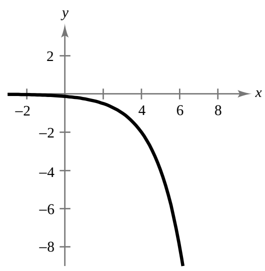
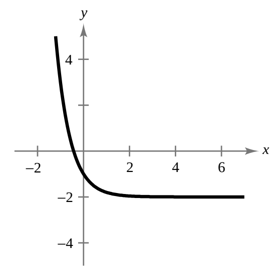
87. $f(x) = -2^{x-3}$



89. $f(x) = 5^{-x} - 2$



91. $s = r\theta$

$$\theta = \frac{s}{r} = \frac{26}{11} \approx 2.3636 \text{ radians}$$

93. Quadrant III

95. Quadrant III

Section 5.3 Solving Trigonometric Equations

- You should be able to identify and solve trigonometric equations.
- A trigonometric equation is a conditional equation. It is true for a specific set of values.
- To solve trigonometric equations, use algebraic techniques such as collecting like terms, taking square roots, factoring, squaring, converting to quadratic form, using formulas, and using inverse functions. Study the examples in this section.
- Use your graphing utility to calculate solutions and verify results.

Solutions to Odd-Numbered Exercises

1. $2\cos x - 1 = 0$

(a) $2\cos\frac{\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

(b) $2\cos\frac{5\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

3. $3\tan^2 2x - 1 = 0$

(a) $$3\left[\tan\left(\frac{2\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{\pi}{6} - 1 = 3\left(\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$$

(b) $$3\left[\tan\left(\frac{10\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{5\pi}{6} - 1 = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$$

5. $2\cos^2 x + 3\cos x + 1 = 0$

(a) $x = \frac{4\pi}{3}$: $2\cos^2\left(\frac{4\pi}{3}\right) + 3\cos\left(\frac{4\pi}{3}\right) + 1 = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1 = 0$

(b) $x = \pi$: $2\cos^2 \pi + 3\cos \pi + 1 = 2(-1)^2 - 3 + 1 = 0$

7. $y = \sin \frac{\pi x}{2} + 1$

From the graph in the textbook we see that the curve has x-intercepts at $x = -1$ and at $x = 3$.

9. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$

From the graph in the textbook we see that the curve has x-intercepts at $x = \pm 2$.

11. $2\cos x + 1 = 0$

$$2\cos x = -1$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3}$$
$$\text{or } x = \frac{4\pi}{3}$$

13. $\sqrt{3}\sec x - 2 = 0$

$$\sqrt{3}\sec x = 2$$
$$\sec x = \frac{2}{\sqrt{3}}$$
$$\cos x = \frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{6} \text{ or } x = \frac{11\pi}{6}$$

15. $3\csc^2 x - 4 = 0$

$$\csc^2 x = \frac{4}{3}$$
$$\csc x = \pm\frac{2}{\sqrt{3}}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

17. $2\sin^2 2x = 1$

$$\sin 2x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$
$$2x = \frac{\pi}{4},\ 2x = \frac{3\pi}{4},\ 2x = \frac{5\pi}{4},\ 2x = \frac{7\pi}{4},$$
$$2x = \frac{9\pi}{4},\ 2x = \frac{11\pi}{4},\ 2x = \frac{13\pi}{4},\ 2x = \frac{15\pi}{4}$$

Thus, $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ (8 solutions)

19. $4\cos^2 x - 3 = 0$

$$\cos^2 x = \frac{3}{4}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

21.

$$\sin^2 x = 3\cos^2 x$$
$$\sin^2 x - 3(1 - \sin^2 x) = 0$$
$$4\sin^2 x = 3$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

23. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$3\tan^2 x - 1 = 0$ or $\tan^2 x - 3 = 0$

$\tan x = \pm\dfrac{1}{\sqrt{3}}$ $\qquad$ $\tan x = \pm\sqrt{3}$

$x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$ $\qquad$ $x = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$

or $x = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$ $\qquad$ or $x = \dfrac{2\pi}{3}, \dfrac{5\pi}{3}$

25. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0$ or $\cos^2 x - 1 = 0$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\qquad$ $\cos x = \pm 1$

$x = 0, \pi$

27. $3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0$ or $3\tan^2 x - 1 = 0$

$x = 0, \pi$ $\qquad$ $\tan x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

29. $\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0$ or $\sec x + 1 = 0$

$\sec x = 2$ $\qquad$ $\sec x = -1$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $\qquad$ $x = \pi$

31. $2\sin x + \csc x = 0$

$2\sin x + \dfrac{1}{\sin x} = 0$

$2\sin^2 x + 1 = 0$

Since $2\sin^2 x + 1 > 0$, there are no solutions.

33. $\csc x + \cot x = 1$

$\dfrac{1}{\sin x} + \dfrac{\cos x}{\sin x} = 1$

$1 + \cos x = \sin x$

$(1 + \cos x)^2 = \sin^2 x$

$1 + 2\cos x + \cos^2 x = 1 - \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$\cos x = 0$ or $\cos x = -1$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\qquad$ $x = \pi$

($3\pi/2$ is extraneous.) $\qquad$ (π is extraneous.)

$x = \pi/2$ is the only solution.

35. $\cos\left(\dfrac{x}{2}\right) = \dfrac{\sqrt{2}}{2}$

$\dfrac{x}{2} = \dfrac{\pi}{4} + 2n\pi$

$x = \dfrac{\pi}{2} + 4n\pi$

$x = \dfrac{\pi}{2}$

37. $\dfrac{1 + \cos x}{1 - \cos x} = 0$

$1 + \cos x = 0$

$\cos x = -1$

$x = \pi$

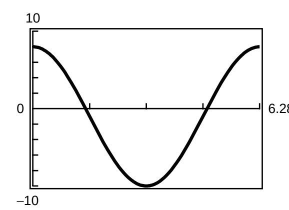
39. $$2\sec^2 x + \tan^2 x - 3 = 0$$
$$2(\tan^2 x + 1) + \tan^2 x - 3 = 0$$
$$3\tan^2 x - 1 = 0$$
$$\tan x = \pm\frac{\sqrt{3}}{3}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

41. $$\sec^2 x + \tan x = 3$$
$$(1 + \tan^2 x) + \tan x = 3$$
$$\tan^2 x + \tan x - 2 = 0$$
$$(\tan x + 2)(\tan x - 1) = 0$$
$\tan x = -2$ or $\tan x = 1$

$x \approx 2.0344, 5.1760$ or $x = \frac{\pi}{4}, \frac{5\pi}{4}$

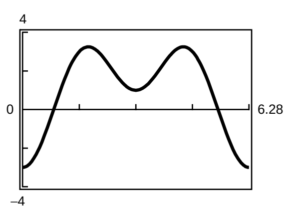
43. $y = 9\cos x - 1$

$x \approx 4.8237, 1.4595$



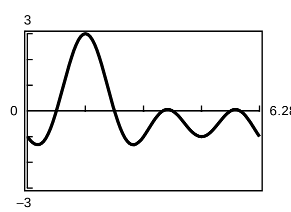
45. $y = 4\sin^2 x - 2\cos x - 1$

$x \approx 0.8614, 5.4218$



47. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$

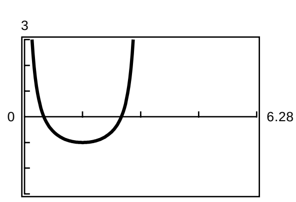
Graph $y = 4\sin^3 x + 2\sin^2 x - 2\sin x - 1$.



By altering the y-range to Ymin $= -.5$ and Ymax $= .5$, you see that there are 6 solutions: 0.7854, 2.3562, 3.6652, 3.9270, 5.4978, 5.7596.

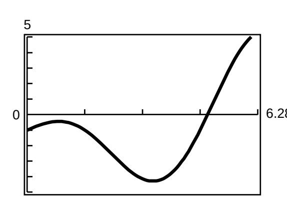
49. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

Graph $y = \dfrac{\cos x}{(1 - \sin x)\tan x} - 3$.



The solutions are approximately $x \approx 0.5236$, $x \approx 2.6180$

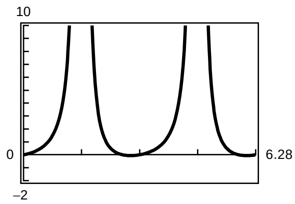
51. $x\cos x - 1 = 0$



$x \approx 4.9172$

53. $\sec^2 x + 0.5 \tan x - 1 = 0$

Graph $y_1 = \dfrac{1}{(\cos x)^2} + 0.5 \tan x - 1.$



The x-intercepts occur at $x = 0$, $x \approx 2.6779$, $x = 3.1416$ and $x \approx 5.8195$.

55. $12 \sin^2 x - 13 \sin x + 3 = 0$

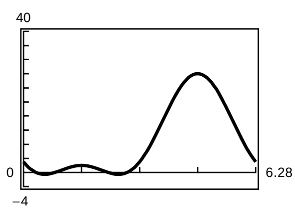
$(3 \sin x - 1)(4 \sin x - 3) = 0$

$3 \sin x - 1 = 0$ or $4 \sin x - 3 = 0$

$\sin x = \frac{1}{3}$ $\quad$ $\sin x = \frac{3}{4}$

$x = 0.3398, \; 2.8018$ $\quad$ $x = 0.8481, \; 2.2935$

Graph $y_1 = 12 \sin^2 x - 13 \sin x + 3.$



The x-intercepts occur at $x \approx 0.3398$, $x \approx 0.8481$, $x \approx 2.2935$, and $x \approx 2.8018$.

57. $y = 3 \tan^2 x + 5 \tan x - 4, \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$x \approx -1.154, 0.535$

59. $y = 4 \cos^2 x - 2 \sin x + 1, \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

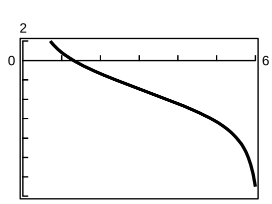
$x \approx 1.110$

61. (a)

x	0	1	2	3	4	5	6
$f(x)$	Undef.	0.83	−1.36	−2.93	−4.46	−6.34	−13.02

The zero is in the interval (1, 2) since f changes signs in the interval.

(b)



The interval is the same as part (a).

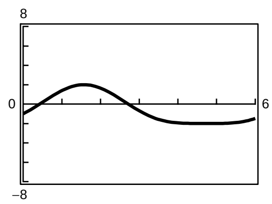
(c) 1.3065

63. (a)

x	0	1	2	3	4	5	6
$f(x)$	–1	1.39	1.65	−0.70	−1.94	−2.00	−1.48

The zeros are in the intervals (0, 1) and (2, 3) since f changes signs in these intervals.

(b)



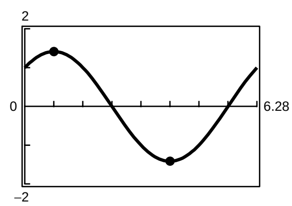
The intervals are the same as part (a).

(c) 0.4271, 2.7145

65. (a) $f(x) = \sin x + \cos x$

Maximum: $\left(\frac{\pi}{4}, \sqrt{2}\right)$

Minimum: $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$



(b) $\cos x - \sin x = 0$

$$\cos x = \sin x$$

$$1 = \frac{\sin x}{\cos x}$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$

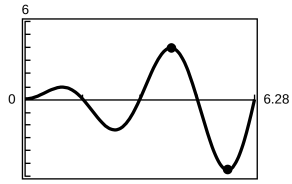
$$f\left(\frac{5\pi}{4}\right) = \sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = -\sin\frac{\pi}{4} + \left(-\cos\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

Therefore, the maximum point in the interval $[0, 2\pi)$ is $\left(\pi/4, \sqrt{2}\right)$ and the minimum point is $\left(5\pi/4, -\sqrt{2}\right)$.

67. (a) $f(x) = x \sin 2x$

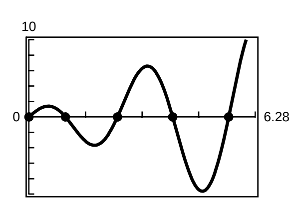
Maximum: $(3.989, 3.958)$

Minimum: $(5.543, -5.520)$



(b) $2x \cos 2x + \sin 2x = 0$

$y = 2x \cos 2x + \sin 2x$



5 solutions: 0, 1.014, 2.457, 3.989, 5.543.

The fourth and fifth correspond to the maximum and minimum found in part (a).

69. $f(x) = \tan \frac{\pi x}{4}$. $\tan 0 = 0$, but 0 is not positive. By graphing $y = \tan \frac{\pi x}{4} - x$, you see that the smallest positive fixed point is $x = 1$.

71. $f(x) = \cos \frac{1}{x}$

(a) The domain of $f(x)$ is all real numbers except 0.

(b) The graph has y-axis symmetry and a horizontal asymptote at $y = 1$.

(c) As $x \longrightarrow 0$, $f(x)$ oscillates between -1 and 1.

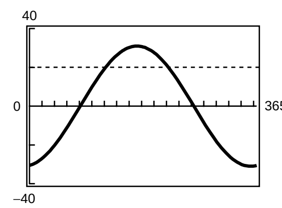
(d) There are an infinite number of solutions in the interval $[-1, 1]$.

(e) The greatest solution appears to occur at $x \approx 0.6366$.

73.
$$y = \frac{1}{12}(\cos 8t - 3 \sin 8t)$$
$$\frac{1}{12}(\cos 8t - 3 \sin 8t) = 0$$
$$\cos 8t = 3 \sin 8t$$
$$\frac{1}{3} = \tan 8t$$
$$8t = 0.32175 + n\pi$$
$$t = 0.04 + \frac{n\pi}{8}$$

In the interval $0 \le t \le 1$, $t = 0.04, 0.43$, and 0.83.

75. $D = 31 \sin\left(\frac{2\pi}{365}t - 1.4\right)$

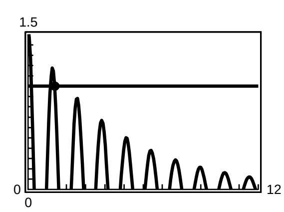


$D > 20°$ for $122 \le t \le 223$

77. Range = 1000 yards = 3000 feet

$$v_0 = 1200 \text{ feet per second}$$
$$r = \tfrac{1}{32} v_0^2 \sin 2\theta$$
$$3000 = \tfrac{1}{32}(1200)^2 \sin 2\theta$$
$$\sin 2\theta = 0.066667$$
$$2\theta \approx 3.8226°$$
$$\theta \approx 1.9113°$$

79. $y_1 = 1.56e^{-0.22t} \cos 4.9t$ intersects $y_2 = -1$ at $t \approx 1.96$

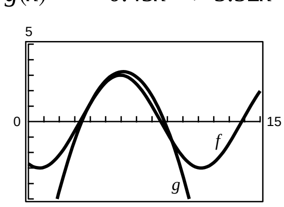


The displacement does not exceed one inch from equilibrium after $t = 1.96$ seconds.

81. $f(x) = 3 \sin(0.6x - 2)$

(a) Zero: $\sin(0.6x - 2) = 0$
$$0.6x - 2 = 0$$
$$0.6x = 2$$
$$x = \frac{2}{0.6} = \frac{10}{3}$$

(b) $g(x) = -0.45x^2 + 5.52x - 13.70$



For $3.5 \le x \le 6$ the approximation appears to be good.

(c) $-0.45x^2 + 5.52x - 13.70 = 0$
$$x = \frac{-5.52 \pm \sqrt{(5.52)^2 - 4(-0.45)(-13.70)}}{2(-0.45)}$$
$x \approx 3.46, 8.81$

The zero of g on $[0, 6]$ is 3.46. The zero is close to the zero $\frac{10}{3} \approx 3.33$ of f.