CHAPTER 5
Analytic Trigonometry

C H A P T E R 5
Analytic Trigonometry

Section 5.1 Using Fundamental Identities

Solutions to Even-Numbered Exercises

2. $\csc\theta = \frac{5}{3}, \tan\theta = \frac{3}{4}$

θ is in Quadrant I.

$\sin\theta = \frac{1}{\csc\theta} = \frac{3}{5}$

$\cos\theta = \frac{\sin\theta}{\tan\theta} = \frac{3}{5} \cdot \frac{4}{3} = \frac{4}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

4. $\tan x = \frac{\sqrt{3}}{3}, \cos x = -\frac{\sqrt{3}}{2}$

x is in Quadrant III.

$\sin x = -\sqrt{1 - \left(-\frac{\sqrt{3}}{2}\right)^2} = -\sqrt{\frac{1}{4}} = -\frac{1}{2}$

$\csc x = \frac{1}{\sin x} = -2$

$\sec x = \frac{1}{\cos x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cot x = \frac{1}{\tan x} = \frac{3}{\sqrt{3}} = \sqrt{3}$

6. $\cot\phi = -5, \sin\phi = \frac{\sqrt{26}}{26}$

ϕ is in Quadrant II

$\cos\phi = \cot\phi \cdot \sin\phi = \frac{-5\sqrt{26}}{26}$

$\tan\phi = \frac{1}{\cot\phi} = -\frac{1}{5}$

$\csc\phi = \frac{1}{\sin\phi} = \frac{26}{\sqrt{26}} = \sqrt{26}$

$\sec\phi = \frac{1}{\cos\phi} = \frac{-26}{5\sqrt{26}} = \frac{-\sqrt{26}}{5}$

8. $\cos\left(\frac{\pi}{2} - x\right) = \frac{3}{5}, \cos x = \frac{4}{5}$

x is in Quadrant I.

$\sin x = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$

$\tan x = \frac{\sin x}{\cos x} = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}$

$\csc x = \frac{1}{\sin x} = \frac{5}{3}$

$\sec x = \frac{1}{\cos x} = \frac{5}{4}$

$x = \frac{1}{\tan x} = \frac{4}{3}$

10. $\csc x = 5, \cos x > 0$

x is in Quadrant I.

$\sin x = \frac{1}{\csc x} = \frac{1}{5}$

$\cos x = \sqrt{1 - \left(\frac{1}{5}\right)^2} = \frac{2\sqrt{6}}{5}$

$\tan x = \frac{\sin x}{\cos x} = \frac{1}{5} \cdot \frac{5}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$

$\sec x = \frac{1}{\cos x} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

$\cot x = \frac{1}{\tan x} = 2\sqrt{6}$

12. $\sec\theta = -3, \tan\theta < 0$

θ is in Quadrant II.

$\cos\theta = \frac{1}{\sec\theta} = -\frac{1}{3}$

$\sin\theta = \sqrt{1 - \left(-\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}}{3} \cdot -\frac{3}{1} = -2\sqrt{2}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$

14. $\tan\theta$ is undefined, $\sin\theta > 0$.

$$\theta = \frac{\pi}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \text{ is undefined} \Longrightarrow \cos\theta = 0$$

$$\sin\theta = \sqrt{1 - 0^2} = 1$$

$$\csc\theta = \frac{1}{\sin\theta} = 1$$

$$\sec\theta = \frac{1}{\cos\theta} \text{ is undefined.}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{0}{1} = 0$$

16. As $x \longrightarrow 0^+$,

$$\cos x \longrightarrow 1 \text{ and } \sec x = \frac{1}{\cos x} \longrightarrow 1.$$

18. As $x \longrightarrow \pi^+$,

$$\sin x \longrightarrow 0 \text{ and } \csc x = \frac{1}{\sin x} \longrightarrow -\infty.$$

20. $\tan x \cos x = \dfrac{\sin x}{\cos x}\cos x = \sin x.$

Matches (f).

22. $(1 - \sin^2 x)\sec x = \cos^2 x \dfrac{1}{\cos x} = \cos x$

Matches (b).

24. $\dfrac{\sin\left[\left(\frac{\pi}{2}\right) - x\right]}{\cos\left[\left(\frac{\pi}{2}\right) - x\right]} = \dfrac{\cos x}{\sin x} = \cot x$

Matches (c).

26.
$$\begin{aligned}\sin^2 x(\csc^2 x - 1) = \sin^2 x \cdot \cot^2 x &= \sin^2 x \frac{\cos^2 x}{\sin^2 x} \\ &= \cos^2 x\end{aligned}$$

Matches (c).

28.
$$\begin{aligned}\cot x \sec x = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} &= \frac{1}{\sin x} \\ &= \csc x\end{aligned}$$

Matches (a).

30.
$$\begin{aligned}\frac{\cos^2\left[\left(\frac{\pi}{2}\right) - x\right]}{\cos x} &= \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x}\sin x \\ &= \tan x \sin x\end{aligned}$$

Matches (d).

32.
$$\begin{aligned}\cos\beta\tan\beta &= \cos\beta\left(\frac{\sin\beta}{\cos\beta}\right) \\ &= \sin\beta\end{aligned}$$

34.
$$\begin{aligned}\sec^2 x(1 - \sin^2 x) &= \sec^2 x - \sec^2 x \sin^2 x \\ &= \sec^2 x - \frac{1}{\cos^2 x} \cdot \sin^2 x \\ &= \sec^2 x - \frac{\sin^2 x}{\cos^2 x} \\ &= \sec^2 x - \tan^2 x \\ &= 1\end{aligned}$$

36. $\dfrac{\sec\theta}{\csc\theta} = \dfrac{1}{\cos\theta}\sin\theta = \tan\theta$

38. $\dfrac{1}{\tan^2 x + 1} = \dfrac{1}{\sec^2 x} = \dfrac{1}{\frac{1}{\cos^2 x}} = \cos^2 x$

40. $\dfrac{\tan^2\theta}{\sec^2\theta} = \dfrac{\sin^2\theta}{\cos^2\theta} \cdot \dfrac{1}{\sec^2\theta} = \dfrac{\sin^2\theta}{\cos^2\theta} \cdot \dfrac{1}{\dfrac{1}{\cos^2\theta}} = \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta} = \sin^2\theta$

42. $\cot\left(\dfrac{\pi}{2} - x\right)\cos x = \tan x \cos x = \dfrac{\sin x}{\cos x} \cdot \cos x = \sin x$

44. $(\cos t)(1 + \tan^2 t) = (\cos t)(\sec^2 t) = \dfrac{\cos t}{\cos^2 t} = \dfrac{1}{\cos t} = \sec t$

46. $\cos\theta\sec\theta - \cos^2\theta = 1 - \cos^2\theta = \sin^2\theta$

48. $\dfrac{\sec^2\theta - \tan^2\theta + \tan\theta}{\sec\theta} = \dfrac{1 + \tan\theta}{\sec\theta} = \dfrac{1}{\sec\theta} + \dfrac{\tan\theta}{\sec\theta}$

$= \cos\theta + \sin\theta$

50. $\dfrac{\tan\theta}{1 + \sec\theta} + \dfrac{1 + \sec\theta}{\tan\theta} = \dfrac{\tan^2\theta + 1 + 2\sec\theta + \sec^2\theta}{(1 + \sec\theta)\tan\theta}$

$= \dfrac{2\sec^2\theta + 2\sec\theta}{(1 + \sec\theta)\tan\theta}$

$= \dfrac{2\sec\theta(\sec\theta + 1)}{(1 + \sec\theta)\tan\theta}$

$= \dfrac{2\sec\theta}{\tan\theta} = 2\csc\theta$

52. $\dfrac{\csc\left(\dfrac{\pi}{2} - \theta\right)}{\tan(-\theta)} = \dfrac{\sec\theta}{-\tan\theta} = -\csc\theta$

54. $(\sec\theta - \tan\theta)(\csc\theta + 1) = \dfrac{1}{\cos\theta}(1 - \sin\theta)\left(\dfrac{1}{\sin\theta} + 1\right)$

$= \dfrac{1}{\cos\theta}(1 - \sin\theta)(1 + \sin\theta)\dfrac{1}{\sin\theta}$

$= \dfrac{1}{\cos\theta \cdot \sin\theta}(1 - \sin^2\theta)$

$= \dfrac{1}{\cos\theta\sin\theta}\cos^2\theta = \cot\theta$

56. $\dfrac{1 + \csc\theta}{\cot\theta + \cos\theta} = \dfrac{1 + \csc\theta}{\cos\theta(\csc\theta + 1)} = \dfrac{1}{\cos\theta} = \sec\theta$

58. $\dfrac{1 + \csc\theta}{\sec\theta} - \cot\theta = \cos\theta + \cot\theta - \cot\theta = \cos\theta$

60. $\dfrac{\sin\theta + \cos\theta}{\sin\theta} - \dfrac{\cos\theta - \sin\theta}{\cos\theta} = 1 + \cot\theta - 1 + \tan\theta = \cot\theta + \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta} = \sec\theta\csc\theta$

62. $\ln|1+\cos\theta| + \ln|1-\cos\theta| = \ln|(1+\cos\theta)(1-\cos\theta)|$
$= \ln|(1-\cos^2\theta)|$
$= \ln|\sin^2\theta|$
$= 2\ln|\sin\theta|$

64. $\sec^2 x\tan^2 x + \sec^2 x = \sec^2 x(\tan^2 x + 1) = \sec^2 x(\sec^2 x) = \sec^4 x$

66. $\dfrac{\csc^2 x - 1}{\csc x - 1} = \dfrac{(\csc x - 1)(\csc x + 1)}{\csc x - 1} = \csc x + 1$

68. $1 - 2\sin^2 x + \sin^4 x = (1-\sin^2 x)^2 = (\cos^2 x)^2 = \cos^4 x$

70. $\sec^3 x - \sec^2 x - \sec x + 1 = \sec^2 x(\sec x - 1) - (\sec x - 1)$
$= (\sec^2 x - 1)(\sec x - 1)$
$= \tan^2 x(\sec x - 1)$

72. $(\cot x + \csc x)(\cot x - \csc x) = \cot^2 x - \csc^2 x$
$= -1$

74. $(3 - 3\sin x)(3 + 3\sin x) = 9 - 9\sin^2 x$
$= 9(1-\sin^2 x)$
$= 9\cos^2 x$

76. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1} = \dfrac{\sec x - 1 - (\sec x + 1)}{(\sec x + 1)(\sec x - 1)}$
$= \dfrac{\sec x - 1 - \sec x - 1}{\sec^2 x - 1}$
$= \dfrac{-2}{\tan^2 x}$
$= -2\left(\dfrac{1}{\tan^2 x}\right)$
$= -2\cot^2 x$

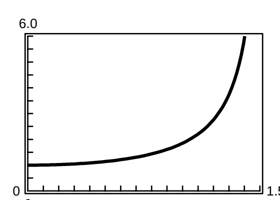
78. $\tan x - \dfrac{\sec^2 x}{\tan x} = \dfrac{\tan^2 x - \sec^2 x}{\tan x}$
$= \dfrac{-1}{\tan x}$
$= -\cot x$

80. $\dfrac{5}{\tan x + \sec x} \cdot \dfrac{\tan x - \sec x}{\tan x - \sec x} = \dfrac{5(\tan x - \sec x)}{\tan^2 x - \sec^2 x}$
$= \dfrac{5(\tan x - \sec x)}{-1}$
$= 5(\sec x - \tan x)$

82. $\dfrac{\tan^2 x}{\csc x + 1} \cdot \dfrac{\csc x - 1}{\csc x - 1} = \dfrac{\tan^2 x(\csc x - 1)}{\csc^2 x - 1}$
$= \dfrac{\tan^2 x(\csc x - 1)}{\cot^2 x}$
$= \tan^2 x(\csc x - 1)\tan^2 x$
$= \tan^4 x(\csc x - 1)$

84. $y_1 = \cos x + \sin x\tan x,\ y_2 = \sec x$

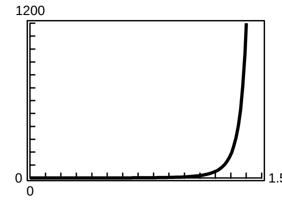
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835
y_2	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835



It appears that $y_1 = y_2$.

86. $y_1 = \sec^4 x - \sec^2, y_2 = \tan^2 x + \tan^4 x$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143
y_2	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143



It appears that $y_1 = y_2$.

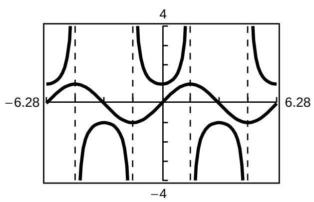
88. $\sin x(\cot x + \tan x) = \cos x + \sin^2 x/\cos x$

$$= (\cos^2 x + \sin^2 x)/\cos x$$

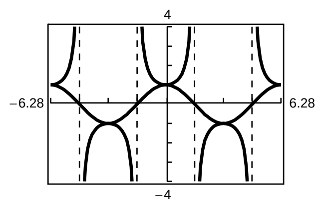
$$= 1/\cos x = \sec x$$

90. $y_1 = \frac{1}{2}\left(\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta}\right)$

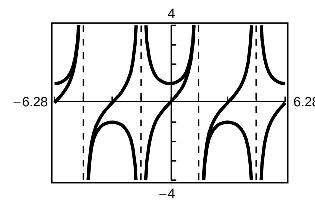
y_1 and $y_2 = \sin\theta$



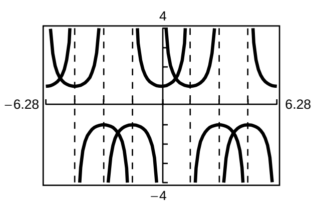
y_1 and $y_2 = \cos\theta$



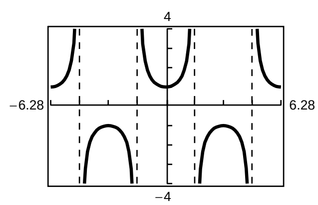
y_1 and $y_2 = \tan\theta$



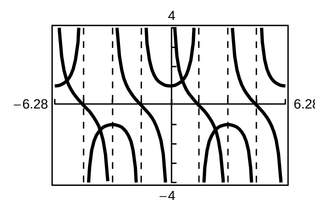
y_1 and $y_2 = \frac{1}{\sin\theta} = \csc\theta$



y_1 and $y_2 = \frac{1}{\cos\theta} = \sec\theta$



y_1 and $y_2 = \frac{1}{\tan\theta} = \cot\theta$



It appears that $\frac{1}{2}\left(\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta}\right) = \sec\theta$.

92. Let $x = 2\sin\theta$.

$$\begin{aligned}\sqrt{16 - 4x^2} &= \sqrt{16 - 4(2\sin\theta)^2}\\ &= \sqrt{16(1 - \sin^2\theta)}\\ &= \sqrt{16\cos^2\theta}\\ &= 4\cos\theta\end{aligned}$$

94. Let $x = 2\sec\theta$.

$$\begin{aligned}\sqrt{x^2 - 4} &= \sqrt{(2\sec\theta)^2 - 4}\\ &= \sqrt{4(\sec^2\theta - 1)}\\ &= \sqrt{4\tan^2\theta}\\ &= 2\tan\theta\end{aligned}$$

96. Let $x = 10\tan\theta$.

$$\begin{aligned}\sqrt{x^2 + 100} &= \sqrt{(10\tan\theta)^2 + 100}\\ &= \sqrt{100(\tan^2\theta + 1)}\\ &= \sqrt{100\sec^2\theta}\\ &= 10\sec\theta\end{aligned}$$

98. $\cos\theta = -\sqrt{1 - \sin^2\theta}$

$$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$$

100. $\tan\theta = \sqrt{\sec^2\theta - 1}$

$$0 \le \theta < \frac{\pi}{2}, \pi \le \theta < \frac{3\pi}{2}$$

102. $\ln|\csc\theta| + \ln|\tan\theta| = \ln|\csc\theta \cdot \tan\theta|$

$$= \ln|\sec\theta|$$

104. $\ln|\cot t| + \ln(1 + \tan^2 t) = \ln\left[|\cot t|(1 + \tan^2 t)\right]$

$$= \ln\frac{(1 + \tan^2 t)}{|\tan t|}$$

$$= \ln\left|\frac{1}{\tan t} + \frac{\tan^2 t}{\tan t}\right|$$

$$= \ln|\cot t + \tan t|$$

106. $\tan^2 \theta + 1 = \sec^2 \theta$

(a) $\theta = 346°$

$(\tan 346°)^2 + 1 \approx 1.0622$

$(\sec 346°)^2 = \left(\dfrac{1}{\cos 346°}\right)^2 \approx 1.0622$

(b) $\theta = 3.1$

$(\tan 3.1)^2 + 1 \approx 1.00173$

$(\sec 3.1)^2 = \left(\dfrac{1}{\cos 3.1}\right)^2 \approx 1.00173$

108. $\sin(-\theta) = -\sin \theta$

(a) $\theta = 250°$

$\sin(-250°) \approx 0.9397$

$-(\sin 250°) \approx 0.9397$

(b) $\theta = \dfrac{1}{2}$

$\sin\left(-\dfrac{1}{2}\right) \approx -0.4794$

$-\left(\sin \dfrac{1}{2}\right) \approx -0.4794$

110. False; $\dfrac{\sin k\theta}{\cos k\theta} = \tan k\theta$

112. False.

$\sin \theta \csc \theta = \sin \theta\left(\dfrac{1}{\sin \theta}\right) = 1,$

provided $\sin \theta \neq 0$.

False for $\theta = 0$.

114. Since $\sin^2 \theta + \cos^2 \theta = 1$ and $\cos^2 \theta = 1 - \sin^2 \theta$:

$\cos \theta = \pm\sqrt{1 - \sin^2 \theta}$

$\tan \theta = \dfrac{\sin \theta}{\cos \theta} = \pm\dfrac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$

$\cot \theta = \dfrac{1}{\tan \theta} = \pm\dfrac{\sqrt{1 - \sin^2 \theta}}{\sin \theta}$

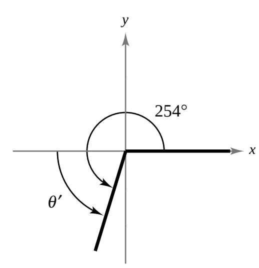
$\sec \theta = \dfrac{1}{\cos \theta} = \pm\dfrac{1}{\sqrt{1 - \sin^2 \theta}}$

$\csc \theta = \dfrac{1}{\sin \theta}$

The sign depends on the choice of θ.

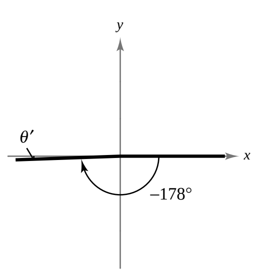
116. $\theta = 254°$

$\theta' = 254° - 180° = 74°$



118. $\theta = -178°$ is coterminal with $182°$

$\theta' = 182° - 180° = 2°$



120. $\theta = \dfrac{13\pi}{15}$

$\theta' = \pi - \dfrac{13\pi}{15} = \dfrac{2\pi}{15}$