83. False. There might not be periodicity, as in the equation $\sin(x^2) = 0$

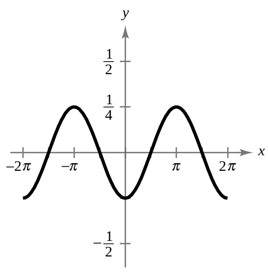
85. $124° = 124°\left(\frac{\pi}{180°}\right)$
≈ 2.164 radians

87. $-0.41° = -0.41\left(\frac{\pi}{180°}\right)$
≈ -0.007 radians

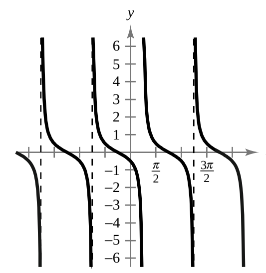
89. $\tan 30° = \frac{14}{x} \implies x = \frac{14}{\tan 30°} = \frac{14}{\frac{\sqrt{3}}{3}} \approx 24.249$

91. $\sin 40° = \frac{16}{x} \implies x = \frac{16}{\sin 40°} \approx 24.892$

93. $f(x) = \frac{1}{4}\sin\left(x - \frac{\pi}{2}\right)$



95. $f(x) = \frac{1}{2}\cot\left(x - \frac{\pi}{4}\right)$



Section 5.4 Sum and Difference Formulas

- You should memorize the sum and difference formulas.

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

- You should be able to use these formulas to find the values of the trigonometric functions of angles whose sums or differences are special angles.
- You should be able to use these formulas to solve trigonometric equations.

Solutions to Odd-Numbered Exercises

1. (a) $\cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\sin\frac{\pi}{3}$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$$

(b) $\cos\frac{\pi}{6} + \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3} + 1}{2}$

3. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right) = \sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$ (b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{-1 - \sqrt{3}}{2}$

5. (a) $\cos(0° + 135°) = \cos 0° \cos 135° - \sin 0° \sin 135°$

$$= \cos 135° = -\frac{\sqrt{2}}{2}$$

(b) $\cos 0° + \cos 135° = 1 - \frac{\sqrt{2}}{2}$

7. (a) $\sin(315° - 60°) = \sin 315 \cos 60° - \cos 315° \sin 60°$

$$= -\frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{-\sqrt{2} - \sqrt{6}}{4}$$

(b) $\sin 315° - \sin 60° = -\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} = -\frac{\sqrt{2} - \sqrt{3}}{2}$

9.
$$\sin 75° = \sin(30° + 45°)$$
$$= \sin 30° \cos 45° + \sin 45° \cos 30°$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$$

$$\cos 75° = \cos(30° + 45°)$$
$$= \cos 30° \cos 45° - \sin 30° \sin 45°$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$$\tan 75° = \tan(30° + 45°)$$
$$= \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°}$$
$$= \frac{\left(\sqrt{3}/3\right) + 1}{1 - \left(\sqrt{3}/3\right)} = \frac{\sqrt{3} + 3}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$
$$= \frac{6\sqrt{3} + 12}{6} = \sqrt{3} + 2$$

11.
$$\sin 105° = \sin(60° + 45°)$$
$$= \sin 60° \cos 45° + \sin 45° \cos 60°$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos 105° = \cos(60° + 45°)$$
$$= \cos 60° \cos 45° - \sin 60° \sin 45°$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan 105° = \tan(60° + 45°)$$
$$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$$
$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$
$$= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

13. $\sin 195^\circ = \sin(225^\circ - 30^\circ)$

$$= \sin 225^\circ \cos 30^\circ - \sin 30^\circ \cos 225^\circ$$
$$= -\sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ$$
$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$\cos 195^\circ = \cos(225^\circ - 30^\circ)$

$$= \cos 225^\circ \cos 30^\circ + \sin 225^\circ \sin 30^\circ$$
$$= -\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$\tan 195^\circ = \tan(225^\circ - 30^\circ)$

$$= \frac{\tan 225^\circ - \tan 30^\circ}{1 + \tan 225^\circ \tan 30^\circ}$$
$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$
$$= \frac{1 - \left(\sqrt{3}/3\right)}{1 + \left(\sqrt{3}/3\right)} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$
$$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

15. $\sin \frac{11\pi}{12} = \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$$= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \sin \frac{\pi}{6} \cos \frac{3\pi}{4}$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$\cos \frac{11\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$$= \cos \frac{3\pi}{4} \cos \frac{\pi}{6} - \sin \frac{3\pi}{4} \sin \frac{\pi}{6}$$
$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$\tan \frac{11\pi}{4} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$$= \frac{\tan(3\pi/4) + \tan(\pi/6)}{1 - \tan(3\pi/4)\tan(\pi/6)}$$
$$= \frac{-1 + \left(\sqrt{3}/3\right)}{1 - (-1)\left(\sqrt{3}/3\right)}$$
$$= \frac{-3 + \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$
$$= \frac{-12 + 6\sqrt{3}}{6} = -2 + \sqrt{3}$$

17. $-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$

$$\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \sin \frac{\pi}{6} \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos \frac{\pi}{6}$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \cos \frac{\pi}{6} \cos \frac{\pi}{4} + \sin \frac{\pi}{6} \sin \frac{\pi}{4}$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \frac{\tan(\pi/6) - \tan(\pi/4)}{1 + \tan(\pi/6)\tan(\pi/4)}$$
$$= \frac{\left(\sqrt{3}/3\right) - 1}{1 + \left(\sqrt{3}/3\right)} = \frac{\sqrt{3} - 3}{\sqrt{3} + 3} \cdot \frac{\sqrt{3} - 3}{\sqrt{3} - 3} = \frac{12 - 6\sqrt{3}}{-6} = -2 + \sqrt{3}$$

19. $\cos 40^\circ \cos 15^\circ - \sin 40^\circ \sin 15^\circ = \cos(40^\circ + 15^\circ) = \cos 55^\circ$

21. $\sin 340^\circ \cos 50^\circ - \cos 340^\circ \sin 50^\circ = \sin(340^\circ - 50^\circ) = \sin 290^\circ$

23. $\dfrac{\tan 325^\circ - \tan 86^\circ}{1 + \tan 325^\circ \tan 86^\circ} = \tan(325^\circ - 86^\circ) = \tan 239^\circ$

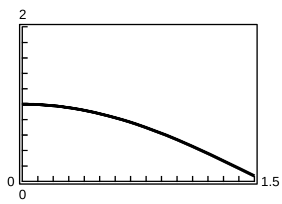
25. $\sin 3 \cos 1.2 - \cos 3 \sin 1.2 = \sin(3 - 1.2) = \sin 1.8$

27. $\cos \dfrac{\pi}{7} \cos \dfrac{\pi}{5} - \sin \dfrac{\pi}{7} \sin \dfrac{\pi}{5} = \cos\left(\dfrac{\pi}{7} + \dfrac{\pi}{5}\right)$

$$= \cos \frac{12\pi}{35}$$

29.

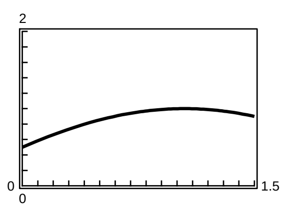
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.9801	0.9211	0.8253	0.6967	0.5403	0.3624	0.1700
y_2	0.9801	0.9211	0.8253	0.6967	0.5403	0.3624	0.1700



$$\begin{aligned} y_1 &= \sin\left(\frac{\pi}{2} + x\right) \\ &= \sin \frac{\pi}{2} \cos x + \sin x \cdot \cos \frac{\pi}{2} \\ &= \cos x \\ &= y_2 \end{aligned}$$

31.

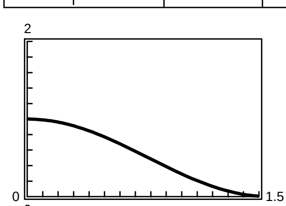
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.6621	0.7978	0.9017	0.9696	0.9989	0.9883	0.9384
y_2	0.6621	0.7978	0.9017	0.9696	0.9989	0.9883	0.9384



$$\begin{aligned} y_1 &= \sin\left(\frac{\pi}{6} + x\right) \\ &= \sin \frac{\pi}{6} \cos x + \sin x \cdot \cos \frac{\pi}{6} \\ &= \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \\ &= \frac{1}{2}(\cos x + \sqrt{3} \sin x) \\ &= y_2 \end{aligned}$$

33.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.9605	0.8484	0.6812	0.4854	0.2919	0.1313	0.0289
y_2	0.9605	0.8484	0.6812	0.4854	0.2919	0.1313	0.0289



$$\begin{aligned} y_1 &= \cos(x + \pi) \cos(x - \pi) \\ &= (\cos x \cdot \cos \pi - \sin x \cdot \sin \pi) \\ &\quad [\cos x \cos \pi + \sin x \sin \pi] \\ &= [-\cos x][-\cos x] \\ &= \cos^2 x \\ &= y_2 \end{aligned}$$

For Exercises 35–37, we have:

$\sin u = \frac{5}{13}$, u is in Quadrant I $\Rightarrow$ $\cos u = \frac{12}{13}$

$\cos v = -\frac{3}{5}$, v is in Quadrant II $\Rightarrow$ $\sin v = \frac{4}{5}$

35. $\sin(u + v) = \sin u \cos v + \cos u \sin v$

$= \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$

$= \frac{33}{65}$

37. $\cos(u + v) = \cos u \cos v - \sin u \sin v$

$= \left(\frac{12}{13}\right)\left(-\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$

$= \frac{-56}{65}$

For Exercises 39–41, we have:

$\sin u = \frac{7}{25}$, u is in Quadrant II $\Rightarrow$ $\cos u = \frac{-24}{25}$

$\cos v = \frac{4}{5}$, v is in Quadrant IV $\Rightarrow$ $\sin v = -\frac{3}{5}$

39. $\cos(u + v) = \cos u \cdot \cos v - \sin u \cdot \sin v$

$$= \left(-\frac{24}{25}\right)\left(\frac{4}{5}\right) - \left(\frac{7}{25}\right)\left(-\frac{3}{5}\right)$$

$$= \frac{-96 + 21}{125} = \frac{-75}{125} = \frac{-3}{5}$$

41. $\sin(v - u) = \sin v \cdot \cos u - \sin u \cdot \cos v$

$$= \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right) - \left(\frac{7}{25}\right)\left(\frac{4}{5}\right)$$

$$= \frac{72 - 28}{125} = \frac{44}{125}$$

43. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = \cos \pi \cos \theta + \sin \pi \sin \theta + \sin \frac{\pi}{2} \cos \theta + \sin \theta \cos \frac{\pi}{2}$

$= (-1)(\cos \theta) + (0)(\sin \theta) + (1)(\cos \theta) + (\sin \theta)(0) = -\cos \theta + \cos \theta = 0$

45. $\tan(x + \pi) - \tan(\pi - x) = \frac{\tan x + \tan \pi}{1 - \tan x \cdot \tan \pi} - \frac{\tan \pi - \tan x}{1 + \tan \pi \tan x}$

$$= \frac{\tan x}{1} - \left(-\frac{\tan x}{1}\right)$$

$= 2 \tan x$

47. $\sin(x + y) + \sin(x - y) = \sin x \cos y + \sin y \cos x + \sin x \cos y - \sin y \cos x = 2 \sin x \cos y$

49. $\cos(x + y)\cos(x - y) = [\cos x \cos y - \sin x \sin y][\cos x \cos y + \sin x \sin y]$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

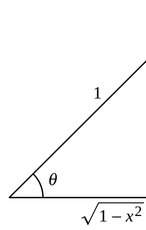
$= \cos^2 x(1 - \sin^2 y) - \sin^2 x \sin^2 y$

$= \cos^2 x - \sin^2 y(\cos^2 x + \sin^2 x)$

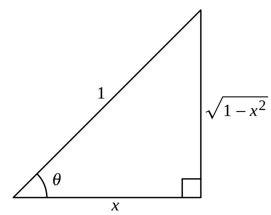
$= \cos^2 x - \sin^2 y$

51. $\sin(\arcsin x + \arccos x) = \sin(\arcsin x)\cos(\arccos x) + \sin(\arccos x)\cos(\arcsin x)$

$$= x \cdot x + \sqrt{1 - x^2} \cdot \sqrt{1 - x^2}$$
$$= x^2 + 1 - x^2$$
$$= 1$$



$\theta = \arcsin x$

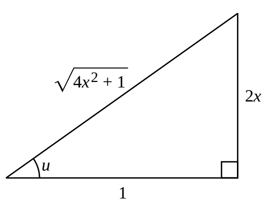


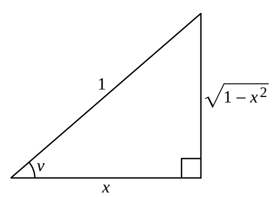
$\theta = \arccos x$

53. Let:

$u = \arctan 2x$ and $v = \arccos x$

$\tan u = 2x$ $\cos v = x$





$$\sin(\arctan 2x - \arccos x) = \sin(u - v)$$
$$= \sin u \cos v - \cos u \sin v$$
$$= \frac{2x}{\sqrt{4x^2 + 1}}(x) - \frac{1}{\sqrt{4x^2 + 1}}\left(\sqrt{1 - x^2}\right)$$
$$= \frac{2x^2 - \sqrt{1 - x^2}}{\sqrt{4x^2 + 1}}$$

55. $$\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$$

$$\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} + \sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} = 1$$
$$2 \sin x(0.5) = 1$$
$$\sin x = 1$$
$$x = \frac{\pi}{2}$$

57.

$$\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4} - \left(\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}\right) = 1$$

$$-2 \sin x\left(\frac{\sqrt{2}}{2}\right) = 1$$

$$-\sqrt{2} \sin x = 1$$

$$\sin x = -\frac{1}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

59.

$$\tan(x + \pi) + 2 \sin(x + \pi) = 0$$

$$\frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} + 2(\sin x \cos \pi + \cos x \sin \pi) = 0$$

$$\frac{\tan x + 0}{1 - \tan x(0)} + 2[\sin x(-1) + \cos x(0)] = 0$$

$$\frac{\tan x}{1} - 2 \sin x = 0$$

$$\frac{\sin x}{\cos x} = 2 \sin x$$

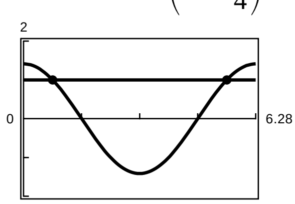
$$\sin x = 2 \sin x \cos x$$

$$\sin x(1 - 2 \cos x) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

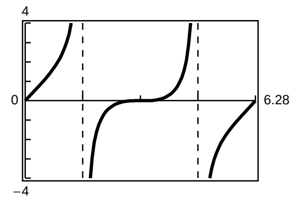
$$x = 0, \pi \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

61. Graph $y_1 = \cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right)$ and $y_2 = 1$.



The points of intersection occur at $x \approx 0.7854$ and $x \approx 5.4978$.

63. $\tan(x + \pi) - \cos\left(x + \dfrac{\pi}{2}\right) = 0$



Answers: (0, 0), (3.14, 0) $\implies x = 0, \pi$

65. $$y_1 + y_2 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$
$$= A\left[\cos\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right) + \sin\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi x}{\lambda}\right)\right] + A\left[\cos\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right) - \sin\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi x}{\lambda}\right)\right]$$
$$= 2A\cos\left(\frac{2\pi}{T}\right)\cos\left(\frac{\pi}{\lambda}\right)$$

67. False. See page 404.

69. False. $$\sin 75^\circ = \sin(30^\circ + \sin 45^\circ)$$
$$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ\ 45^\circ$$
$$= \frac{1}{2}\,\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

71. $$\cos(n\pi + \theta) = \cos n\pi \cos\theta - \sin n\pi \sin\theta$$
$$= (-1)^n(\cos\theta) - (0)(\sin\theta)$$
$$= (-1)^n(\cos\theta), \text{ where } n \text{ is an integer.}$$

73. $$C = \arctan\frac{b}{a} \implies \tan C = \frac{b}{a} \implies \sin C = \frac{b}{\sqrt{a^2 + b^2}}, \cos C = \frac{a}{\sqrt{a^2 + b^2}}$$
$$\sqrt{a^2 + b^2}\sin(B\theta + C) = \sqrt{a^2 + b^2}\left(\sin B\theta \cdot \frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}} \cdot \cos B\theta\right) = a\sin B\theta + b\cos B\theta$$

75. $\sin\theta + \cos\theta$

$a = 1,\ b = 1,\ B = 1$

(a) $C = \arctan\dfrac{b}{a} = \arctan 1 = \dfrac{\pi}{4}$

$$\sin\theta + \cos\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$
$$= \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan 1 = \dfrac{\pi}{4}$

$$\sin\theta + \cos\theta = \sqrt{a^2 + b^2}\cos(B\theta - C)$$
$$= \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$$

77. $12\sin 3\theta + 5\cos 3\theta$

$a = 12,\ b = 5,\ B = 3$

(a) $C = \arctan\dfrac{b}{a} = \arctan\dfrac{5}{12} \approx 0.3948$

$$12\sin 3\theta + 5\cos 3\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$
$$\approx 13\sin(3\theta + 0.3948)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan\dfrac{12}{5} \approx 1.1760$

$$12\sin 3\theta + 5\cos 3\theta = \sqrt{a^2 + b^2}\cos(B\theta - C)$$
$$\approx 13\cos(3\theta - 1.1760)$$

79. $C = \arctan \frac{b}{a} = \frac{\pi}{2} \implies a = 0$

$\sqrt{a^2 + b^2} = 2 \implies b = 2$

$B = 1$

$2 \sin\left(\theta + \frac{\pi}{2}\right) = (0)(\sin \theta) + (2)(\cos \theta) = 2 \cos \theta$

81. From the figure, it appears that $u + v = w$. Assume that u, v, and w are all in Quadrant I. From the figure:

$\tan u = \frac{s}{3s} = \frac{1}{3}$

$\tan v = \frac{s}{2s} = \frac{1}{2}$

$\tan w = \frac{s}{s} = 1$

$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{1/3 + 1/2}{1 - (1/3)(1/2)} = \frac{5/6}{1 - (1/6)} = 1 = \tan w.$

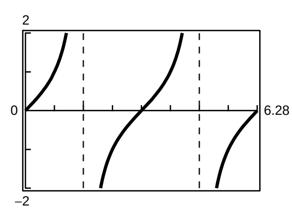
Thus, $\tan(u + v) = \tan w$. Because u, v, and w are all in Quadrant I, we have

$$\arctan[\tan(u + v)] = \arctan[\tan w]$$
$$u + v = w.$$

83. $\tan(\pi + \theta) = \frac{\tan \pi + \tan \theta}{1 - \tan \pi \tan \theta}$

$= \frac{0 + \tan \theta}{1 - (0) \tan \theta}$

$= \tan \theta$



85. $f(h) = \frac{\cos\left(\frac{\pi}{6} + h\right) - \cos \frac{\pi}{6}}{h}$

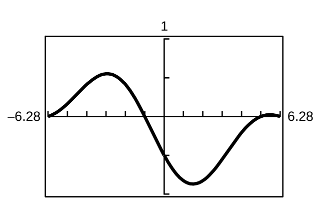
$g(h) = \cos \frac{\pi}{6}\left(\frac{\cos h - 1}{h}\right) - \sin \frac{\pi}{6}\left(\frac{\sin h}{h}\right)$

(a) The domains are both $(-\infty, 0)$, $(0, \infty)$.

(b)

h	0.01	0.02	0.05	0.1	0.2	0.5
$f(h)$	−0.5043	−0.5086	−0.5214	−0.5424	−0.5830	−0.6915
$g(h)$	−0.5043	−0.5086	−0.5214	−0.5424	−0.5830	−0.6915

(c)



(d) The values tend to $y = -\frac{1}{2}$.