graph of f shifted 2 units to the right

79. (a) ${}_{12}C_5 = 792$

(b) $({}_6C_5)^2 = 36$

(c) ${}_{11}C_5 + {}_{11}C_4 = 792$

(d) ${}_6C_5 + {}_6C_5 = 12$

(a) and (c) are equal.

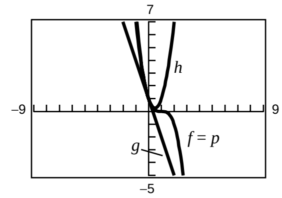
81. $f(x) = (1 - x)^3$

$g(x) = 1 - 3x$

$h(x) = 1 - 3x + 3x^2$

$p(x) = 1 - 3x + 3x^2 - x^3$

Since $p(x)$ is the expansion of $f(x)$, they have the same graph.



83. ${}_7C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = 35\left(\frac{1}{16}\right)\left(\frac{1}{8}\right) \approx 0.273$

85. ${}_8C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^4 = 70\left(\frac{1}{81}\right)\left(\frac{16}{81}\right) \approx 0.171$

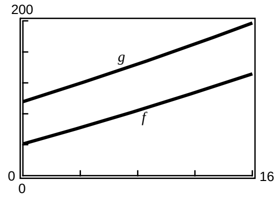
87. (a) $g(t) = f(t + 10)$

$= 0.0348(t + 10)^2 + 5.1083(t + 10) + 41.0250$

$= 0.0348(t^2 + 20t + 100) + 5.1083t + 51.083 + 41.0250$

$= 0.0348t^2 + 5.8043t + 95.588$

(b)



89. False. The x^4y^8 term is

$${}_{12}C_4x^4(-2y)^8 = 495x^4(-2)^8y^8 = 126{,}720x^4y^8$$

[Note 7920 is the coefficient of x^8y^4]

91. Answers will vary. See page 658.

93. There are $n + 1$ terms in the expansion of $(x + y)^n$.

95. $${}_nC_{n-r} = \frac{n!}{[n - (n - r)]!(n - r)!} = \frac{n!}{r!(n - r)!} = \frac{n!}{(n - r)!r!} = {}_nC_r$$

97. $${}_nC_r + {}_nC_{r-1} = \frac{n!}{(n - r)!r!} + \frac{n!}{(n - r + 1)!(r - 1)!}$$
$$= \frac{n!(n - r + 1)}{(n - r)!r!(n - r + 1)} + \frac{n!}{(n - r + 1)!(r - 1)!}\frac{r}{r}$$
$$= \frac{n!(n - r + 1)}{(n - r + 1)!r!} + \frac{n!r}{(n - r + 1)!r!}$$
$$= \frac{n!(n - r + 1 + r)}{(n - r + 1)!r!}$$
$$= \frac{n!(n + 1)}{(n - r + 1)!r!}$$
$$= \frac{(n + 1)!}{(n + 1 - r)!r!} = {}_{n+1}C_r$$

99. $g(x) = f(x) + 8$

$g(x)$ is shifted 8 units up from $f(x)$.

101. $g(x) = f(-x)$

$g(x)$ is the reflection of $f(x)$ in the y-axis.

103. $$A + B = \begin{bmatrix} -2 & -1 & -3 \\ 8 & 0 & 4 \\ -1 & -2 & 7 \end{bmatrix}$$

105. $$-3A - 5B = \begin{bmatrix} 6 & 11 & 15 \\ -30 & -2 & -16 \\ 13 & 10 & -33 \end{bmatrix}$$

107. $$AB = \begin{bmatrix} 9 & 11 & 12 \\ -13 & -25 & -5 \\ -5 & -18 & -6 \end{bmatrix}$$

109. $$\begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix}^{-1} = \frac{1}{-24 + 25}\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$$