78. $|\tan\theta| = \sqrt{\sec^2\theta - 1}$.

One such value is $x = \dfrac{3\pi}{4}$.

80. When n is even,

$$\cos\left[\frac{(2n+1)\pi}{2}\right] = \cos\frac{\pi}{2} = 0.$$

When n is odd,

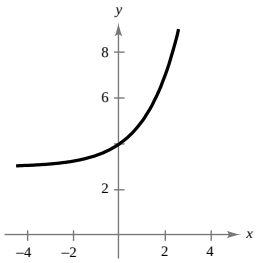
$$\cos\left[\frac{(2n+1)\pi}{2}\right] = \cos\frac{3\pi}{2} = 0.$$

Thus, $\cos\left[\dfrac{(2n+1)\pi}{2}\right] = 0$ for all integers n.

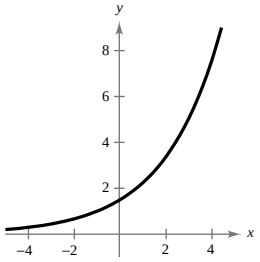
82. $(x-1)(x-8i)(x+8i) = (x-1)(x^2+64) = x^3 - x^2 + 64x - 64$

84.
$$\begin{aligned}(x-4)[x-(6+i)][x-(6-i)] &= (x-4)[(x-6)-i][(x-6)+i] \\ &= (x-4)[(x-6)^2+1] \\ &= (x-4)(x^2-12x+37) \\ &= x^3 - 16x^2 + 85x - 148\end{aligned}$$

86. $f(x) = 2^x + 3$



88. $f(x) = \left(\frac{3}{2}\right)^{x+1}$



90. $s = r\theta \implies \theta = \dfrac{s}{r} = \dfrac{9}{7} \approx 1.286$ radians

92. $\csc\theta > 0$ and $\tan\theta < 0 \implies$ Quadrant II

94. $\sec\theta > 0$ and $\sin\theta < 0 \implies$ Quadrant IV

Section 5.3 Solving Trigonometric Equations

Solutions to Even-Numbered Exercises

2. $\csc x - 2 = 0$

(a) $x = \dfrac{\pi}{6}$

$$\csc\frac{\pi}{6} - 2 = \frac{1}{\sin(\pi/6)} - 2 = 2 - 2 = 0$$

(b) $x = \dfrac{5\pi}{6}$

$$\csc\frac{5\pi}{6} - 2 = \frac{1}{\sin(5\pi/6)} - 2 = 2 - 2 = 0$$

4. $4\cos^2 2x - 2 = 0$

(a) $x = \dfrac{\pi}{8}$: $4\cos^2\left(2 \cdot \dfrac{\pi}{8}\right) - 2 = 4\cos^2\left(\dfrac{\pi}{4}\right) - 2 = 4\left(\dfrac{\sqrt{2}}{2}\right)^2 - 2 = 0$

(b) $x = \dfrac{7\pi}{8}$: $4\cos^2\left(2 \cdot \dfrac{7\pi}{8}\right) - 2 = 4\cos^2\left(\dfrac{7\pi}{4}\right) - 2 = 4\left(\dfrac{\sqrt{2}}{2}\right)^2 - 2 = 0$

6. $\sec^4 x - 3\sec^2 x - 4 = 0$

(a) $x = \dfrac{2\pi}{3}$ $\sec\dfrac{2\pi}{3} = -2$ and $\sec^4 x - 3\sec^2 x - 4 = (-2)^4 - 3(-2)^2 - 4 = 0$

(b) $x = \dfrac{5\pi}{3}$ $\sec\dfrac{5\pi}{3} = 2$ and $\sec^4 x - 3\sec^2 x - 4 = 2^4 - 3(2)^2 - 4 = 0$

8. $y = \sin \pi x + \cos \pi x$

$$\sin \pi x + \cos \pi x = 0$$
$$\cos \pi x = -\sin \pi x$$
$$1 = \frac{-\sin \pi x}{\cos \pi x}$$
$$1 = -\tan \pi x$$
$$-1 = \tan \pi x$$
$$\pi x = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$$
$$x = -\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{11}{4}$$

10. $y = \sec^4\left(\dfrac{\pi x}{8}\right) - 4$

$$\sec^4\left(\frac{\pi x}{8}\right) - 4 = 0$$
$$\frac{1}{\cos^4(\pi x/8)} + \ = 4$$
$$\cos^4\left(\frac{\pi x}{8}\right) = \frac{1}{4}$$
$$\cos\left(\frac{\pi x}{8}\right) = \sqrt[4]{\frac{1}{4}}$$
$$\cos\left(\frac{\pi x}{8}\right) = \frac{\sqrt{2}}{2}$$
$$\frac{\pi x}{8} = -\frac{\pi}{4}, \frac{\pi}{4}$$
$$x = -2, 2$$

12. $\sqrt{2}\sin x + 1 = 0$

$$\sin x = -\frac{1}{\sqrt{2}}$$
$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

14. $\cot x + 1 = 0$

$$\cot x = -1$$
$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

16. $\csc^2 x - 2 = 0$

$$\csc x = \pm\sqrt{2}$$
$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

18. $\tan^2 3x = 3$

$$\tan 3x = \pm\sqrt{3}$$
$$3x = \frac{\pi}{3} + n\pi \quad \text{or} \quad 3x = \frac{2\pi}{3} + n\pi$$
$$x = \frac{\pi}{9} + \frac{n\pi}{3} \quad \text{or} \quad x = \frac{2\pi}{9} + \frac{n\pi}{3}$$
$$x = \frac{\pi + 3n\pi}{9} \quad \text{or} \quad x = \frac{2\pi + 3n\pi}{9}$$
$$x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$$

(12 solutions)

20. $\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = 0$

22. $\tan 3x(\tan x - 1) = 0$

$\tan 3x = 0$ or $\tan x - 1 = 0$

$3x = n\pi$ $\tan x = 1$

$x = \frac{n\pi}{3}$ $x = \frac{\pi}{4} + n\pi$

$0, x = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{4}$

24. $\cos 2x(2\cos x + 1) = 0$

$\cos 2x = 0$ or $2\cos x + 1 = 0$

$2x = \frac{\pi}{2} + n\pi$ $\cos x = -\frac{1}{2}$

$x = \frac{\pi}{4} + \frac{n\pi}{2}$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

26. $\tan^2 x - 1 = 0$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

28. $2\sin^2 x = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$ or $2\cos x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

30. $\sec x \csc x = 2\csc x$

$\sec x \csc x - 2\csc x = 0$

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ or $\sec x - 2 = 0$

No solution $\sec x = 2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

32. $\sin 2x = -\frac{\sqrt{3}}{2}$

$2x = \frac{4\pi}{3} + 2n\pi$ or $2x = \frac{5\pi}{3} + 2n\pi$

$x = \frac{2\pi}{3} + n\pi$ $x = \frac{5\pi}{6} + n\pi$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$ $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

34. $\tan 3x = 1$

$3x = \frac{\pi}{4} + 2n\pi$ or $3x = \frac{5\pi}{4} + 2n\pi$

$x = \frac{\pi}{12} + \frac{2n\pi}{3}$ $x = \frac{5\pi}{12} + \frac{2n\pi}{3}$

$x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{17\pi}{12}$ $x = \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{4}$

36. $\sec 4x = 2$

$4x = \frac{\pi}{3} + 2n\pi$ or $4x = \frac{5\pi}{3} + 2n\pi$

$x = \frac{\pi}{12} + \frac{n\pi}{2}$ $x = \frac{5\pi}{12} + \frac{n\pi}{2}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$ $x = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$

38. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0$ or $\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$ $\sin x = -1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $x = \frac{3\pi}{2}$

40. $\cos x + \sin x \tan x = 2$

$$\cos x + \sin x\left(\frac{\sin x}{\cos x}\right) = 2$$

$$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$$

$$\frac{1}{\cos x} = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

42. $\csc^2 x - 4\cot x = -2$

$(1 + \cot^2 x) - 4\cot x = -2$

$\cot^2 x - 4\cot x + 3 = 0$

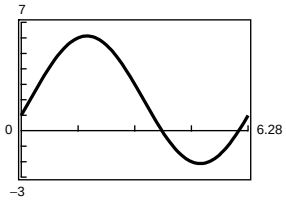
$(\cot x - 3)(\cot x - 1) = 0$

$\cot x = 3$ or $\cot x = 1$

$x \approx 0.3218, 3.4633$ or

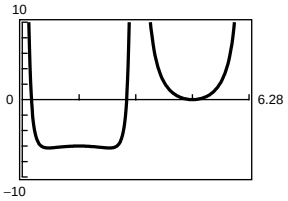
$x = \frac{\pi}{4}, \frac{5\pi}{4}$

44. $y = 4\sin x - \cos x + 2$



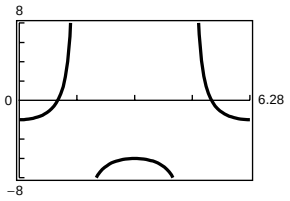
$x \approx 3.8930, 6.0217$

46. $y = \dfrac{1}{\sin^2 x} - \dfrac{3}{\sin x} - 4$



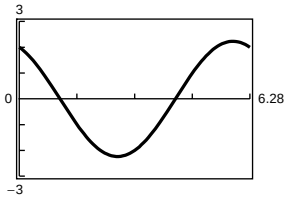
$x \approx 0.2527, 2.8889, 4.7124$

48. $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} - 4 = 0$



$x \approx 1.0472, 5.2360$

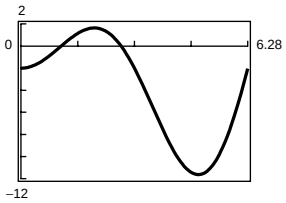
50. $2\cos x - \sin x = 0$



$x \approx 1.1071, 4.2487$

52. $2x \sin x - 2 = 0$

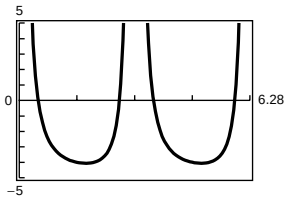
$y = 2x \sin x - 2$



$x \approx 1.1142, 2.7726$

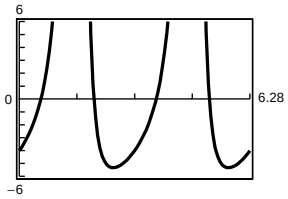
54. $\csc^2 x + 0.5\cot x - 5 = 0$

$y_1 = \left(\dfrac{1}{\sin x}\right)^2 + \dfrac{1}{2\tan x} - 5$



$x \approx 0.5153, 2.7259, 3.6569, 5.8675$

56. $3\tan^2 x + 4\tan x - 4 = 0$



$x \approx 0.5880, 2.0344, 3.7296, 5.1760$

58. $y = \cos^2 x - 2\cos x - 1 = 0, \quad [0, \pi]$

$x \approx 1.998$

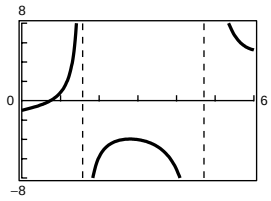
60. $y = 2\sec^2 x + \tan x - 6 = 0, \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x \approx -1.035, 0.870$

62. (a)

x	0	1	2	3	4	5	6
$f(x)$	-1	0.85	-5.81	-4.03	-7.12	16.63	5.25

Any zeros are in the intervals (0, 1), (1, 2), and (4, 5) because f changes signs in the intervals.

(b) 

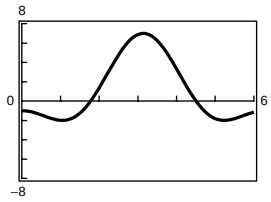
The only interval is (0, 1). It differs from that in part (a) because of the vertical asymptotes.

(c) 0.7391

64. (a)

x	0	1	2	3	4	5	6
$f(x)$	-1	-1.99	1.36	6.88	3.32	-1.81	-1.15

Any zeros are in the intervals (1, 2) and (4, 5) because f changes signs in these intervals.

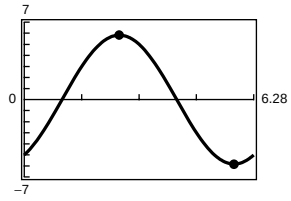
(b) 

The intervals are the same as in part (a).

(c) 1.7794, 4.5038

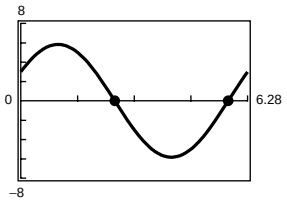
66. (a) $f(x) = 3 \sin x - 5 \cos x$

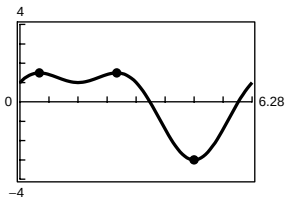
Maximum: (2.6012, 5.8310)
Minimum: (5.7428, -5.8310)



(b) $3 \cos x + 5 \sin x = 0$

$x \approx 2.601, 5.743$



68. (a) 

Maximum: (0.5236, 1.5), (2.6180, 1.5)
Minimum: (4.7124, -3.0)

(b) $2 \cos x - 4 \sin x \cos x = 0$

$2 \cos x(1 - 2 \sin x) = 0$

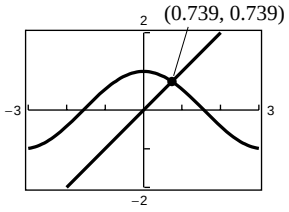
$\cos x = 0 \implies x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

$1 - 2 \sin x = 0 \implies \sin x = \dfrac{1}{2} \implies x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

The zeros are 0.5236, 2.618, 4.712 and 1.571.

The first three correspond to the values in (a).

70. Graph $y = \cos x$ and $y = x$ on the same set of axes. Their point of intersection gives the value of c such that $f(c) = c \implies \cos c = c$.



$c \approx 0.739$

72. $f(x) = \dfrac{\sin x}{x}$

(a) Domain: all real numbers except $x = 0$.

(b) The graph has y-axis symmetry.

Horizontal asymptote: $y = 0$

(c) As $x \to 0, f(x) \to 1$.

(d) $\sin x/x = 0$ has four solutions in the interval $[-8, 8]$.

$$(\sin x)\left(\frac{1}{x}\right) = 0$$

$$\sin x = 0$$

$$x = -2\pi, -\pi, \pi, 2\pi$$

74. $S = 74.50 + 43.75 \sin \dfrac{\pi t}{6}$

t	1	2	3	4	5	6	7	8	9	10	11	12
S	96.4	112.4	118.3	112.4	96.4	74.5	52.6	36.6	30.8	36.6	52.6	74.5

Sales exceed 100,000 units during February, March, and April.

76. $r = \dfrac{1}{32} v_0^2 \sin 2\theta,\ r = 300,\ v_0 = 100$

$$300 = \frac{1}{32}(100)^2 \sin 2\theta$$

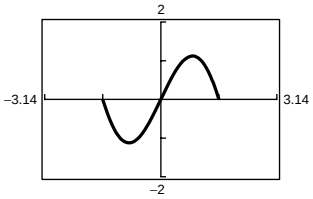
$$\sin 2\theta = 0.96$$

$2\theta \approx 1.287$ or $2\theta = \pi - 1.287 \approx 1.855$

$\theta \approx 0.6435 \approx 37°$ $\theta \approx 0.928 \approx 53°$

78. $A = 2x \cos x,\ -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

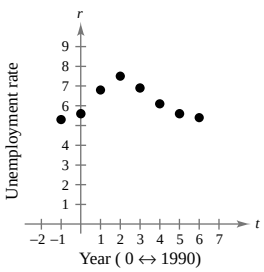
(a)



The maximum area of $A \approx 1.12$ occurs when $x \approx 0.86$.

(b) $A \geq 1$ for $0.6 < x < 1.1$

80. (a)



(b) iii (c) constant: 6.34%

(d) Approximately 6 years

(e) 2001

82. False. $\sin x - x = 0$ has 1 solution, $x = 0$.

84. Answers will vary.

86. $486° = 486°\left(\dfrac{\pi}{180°}\right) \approx 8.482$ radians