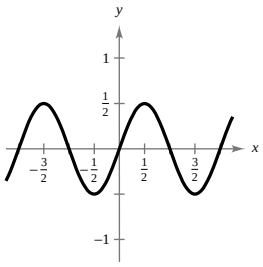
122. $f(x) = \frac{1}{2}\sin \pi x$

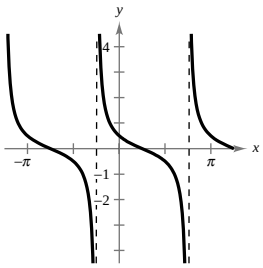
Amplitude: $\frac{1}{2}$

Period: 2



124. $f(x) = \frac{1}{2}\cot\left(x + \frac{\pi}{4}\right)$

Asymptotes: $x = -\frac{\pi}{4}, \frac{3\pi}{4}$



126. $A = 90° - B = 90° - 80° = 10°$

$$\sin A = \frac{a}{c} \implies c = \frac{a}{\sin A} = \frac{16}{\sin 10°} \approx 92.14$$

$$\tan B = \frac{b}{a} \implies b = a \tan B = 16 \tan 80° \approx 90.74$$

128. $c = \sqrt{a^2 + b^2} = \sqrt{14^2 + 8^2} = \sqrt{260} = 2\sqrt{65} \approx 16.12$

$$\sin A = \frac{a}{c} = \frac{14}{2\sqrt{65}} \implies A \approx 60.26°$$

$$\sin B = \frac{b}{c} = \frac{8}{2\sqrt{65}} \implies B \approx 29.74°$$

Section 5.2 Verifying Trigonometric Identities

Solutions to Even-Numbered Exercises

2. $\tan y \cot y = \tan y\left(\frac{1}{\tan y}\right) = 1$

4. $\cot^2 y(\sec^2 y - 1) = \cot^2 y \tan^2 y = 1$

6. $\cos^2 \beta - \sin^2 \beta = \cos^2 \beta - (1 - \cos^2 \beta)$
$= 2\cos^2 \beta - 1$

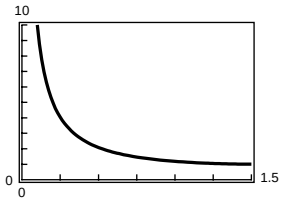
8. $2 - \csc^2 z = 2 - (\cot^2 z + 1) = 1 - \cot^2 z$

10. $\cos t(\csc^2 t - 1) = \cos t \cot^2 t$
$= \sin t\left(\frac{\cos t}{\sin t}\right)\cot^2 t$
$= \frac{1}{\csc t}\cot^3 t$
$= \frac{\cot^3 t}{\csc t}$

12.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148
y_2	5.0355	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148

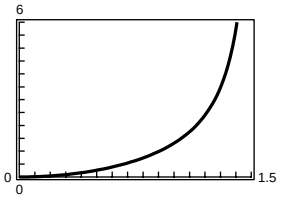
$$y_1 = \frac{\csc x - 1}{1 - \sin x} = \frac{\dfrac{1}{\sin x} - 1}{1 - \sin x}$$
$$= \frac{1 - \sin x}{\sin x}\,\frac{1}{1 - \sin x}$$
$$= \frac{1}{\sin x}$$
$$= \csc x$$
$$= y_2$$



14.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135
y_2	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135

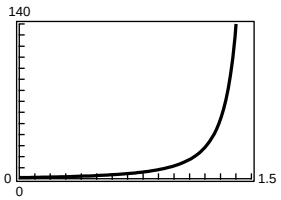
$$y_1 = \sec x - \cos x = \frac{1}{\cos x} - \cos x$$
$$= \frac{1 - \cos^2 x}{\cos x}$$
$$= \frac{\sin^2 x}{\cos x}$$
$$= \sin x\left(\frac{\sin x}{\cos x}\right)$$
$$= \sin x \tan x$$
$$= y_2$$



16.

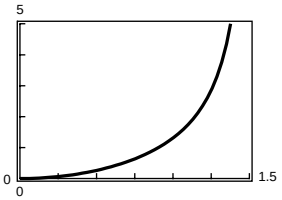
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.4958	2.2756	3.5939	6.0760	11.6160	28.4287	136.4545
y_2	1.4958	2.2756	3.5939	6.0760	11.6160	28.4287	136.4545

$$y_1 = \frac{\sec x + \tan x}{\sec x - \tan x} = \frac{(\sec x + \tan x)^2}{(\sec x - \tan x)(\sec x + \tan x)}$$
$$= \frac{(\sec x + \tan x)^2}{\sec^2 x - \tan^2 x}$$
$$= (\sec x + \tan x)^2$$
$$= y_2$$



18.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135
y_2	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135



$$y_1 = \frac{1}{\cos x} - \frac{1}{\sec x} = \sec x - \cos x = y_2$$

20. There are two errors in line 1:

$\sec(-\theta) = \sec\theta$ and $\sin(-\theta) = -\sin\theta$

22. Line 2: $\dfrac{\dfrac{\sin^2 x - \cos^2 x}{\cos x \sin x}}{\dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}}$

Line 5: $(1 - \cos^2 x) - \cos^2 x$

24.
$$\begin{aligned}\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) &= \sec^4 x(\sec x \tan x)(\sec^2 x - 1)\\ &= \sec^4 x(\sec x \tan x)\tan^2 x\\ &= \sec^5 x \tan^3 x\end{aligned}$$

26.
$$\frac{\sin\left[\left(\frac{\pi}{2}\right) - x\right]}{\cos\left[\left(\frac{\pi}{2}\right) - x\right]} = \frac{\cos x}{\sin x} = \cot x$$

28.
$$\begin{aligned}(1 + \sin y)[1 + \sin(-y)] &= (1 + \sin y)(1 - \sin y)\\ &= 1 - \sin^2 y\\ &= \cos^2 y\end{aligned}$$

30.
$$\begin{aligned}\frac{1 + \csc(-\theta)}{\cos(-\theta) + \cot(-\theta)} &= \frac{1 - \csc\theta}{\cos\theta - \cot\theta}\\ &= \frac{1 - \csc\theta}{\cos\theta\left(1 - \frac{1}{\sin\theta}\right)}\\ &= \frac{1 - \csc\theta}{\cos\theta(1 - \csc\theta)}\\ &= \frac{1}{\cos\theta}\\ &= \sec\theta\end{aligned}$$

32.
$$\begin{aligned}\frac{\tan x + \tan y}{1 - \tan x \tan y} &= \frac{\frac{1}{\cot x} + \frac{1}{\cot y}}{1 - \frac{1}{\cot x}\cdot\frac{1}{\cot y}} \cdot \frac{\cot x \cot y}{\cot x \cot y}\\ &= \frac{\cot y + \cot x}{\cot x \cot y - 1}\end{aligned}$$

34.
$$\begin{aligned}\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} &= \frac{(\cos x - \cos y)(\cos x + \cos y) + (\sin x - \sin y)(\sin x + \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}\\ &= \frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}\\ &= \frac{(\cos^2 x + \sin^2 x) - (\cos^2 y + \sin^2 y)}{(\sin x + \sin y)(\cos x + \cos y)}\\ &= 0\end{aligned}$$

36. $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}\cdot\dfrac{1-\cos\theta}{1-\cos\theta}}$

$= \sqrt{\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\dfrac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \dfrac{1-\cos\theta}{|\sin\theta|}$

38. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = \sec^2 y - \tan^2 y = 1$

40. $\csc^2\left(\dfrac{\pi}{2} - x\right) - 1 = \sec^2 x - 1 = \tan^2 x$

42. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x - \csc x \sin x + 1 - \dfrac{\cos x}{\sin x} + \cot x$

$= \csc^2 x - 1 + 1 - \cot x + \cot x$

$= \csc^2 x$

44. $4\tan^4 x + \tan^2 x - 3 = (\tan^2 x + 1)(4\tan^2 x - 3)$

$= \sec^2 x(4\tan^2 x - 3)$

46. $\sin x(1 - 2\cos^2 x + \cos^4 x) = \sin x(1 - \cos^2 x)^2$

$= \sin x(\sin^2 x)^2$

$= \sin^5 x$

48. $\csc^4\theta - \cot^4\theta = (\csc^2\theta - \cot^2\theta)(\csc^2\theta + \cot^2\theta)$

$= \csc^2\theta + \cot^2\theta$

$= \csc^2\theta + (\csc^2\theta - 1)$

$= 2\csc^2\theta - 1$

50. $\dfrac{\cot\alpha}{\csc\alpha - 1}\cdot\dfrac{\csc\alpha + 1}{\csc\alpha + 1} = \dfrac{\cot\alpha(\csc\alpha + 1)}{\csc^2\alpha - 1}$

$= \dfrac{\cot\alpha(\csc\alpha + 1)}{\cot^2\alpha}$

$= \dfrac{\csc\alpha + 1}{\cot\alpha}$

52. $\dfrac{\sin^3\beta + \cos^3\beta}{\sin\beta + \cos\beta} = \dfrac{(\sin\beta + \cos\beta)(\sin^2\beta - \sin\beta\cos\beta + \cos^2\beta)}{\sin\beta + \cos\beta}$

$= \sin^2\beta + \cos^2\beta - \sin\beta\cos\beta$

$= 1 - \sin\beta\cos\beta$

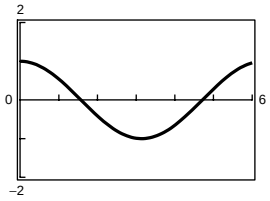
54. The function appears to be $y = \cos x$. Analytically,

$y = \dfrac{\cos x}{1 - \tan x} + \dfrac{\sin x \cdot \cos x}{\sin x - \cos x}$

$= \dfrac{\cos x}{1 - (\sin x/\cos x)} + \dfrac{\sin x\cos x}{\sin x - \cos x}$

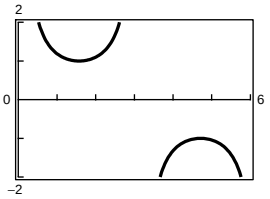
$= \dfrac{\cos^2 x}{\cos x - \sin x} - \dfrac{\sin x\cos x}{\cos x - \sin x}$

$= \dfrac{\cos x(\cos x - \sin x)}{\cos x - \sin x} = \cos x.$



56. The function appears to be $y = \csc t$. Analytically,

$$\begin{aligned} y &= \sin t + \frac{\cot^2 t}{\csc t} \\ &= \frac{1 + \cot^2 t}{\csc t} \\ &= \frac{\csc^2 t}{\csc t} = \csc t. \end{aligned}$$



58. $\ln|\sec\theta| = \ln\left|\dfrac{1}{\cos\theta}\right| = \ln|\cos\theta|^{-1} = -\ln|\cos\theta|$

60.
$$\begin{aligned} -\ln|\csc\theta + \cot\theta| &= -\ln\left|\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right| \\ &= \ln\left|\frac{1 + \cos\theta}{\sin\theta}\right|^{-1} \\ &= \ln\left|\frac{\sin\theta}{1 + \cos\theta}\right| \\ &= \ln\left|\frac{\sin\theta}{1 + \cos\theta} \cdot \frac{1 - \cos\theta}{1 - \cos\theta}\right| \\ &= \ln\left|\frac{\sin\theta(1 - \cos\theta)}{1 - \cos^2\theta}\right| \\ &= \ln\left|\frac{\sin\theta(1 - \cos\theta)}{\sin^2\theta}\right| \\ &= \ln\left|\frac{1 - \cos\theta}{\sin\theta}\right| \\ &= \ln|\csc\theta - \cot\theta| \end{aligned}$$

62.
$$\begin{aligned} \cos^2 14^\circ + \cos^2 76^\circ &= \sin^2(90^\circ - 14^\circ) + \cos^2 76^\circ \\ &= \sin^2 76^\circ + \cos^2 76^\circ = 1 \end{aligned}$$

64.
$$\begin{aligned} \sin^2 12^\circ + \sin^2 40^\circ + \sin^2 50^\circ + \sin^2 78^\circ &= \sin^2 12^\circ + \sin^2 78^\circ + \sin^2 40^\circ + \sin^2 50^\circ \\ &= \cos^2(90^\circ - 12^\circ) + \sin^2 78^\circ + \cos^2(90^\circ - 40^\circ) + \sin^2 50^\circ \\ &= \cos^2 78^\circ + \sin^2 78^\circ + \cos^2 50^\circ + \sin^2 50^\circ \\ &= 1 + 1 = 2 \end{aligned}$$

66.
$$\begin{aligned} \sec^4 x(\sec x \tan x) - \sec^2 x(\sec x \tan x) &= \sec^2 x(\sec x \tan x)(\sec^2 x - 1) \\ &= \sec^2 x(\sec x \tan x)\tan^2 x \\ &= \sec^3 x \tan^3 x \end{aligned}$$

68.
$$\begin{aligned} 1 - 2\cos^2 x + 2\cos^4 x &= [1 - 2\cos^2 x + \cos^4 x] + \cos^4 x \\ &= (1 - \cos^2 x)^2 + \cos^4 x \\ &= \sin^4 x + \cos^4 x \end{aligned}$$

70. $s = \dfrac{h\sin(90^\circ - \theta)}{\sin\theta} = h\dfrac{\cos\theta}{\sin\theta} = h\cot\theta$

72. true

74. False. For example, $\ln\cos 0 = \ln 1 = 0$ whereas $\cos(\ln 0)$ is not defined.

76. $\sqrt{\tan^2\theta} = |\tan\theta|$

$\sqrt{\tan^2\theta} \neq \tan\theta$ if θ lies in Quadrant II or IV.

One such angle is $\theta = \dfrac{3\pi}{4}$.