Section 9.7 Probability

You should know the following basic principles of probability.

- If an event E has $n(E)$ equally likely outcomes and its sample space has $n(S)$ equally likely outcomes, then the probability of event E is

  $$P(E) = \frac{n(E)}{n(S)}, \text{ where } 0 \le P(E) \le 1.$$

- If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

  If A and B are not mutually exclusive events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- If A and B are independent events, then the probability that both A and B will occur is $P(A)P(B)$.
- The probability of the complement of an event A is $P(A') = 1 - P(A)$.

Solutions to Odd-Numbered Exercises

1. $\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

3. $\{ABC, ACB, BAC, BCA, CAB, CBA\}$

5. $\{(A, B), (A, C), (A, D), (A, E), (B, C), (B, D), (B, E), (C, D), (C, E), (D, E)\}$

7. $E = \{HTT, THT, TTH\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

9. $E = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$$

11. $E = \{K, K, K, K, Q, Q, Q, Q, J, J, J, J\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

13. $E = \{K, K, Q, Q, J, J\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

15. $E = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

17. not $E = \{(6, 6)\}$

$n(E) = n(S) - n(\text{not } E) = 36 - 1 = 35$

$$P(E) = \frac{n(E)}{n(S)} = \frac{35}{36}$$

19. sum is 3 or 5: $E = \{(1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1)\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

21. $P(E) = \dfrac{{}_3C_2}{{}_6C_2} = \dfrac{3}{15} = \dfrac{1}{5}$

23. $P(E) = \dfrac{{}_4C_2}{{}_6C_2} = \dfrac{6}{15} = \dfrac{2}{5}$

25. $P(E') = 1 - P(E) = 1 - 0.7 = 0.3$

27. $P(E') = 1 - P(E) = 1 - \frac{1}{3} = \frac{2}{3}$

29. $P(E) = 1 - P(E') = 1 - p = 1 - 0.15 = 0.85$

31. $P(E) = 1 - P(E') = 1 - \frac{13}{20} = \frac{7}{20}$

33. (a) $0.06(1.3) = 0.078$ or 78,000

(b) $\frac{0.30}{1.00} = 0.3$

(c) $\frac{0.21 + 0.16}{1.00} = 0.37$

(d) $\frac{0.07 + 0.03}{1.00} = 0.10$

35. (a) $\frac{290}{500} = 0.58$

(b) $\frac{478}{500} = 0.956$

(c) $\frac{2}{500} = 0.004$

37. (a) $\frac{672}{1254}$

(b) $\frac{582}{1254}$

(c) $\frac{672 - 124}{1254} = \frac{548}{1254}$

39. $p + p + 2p = 1$

$p = 0.25$

Taylor: $0.50 = \frac{1}{2}$

Moore: $0.25 = \frac{1}{4}$

Perez: $0.25 = \frac{1}{4}$

41. (a) $\frac{{}_{15}C_{10}}{{}_{20}C_{10}} = \frac{3003}{184{,}756} = \frac{21}{1292} \approx 0.016$

(b) $\frac{{}_{15}C_8 \cdot {}_5C_2}{{}_{20}C_{10}} = \frac{64{,}350}{184{,}756} = \frac{225}{646} \approx 0.348$

(c) $\frac{{}_{15}C_9 \cdot {}_5C_1}{{}_{20}C_{10}} + \frac{{}_{15}C_{10}}{{}_{20}C_{10}} = \frac{25{,}025 + 3003}{184{,}756} = \frac{28{,}028}{184{,}756} = \frac{49}{323} \approx 0.152$

43. Total ways to insert letters: $4! = 24$ ways

4 correct: 1 way
3 correct: not possible
2 correct: 6 ways
1 correct: 8 ways
0 correct: 9 ways

(a) $\frac{8}{24} = \frac{1}{3}$

(b) $\frac{8 + 6 + 1}{24} = \frac{15}{24} = \frac{5}{8}$

45. (a) $\frac{1}{{}_5P_5} = \frac{1}{120}$

(b) $\frac{1}{{}_4P_4} = \frac{1}{24}$

47. (a) $\frac{{}_{74}C_8}{{}_{84}C_8} = 0.3457$

(b) $\frac{{}_{20}C_8}{{}_{84}C_8} = 2.89 \times 10^{-6}$

49. (a) $\frac{{}_9C_4}{{}_{12}C_4} = \frac{126}{495} = \frac{14}{55}$ (4 good units)

(b) $\frac{({}_9C_2)({}_3C_2)}{{}_{12}C_4} = \frac{108}{495} = \frac{12}{55}$ (2 good units)

(c) $\frac{({}_9C_3)({}_3C_1)}{{}_{12}C_4} = \frac{252}{495} = \frac{28}{55}$ (3 good units)

At least 2 good units: $\frac{12}{55} + \frac{28}{55} + \frac{14}{55} = \frac{54}{55}$

51. (a) $P(EE) = \frac{15}{30} \cdot \frac{15}{30} = \frac{1}{4}$

(b) $P(EO) + P(OE) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(c) $P(N_1 < 10, N_2 < 10) = \frac{9}{30} \cdot \frac{9}{30} = \frac{9}{100}$

(d) $P(N_1N_1) = \frac{30}{30} \cdot \frac{1}{30} = \frac{1}{30}$

53. (a) $P(SS) = (0.985)^2 \approx 0.9702$

(b) $P(S) = 1 - P(FF) = 1 - (0.015)^2 \approx 0.9998$

(c) $P(FF) = (0.015)^2 \approx 0.0002$

55. (a) $\left(\frac{1}{5}\right)^6 = \frac{1}{15{,}625}$

(b) $\left(\frac{4}{5}\right)^6 = \frac{4096}{15{,}625} = 0.26144$

(c) $1 - 0.262144 = 0.737856 = \frac{11{,}529}{15{,}625}$

57. $(0.32)^2 = 0.1024$

59. $1 - \frac{(45)^2}{(60)^2} = 1 - \left(\frac{45}{60}\right)^2 = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$

61. True. $P(E) + P(E') = 1$

63. (a) As you consider successive people with distinct birthdays, the probabilities must decrease to take into account the birth dates already used. Since the birth dates of people are independent events, multiply the respective probabilities of distinct birthdays.

(b) $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365}$

(c) $P_1 = \frac{365}{365} = 1$

$$P_2 = \frac{365}{365} \cdot \frac{364}{365} = \frac{364}{365} P_1 = \frac{365 - (2 - 1)}{365} P_1$$

$$P_3 = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} = \frac{363}{365} P_2 = \frac{365 - (3 - 1)}{365} P_2$$

$$P_n = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \ldots \cdot \frac{365 - (n - 1)}{365} = \frac{365 - (n - 1)}{365} P_{n-1}$$

(d) Q_n is the probability that the birthdays are *not* distinct which is equivalent to at least 2 people having the same birthday.

(e)

n	10	15	20	23	30	40	50
P_n	0.88	0.75	0.59	0.49	0.29	0.11	0.03
Q_n	0.12	0.25	0.41	0.51	0.71	0.89	0.97

(f) 23, See the chart above.