127. $\sin A = \dfrac{a}{c} \Rightarrow a = c \cdot \sin A = 20 \sin 28° \approx 9.39$

$B = 90° - A° = 62°$

$\cos A = \dfrac{b}{c} \Rightarrow b = c \cdot \cos A \approx 17.66$

129. $a = \sqrt{c^2 - b^2} = \sqrt{12.54^2 - 6.2^2} \approx 10.90$

$\sin B = \dfrac{b}{c} = \dfrac{6.2}{12.54} \Rightarrow B \approx 29.63°$

$A = 90° - 29.63° = 60.37°$

Section 5.2 Verifying Trigonometric Identities

- You should know the difference between an expression, a conditional equation, and an identity.
- You should be able to solve trigonometric identities, using the following techniques.
  (a) Work with *one* side at a time. Do not "cross" the equal sign.
  (b) Use algebraic techniques such as combining fractions, factoring expressions, rationalizing denominators, and squaring binomials.
  (c) Use the fundamental identities.
  (d) Convert all the terms into sines and cosines.

Solutions to Odd-Numbered Exercises

1. $\sin t \csc t = \sin t\left(\dfrac{1}{\sin t}\right) = 1$

3. $\dfrac{\csc^2 x}{\cot x} = \dfrac{1}{\sin^2 x} \cdot \dfrac{\sin x}{\cos x} = \dfrac{1}{\sin x \cdot \cos x}$

$= \csc x \cdot \sec x$

5. $\cos^2 \beta - \sin^2 \beta = (1 - \sin^2 \beta) - \sin^2 \beta$

$= 1 - 2 \sin^2 \beta$

7. $\tan^2 \theta + 6 = (\sec^2 \theta - 1) + 6$

$= \sec^2 \theta + 5$

9. $\cos x + \sin x \tan x = \cos x + \sin x \cdot \dfrac{\sin x}{\cos x}$

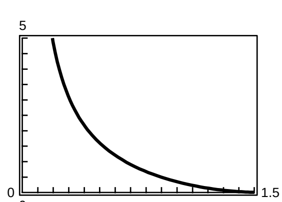
$= \dfrac{\cos^2 x + \sin^2 x}{\cos x}$

$= \dfrac{1}{\cos x}$

$= \sec x$

11.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.835	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.835	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293



$\dfrac{1}{\sec x \tan x} = \cos x \cdot \dfrac{\cos x}{\sin x}$

$= \dfrac{\cos^2 x}{\sin x}$

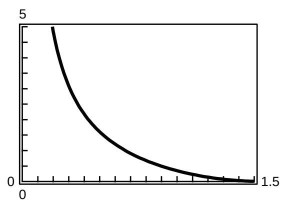
$= \dfrac{1 - \sin^2 x}{\sin x}$

$= \dfrac{1}{\sin x} - \sin x$

$= \csc x - \sin x$

13.

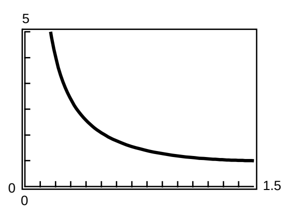
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.835	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.835	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293



$$\begin{aligned}\csc x - \sin x &= \frac{1}{\sin x} - \sin x\\ &= \frac{1 - \sin^2 x}{\sin x}\\ &= \frac{\cos^2 x}{\sin x}\\ &= \cos x \cdot \frac{\cos x}{\sin x}\\ &= \cos x \cdot \cot x\end{aligned}$$

15.

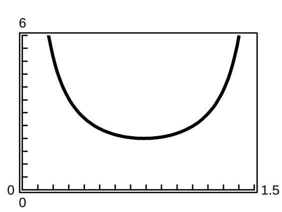
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148
y_2	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148



$$\begin{aligned}\sin x + \cos x \cot x &= \sin x + \cos x \frac{\cos x}{\sin x}\\ &= \frac{\sin^2 x + \cos^2 x}{\sin x}\\ &= \frac{1}{\sin x}\\ &= \csc x\end{aligned}$$

17.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.1359	2.7880	2.1458	2.0009	2.1995	2.9609	5.9704
y_2	5.1359	2.7880	2.1458	2.0009	2.1995	2.9609	5.9704



$$\begin{aligned}\frac{1}{\tan x} + \frac{1}{\cot x} &= \frac{\cot x + \tan x}{\tan x \cdot \cot x}\\ &= \cot x + \tan x\end{aligned}$$

19. The error is in line 1: $\cot(-x) \neq \cot x$.

21. Missing step: $(\sec^2 x - 1)^2 = (\tan^2 x)^2 = \tan^4 x$

23. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \sin^{1/2} x \cos x(1 - \sin^2 x) = \sin^{1/2} x \cos x \cdot \cos^2 x = \cos^3 x\sqrt{\sin x}$

25.
$$\begin{aligned}\tan\left(\frac{\pi}{2} - x\right)\sec x &= \cot x \cdot \sec x\\ &= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}\\ &= \frac{1}{\sin x}\\ &= \csc x\end{aligned}$$

27. $$\frac{\sec(-x)}{\csc(-x)} = \frac{\dfrac{1}{\cos(-x)}}{\dfrac{1}{\sin(-x)}} = \frac{\sin(-x)}{\cos(-x)} = -\frac{\sin x}{\cos x} = -\tan x$$

29. $\dfrac{\cos(-\theta)}{1+\sin(-\theta)} = \dfrac{\cos\theta}{1-\sin\theta}\cdot\dfrac{1+\sin\theta}{1+\sin\theta}$

$= \dfrac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$= \dfrac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}$

$= \sec\theta+\tan\theta$

31. $\dfrac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y} = \dfrac{\dfrac{\sin x\cos y}{\cos x\cos y}+\dfrac{\cos x\sin y}{\cos x\cos y}}{\dfrac{\cos x\cos y}{\cos x\cos y}-\dfrac{\sin x\sin y}{\cos x\cos y}} = \dfrac{\tan x+\tan y}{1-\tan x\tan y}$

33. $\dfrac{\tan x+\cot y}{\tan x\cot y} = \dfrac{\dfrac{1}{\cot x}+\dfrac{1}{\tan y}}{\dfrac{1}{\cot x}\cdot\dfrac{1}{\tan y}} = \dfrac{\dfrac{\tan y+\cot x}{\cot x\cdot\tan y}}{\dfrac{1}{\cot x\cdot\tan y}} = \tan y+\cot x$

35. $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}\cdot\dfrac{1+\sin\theta}{1+\sin\theta}}$

$= \sqrt{\dfrac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\dfrac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \dfrac{1+\sin\theta}{|\cos\theta|}$

Note: Check your answer with a graphing utility. What happens if you leave off the absolute value?

37. $\cos^2 x+\cos^2\left(\dfrac{\pi}{2}-x\right) = \cos^2 x+\sin^2 x = 1$

39. $\sec x\cdot\sin\left(\dfrac{\pi}{2}-x\right) = \sec x\cdot\cos x = 1$

41. $2\sec^2 x-2\sec^2 x\sin^2 x-\sin^2 x-\cos^2 x = 2\sec^2 x(1-\sin^2 x)-(\sin^2 x+\cos^2 x)$

$= 2\sec^2 x(\cos^2 x)-1$

$= 2\cdot\dfrac{1}{\cos^2 x}\cdot\cos^2 x-1$

$= 2-1$

$= 1$

43. $2+\cos^2 x-3\cos^4 x = (1-\cos^2 x)(2+3\cos^2 x)$

$= \sin^2 x(2+3\cos^2 x)$

45. $\csc^4 x - 2\csc^2 x + 1 = (\csc^2 x - 1)^2$

$= (\cot^2 x)^2 = \cot^4 x$

47. $\sec^4 \theta - \tan^4 \theta = (\sec^2 \theta + \tan^2 \theta)(\sec^2 \theta - \tan^2 \theta)$

$= (1 + \tan^2 \theta + \tan^2 \theta)(1)$

$= 1 + 2\tan^2 \theta$

49. $\dfrac{\sin \beta}{1 - \cos \beta} \cdot \dfrac{1 + \cos \beta}{1 + \cos \beta} = \dfrac{\sin \beta(1 + \cos \beta)}{1 - \cos^2 \beta}$

$= \dfrac{\sin \beta(1 + \cos \beta)}{\sin^2 \beta} = \dfrac{1 + \cos \beta}{\sin \beta}$

51. $\dfrac{\tan^3 \alpha - 1}{\tan \alpha - 1} = \dfrac{(\tan \alpha - 1)(\tan^2 \alpha + \tan \alpha + 1)}{\tan \alpha - 1} = \tan^2 \alpha + \tan \alpha + 1$

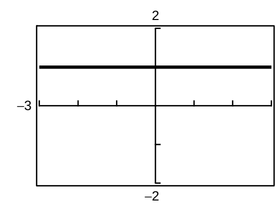
53. It appears that $y_1 = 1$. Analytically,

$\dfrac{1}{\cot x + 1} + \dfrac{1}{\tan x + 1} = \dfrac{\tan x + 1 + \cot x + 1}{(\cot x + 1)(\tan x + 1)}$

$= \dfrac{\tan x + \cot x + 2}{\cot x \tan x + \cot x + \tan x + 1}$

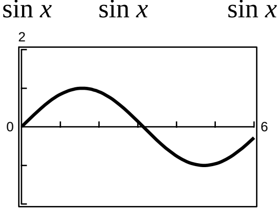
$= \dfrac{\tan x + \cot x + 2}{\tan x + \cot x + 2}$

$= 1.$



55. It appears that $y_1 = \sin x$. Analytically,

$\dfrac{1}{\sin x} - \dfrac{\cos^2 x}{\sin x} = \dfrac{1 - \cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} = \sin x.$



57. $\ln|\cot \theta| = \ln\left|\dfrac{\cos \theta}{\sin \theta}\right|$

$= \ln\dfrac{|\cos \theta|}{|\sin \theta|}$

$= \ln|\cos \theta| - \ln|\sin \theta|$

59. $-\ln(1 + \cos \theta) = \ln(1 + \cos \theta)^{-1}$

$= \left[\ln \dfrac{1}{1 + \cos \theta} \cdot \dfrac{1 - \cos \theta}{1 - \cos \theta}\right]$

$= \ln \dfrac{1 - \cos \theta}{1 - \cos^2 \theta}$

$= \ln \dfrac{1 - \cos \theta}{\sin^2 \theta}$

$= \ln(1 - \cos \theta) - \ln \sin^2 \theta$

$= \ln(1 - \cos \theta) - 2\ln|\sin \theta|$

61. $\sin^2 25 + \sin^2 65 = \sin^2 25 + \cos^2 25° = 1$

63. $\cos^2 20° + \cos^2 52° + \cos^2 38° + \cos^2 70° = \cos^2 20° + \cos^2 52^2 + \sin^2(90° - 38°) + \sin^2(90° - 70°)$
$= \cos^2 20° + \cos^2 52^2 + \sin^2 52° + \sin^2 20°$
$= (\cos^2 20° + \sin^2 20°) + (\cos^2 52° + \sin^2 52°)$
$= 1 + 1$
$= 2$

65. $\tan^5 x = \tan^3 x \cdot \tan^2 x$
$= \tan^3 x(\sec^2 x - 1)$
$= \tan^3 x \sec^2 x - \tan^3 x$

67. $(\sin^2 x - \sin^4 x)\cos x = \sin^2 x(1 - \sin^2 x)\cos x$
$= \sin^2 x \cdot \cos^2 x \cdot \cos x$
$= \cos^3 x \sin^2 x$

69. $\mu W \cos\theta = W \sin\theta$

$$\mu = \frac{W\sin\theta}{W\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta, W \neq 0$$

71. $\cos x - \csc x \cdot \cot x = \cos x - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$
$= \cos x\left[1 - \frac{1}{\sin^2 x}\right]$
$= \cos x(1 - \csc^2 x)$
$= \cos x(-\cot^2 x)$
$= -\cos x \cdot \cot^2 x$

73. True. $f(x) = \cos x$ and $g(x) = \sec x$ are even

75. False. For example, $\sin(1^2) \neq \sin^2(1)$

77. $\sin\theta = \sqrt{1 - \cos^2\theta}$.
True identity is $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$.
For example, $\sin\theta \neq \sqrt{1 - \cos^2\theta}$ for $\theta = \frac{3\pi}{2}$:
$\sin\left(\frac{3\pi}{2}\right) = -1 \neq \sqrt{1 - 0} = 1$

79. $\sqrt{\sin^2 x + \cos^2 x} \neq \sin x + \cos x$
The left side is 1 for any x, but the right side is not necessarily 1. For example, the equation is not true for $x = \pi/4$.

81. $\sin\left[\frac{(12n + 1)\pi}{6}\right] = \sin\left[\frac{1}{6}(12n\pi + \pi)\right]$
$= \sin\left(2n\pi + \frac{\pi}{6}\right)$
$= \sin\frac{\pi}{6} = \frac{1}{2}$

Thus, $\sin\left[\frac{(12n + 1)\pi}{6}\right] = \frac{1}{2}$ for all integers n.

83. $(x - i)(x + i)(x - 4i)(x + 4i) = (x^2 + 1)(x^2 + 16)$
$= x^4 + 17x^2 + 16$

85. $x^2(x - 2)(x - (1 - i))(x - (1 + i)) = (x^3 - 2x^2)((x - 1)^2 + 1)$
$= (x^3 - 2x^2)(x^2 - 2x + 2)$
$= x^5 - 4x^4 + 6x^3 - 4x^2$