86. $\tan(u+v) = \dfrac{\sin(u+v)}{\cos(u+v)} = \dfrac{\sin u \cos v + \sin v \cos u}{\cos u \cos v - \sin u \sin v}$

$$= \frac{\sin u \cos v + \sin v \cos u}{\cos u \cos v - \sin u \sin v} \cdot \frac{\dfrac{1}{\cos u \cos v}}{\dfrac{1}{\cos u \cos v}} = \frac{\dfrac{\sin u}{\cos u} + \dfrac{\sin v}{\cos v}}{1 - \dfrac{\sin u \sin v}{\cos u \cos v}} = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

88. $y = 0$: $x^2 - 3x - 40 = (x-8)(x+5) = 0 \implies$ x-intercepts: $(8, 0), (-5, 0)$

$x = 0 \implies y = -40$. y-intercept: $(0, -40)$

90. $y = 0$: $2x\sqrt{x+7} = 0 \implies x = 0, -7$. x-intercepts: $(0, 0), (-7, 0)$

$x = 0 \implies y = 0$. y-intercept: $(0, 0)$

92. $\arctan\left(-\sqrt{3}\right) = -\dfrac{\pi}{3}$ because $\tan\left(-\dfrac{\pi}{3}\right) = -\sqrt{3}$

94. $\arctan 0 = 0$

Section 5.5 Multiple-Angle and Product-Sum Formulas

Solutions to Even-Numbered Exercises

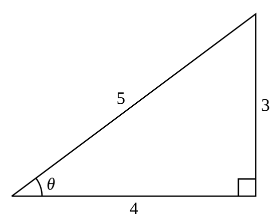


Figure for Exercises 2–8

2. $\tan\theta = \frac{3}{4}$

4. $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

6. $\sec 2\theta = \dfrac{1}{\cos 2\theta}$

$$= \frac{1}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{1}{(4/5)^2 - (3/5)^2}$$

$$= \frac{1}{(16/25) - (9/25)}$$

$$= \frac{25}{7}$$

8. $\cot 2\theta = \dfrac{1}{\tan 2\theta}$

$$= \frac{1 - \tan^2\theta}{2\tan\theta}$$

$$= \frac{1 - (3/4)^2}{2(3/4)}$$

$$= \frac{7/16}{3/2}$$

$$= \frac{7}{24}$$

10.

$$\begin{aligned}
\sin 2x + \cos x &= 0\\
2\sin x\cos x + \cos x &= 0\\
\cos x(2\sin x + 1) &= 0
\end{aligned}$$

$\cos x = 0$ or $2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

12.

$$\begin{aligned}
\sin 2x \sin x &= \cos x\\
2\sin x\cos x\sin x - \cos x &= 0\\
\cos x(2\sin^2 x - 1) &= 0
\end{aligned}$$

$\cos x = 0$ or $2\sin^2 x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

14.

$$\begin{aligned}
\tan 2x - \cot x &= 0\\
\frac{2\tan x}{1-\tan^2 x} &= \cot x\\
2\tan x &= \cot x(1-\tan^2 x)\\
2\tan x &= \cot x - \cot x\tan^2 x\\
2\tan x &= \cot x - \tan x\\
3\tan x &= \cot x\\
3\tan x - \cot x &= 0\\
3\tan x - \frac{1}{\tan x} &= 0\\
\frac{3\tan^2 x - 1}{\tan x} &= 0\\
\frac{1}{\tan x}(3\tan^2 x - 1) &= 0\\
\cot x(3\tan^2 x - 1) &= 0
\end{aligned}$$

$\cot x = 0$ or $3\tan^2 x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

16.

$$\begin{aligned}
(\sin 2x + \cos 2x)^2 &= 1\\
\sin^2 2x + 2\sin 2x\cos 2x + \cos^2 2x &= 1\\
2\sin 2x\cos 2x &= 0\\
\sin 4x &= 0\\
4x &= n\pi\\
x &= \frac{n\pi}{4}\\
x &= 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}
\end{aligned}$$

18. $6 \sin x \cos x + 4 = 3(2 \sin x \cos x) + 4$
$= 3 \sin 2x + 4$

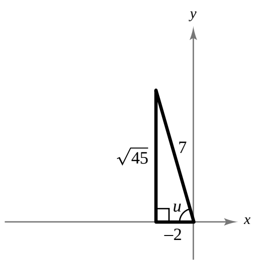
20. $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x$
$= \cos 2x$

22. $\cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi$ Quadrant II

$$\sin 2u = 2 \sin u \cos u = 2\left(\frac{\sqrt{45}}{7}\right)\left(-\frac{2}{7}\right) - \frac{12\sqrt{5}}{49}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{49} - \frac{45}{49} = -\frac{41}{49}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{\sqrt{45}}{2}\right)}{1 - \frac{45}{4}} = \frac{-\sqrt{45}}{\frac{4-45}{4}} = \frac{12\sqrt{5}}{41}$$

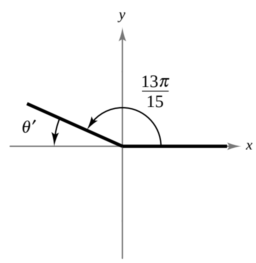


24. $\cot u = -6, \frac{3\pi}{2} < u < 2\pi$ Quadrant IV

$$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{1}{\sqrt{37}}\right)\left(\frac{6}{\sqrt{37}}\right) = -\frac{12}{37}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{36}{37} - \frac{1}{37} = \frac{35}{37}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{1}{6}\right)}{1 - \left(-\frac{1}{6}\right)^2} = \frac{-\frac{2}{6}}{\frac{35}{36}} = -\frac{12}{35}$$



26.
$$\begin{aligned}\sin^4 x &= (\sin^2 x)(\sin^2 x)\\ &= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - \cos 2x}{2}\right)\\ &= \frac{1 - 2\cos 2x + \cos^2 2x}{4}\\ &= \frac{1 - 2\cos 2x + \left(\frac{1 + \cos 4x}{2}\right)}{4}\\ &= \frac{2 - 4\cos 2x + 1 + \cos 4x}{8}\\ &= \frac{1}{8}(3 - 4\cos 2x + \cos 4x)\end{aligned}$$

28.
$$\begin{aligned}\cos^6 x &= (\cos^2 x)^3 = \left(\frac{1 + \cos 2x}{2}\right)^3\\ &= \frac{1}{8}[1 + 3\cos 2x + 3\cos^2 2x + \cos^3 2x]\\ &= \frac{1}{8}\left[1 + 3\cos 2x + 3 \cdot \frac{1 + \cos 4x}{2} + \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]\\ &= \frac{1}{8}\left[\frac{5}{2} + 3\cos 2x + \frac{3}{2}\cos 4x + \frac{1}{2}\cos 2x + \frac{1}{2}\cos 2x \cdot \cos 4x\right]\\ &= \frac{1}{8}\left[\frac{5}{2} + \frac{7}{2}\cos 2x + \frac{3}{2}\cos 4x + \frac{1}{2}\frac{1}{2}(\cos 2x + \cos 6x)\right]\\ &= \frac{1}{32}[10 + 15\cos 2x + 6\cos 4x + \cos 6x]\end{aligned}$$

30. $\sin^4 x \cos^2 x = \sin^2 x \sin^2 x \cos^2 x$

$$= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$$

$$= \frac{1}{8}(1 - \cos 2x)(1 - \cos^2 2x)$$

$$= \frac{1}{8}(1 - \cos 2x - \cos^2 2x + \cos^3 2x)$$

$$= \frac{1}{8}\left[1 - \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) + \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]$$

$$= \frac{1}{16}[2 - 2\cos 2x - 1 - \cos 4x + \cos 2x + \cos 2x \cos 4x]$$

$$= \frac{1}{16}\left[1 - \cos 2x - \cos 4x + \frac{1}{2}\cos 2x + \frac{1}{2}\cos 6x\right]$$

$$= \frac{1}{32}[2 - 2\cos 2x - 2\cos 4x + \cos 2x + \cos 6x]$$

$$= \frac{1}{32}[2 - \cos 2x - 2\cos 4x + \cos 6x]$$

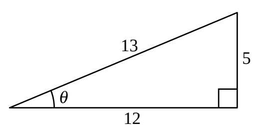


Figure for Exercises 32–38

32. $\sin\frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}}$

$$= \sqrt{\frac{1 - (12/13)}{2}}$$

$$= \sqrt{\frac{1/13}{2}}$$

$$= \frac{1}{\sqrt{26}}$$

$$= \frac{\sqrt{26}}{26}$$

34. $\sec\frac{\theta}{2} = \frac{1}{\cos\frac{\theta}{2}}$

$$= \frac{1}{\sqrt{\frac{1 + \cos\theta}{2}}}$$

$$= \frac{\sqrt{2}}{\sqrt{1 + \frac{12}{13}}}$$

$$= \frac{\sqrt{26}}{5}$$ (see #31)

36. $\cot\frac{\theta}{2} = \frac{1}{\tan\frac{\theta}{2}}$

$$= \frac{\sin\theta}{1 - \cos\theta}$$

$$= \frac{\frac{5}{13}}{1 - \frac{12}{13}}$$

$$= \frac{5}{13}\left(\frac{13}{1}\right) = 5$$

38. $2\cos\frac{\theta}{2}\tan\frac{\theta}{2} = 2\sin\frac{\theta}{2}$

$$= 2\left(\frac{1}{\sqrt{26}}\right)$$

$$= \frac{2}{\sqrt{26}}$$

$$= \frac{\sqrt{26}}{13}$$ (see #32)

40. $\sin 165° = \sin\left(\frac{1}{2} \cdot 330°\right) = \sqrt{\frac{1 - \cos 330°}{2}} = \sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\cos 165° = \cos\left(\frac{1}{2} \cdot 330°\right) = -\sqrt{\frac{1 + \cos 330°}{2}} = -\sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\tan 165° = \tan\left(\frac{1}{2} \cdot 330°\right) = \frac{\sin 330°}{1 + \cos 330°} = \frac{-1/2}{1 + (\sqrt{3}/2)} = \frac{-1}{2 + \sqrt{3}} = \sqrt{3} - 2$

42. $157° \, 30' = 157.5° = \frac{1}{2}(315°)$ Quadrant II

$\sin(157° \, 30') = \sin\left(\frac{1}{2} \cdot 315°\right) = \sqrt{\frac{1 - \cos 315°}{2}} = \sqrt{\frac{1 - \sqrt{2}/2}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

$\cos(157° \, 30') = \cos\left(\frac{1}{2} \cdot 315°\right) = -\sqrt{\frac{1 + \cos 315°}{2}} = -\sqrt{\frac{1 + \sqrt{2}/2}{2}} = -\frac{\sqrt{2 + \sqrt{2}}}{2}$

$\tan(157° \, 30') = \tan\left(\frac{1}{2} \cdot 315°\right) = \frac{\sin 315°}{1 + \cos 315°} = \frac{-\sqrt{2}/2}{1 + \sqrt{2}/2} = \frac{-\sqrt{2}}{2 + \sqrt{2}} = 1 - \sqrt{2}$

44. $\sin \frac{\pi}{12} = \sin\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1 - \cos(\pi/6)}{2}} = \sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\cos \frac{\pi}{12} = \cos\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1 + \cos(\pi/6)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\tan \frac{\pi}{12} = \tan\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \frac{\sin(\pi/6)}{1 + \cos(\pi/6)} = \frac{1/2}{1 + (\sqrt{3}/2)} = 2 - \sqrt{3}$

46. $\frac{7\pi}{12} = \frac{1}{2}\left(\frac{7\pi}{6}\right)$ Quadrant II

$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right) = \sqrt{\frac{1 - \cos(7\pi/6)}{2}} = \sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right) = -\sqrt{\frac{1 + \cos(7\pi/6)}{2}} = -\sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = -\frac{\sqrt{2 - \sqrt{3}}}{2}$

$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right) = \frac{\sin \frac{7\pi}{6}}{1 + \cos \frac{7\pi}{6}} = \frac{-\frac{1}{2}}{1 - \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3} - 2} = -2 - \sqrt{3}$

48. $\cos u = \frac{7}{25}, 0 < u < \frac{\pi}{2}$ Quadrant I $\sin u = \sqrt{1 - \cos^2 u} = \sqrt{1 - \frac{49}{625}} = \frac{24}{25}$

$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \frac{7}{25}}{2}} = \sqrt{\frac{9}{25}} = 0.6 = \frac{3}{5}$

$\cos \frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 + \frac{7}{25}}{2}} = \sqrt{\frac{16}{25}} = 0.8 = \frac{4}{5}$

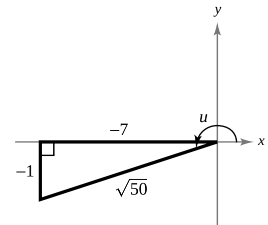
$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{7}{25}}{\frac{24}{25}} = \frac{18}{24} = \frac{3}{4}$

50. $\cot u = 7,\ \pi < u < \dfrac{3\pi}{2}$ Quadrant III

$$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+(7/\sqrt{50})}{2}} = \sqrt{\frac{\sqrt{50}+7}{2\sqrt{50}}} = \frac{\sqrt{50+7\sqrt{50}}}{10}$$

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1-7/\sqrt{50}}{2}} = -\sqrt{\frac{\sqrt{50}-7}{2\sqrt{50}}} = -\frac{\sqrt{50-7\sqrt{50}}}{10}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1+\dfrac{7}{\sqrt{50}}}{\dfrac{-1}{\sqrt{50}}} = -\left(\sqrt{50}+7\right)$$



(Not drawn to scale)

52. $\sqrt{\dfrac{1+\cos 4x}{2}} = \left|\cos\dfrac{4x}{2}\right| = |\cos 2x|$

54. $-\sqrt{\dfrac{1-\cos(x-1)}{2}} = -\left|\sin\left(\dfrac{x-1}{2}\right)\right|$

56. $h(x) = \sin\dfrac{x}{2} + \cos x - 1$

$$\sin\frac{x}{2} + \cos x - 1 = 0$$
$$\pm\sqrt{\frac{1-\cos x}{2}} = 1 - \cos x$$
$$\frac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$$
$$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$$
$$2\cos^2 x - 3\cos x + 1 = 0$$
$$(2\cos x - 1)(\cos x - 1) = 0$$
$$2\cos x - 1 = 0 \quad\text{or}\quad \cos x - 1 = 0$$
$$\cos x = \frac{1}{2} \qquad \cos x = 1$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = 0$$

58. $g(x) = \tan\dfrac{x}{2} - \sin x$

$$\tan\frac{x}{2} - \sin x = 0$$
$$\frac{1-\cos x}{\sin x} = \sin x$$
$$1 - \cos x = \sin^2 x$$
$$1 - \cos x = 1 - \cos^2 x$$
$$\cos^2 x - \cos x = 0$$
$$\cos x(\cos x - 1) = 0$$
$$\cos x = 0 \quad\text{or}\quad \cos x - 1 = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \cos x = 1$$
$$x = 0$$

0, $\pi/2$, and $3\pi/2$ are all solutions to the equation.

60. $4\sin\dfrac{\pi}{3}\cos\dfrac{5\pi}{6} = 4\cdot\dfrac{1}{2}\left[\sin\left(\dfrac{\pi}{3}+\dfrac{5\pi}{6}\right) + \sin\left(\dfrac{\pi}{3}-\dfrac{5\pi}{6}\right)\right]$

$$= 2\left[\sin\frac{7\pi}{6} + \sin\left(-\frac{\pi}{2}\right)\right]$$
$$= 2\left(\sin\frac{7\pi}{6} - \sin\frac{\pi}{2}\right)$$

62.

$$5\sin 3\alpha\sin 4\alpha = 5\cdot\tfrac{1}{2}[\cos(3\alpha-4\alpha) - \cos(3\alpha+4\alpha)]$$
$$= \tfrac{5}{2}[\cos(-\alpha) - \cos(7\alpha)]$$
$$= \tfrac{5}{2}[\cos\alpha - \cos 7\alpha]$$

64. $\cos 2\theta\cos 4\theta = \tfrac{1}{2}[\cos(2\theta-4\theta) + \cos(2\theta+4\theta)]$

$$= \tfrac{1}{2}[\cos(-2\theta) + \cos 6\theta]$$
$$= \tfrac{1}{2}(\cos 2\theta + \cos 6\theta)$$

66. $\cos 120^\circ + \cos 30^\circ = 2\cos\left(\dfrac{120^\circ + 30^\circ}{2}\right)\cos\left(\dfrac{120^\circ - 30^\circ}{2}\right)$

$$= 2\cos 75^\circ\cos 45^\circ$$

68. $\sin x + \sin 7x = 2\sin\left(\frac{x+7x}{2}\right)\cos\left(\frac{x-7x}{2}\right)$

$= 2\sin 4x\cos(-3x)$

$= 2\sin 4x\cos 3x$

70. $\cos(\phi + 2\pi) + \cos\phi = 2\cos\left(\frac{\phi + 2\pi + \phi}{2}\right)\cos\left(\frac{\phi + 2\pi - \phi}{2}\right) = 2\cos(\phi + \pi)\cos\pi = -2\cos(\phi + \pi)$

72. $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right) = 2\sin\left(\frac{x + (\pi/2) + x - (\pi/2)}{2}\right)\cos\left(\frac{x + (\pi/2) - x + (\pi/2)}{2}\right)$

$= 2\sin x\cos\frac{\pi}{2} = 0$

74. $h(x) = \cos 2x - \cos 6x$

$\cos 2x - \cos 6x = 0$

$-2\sin 4x\sin(-2x) = 0$

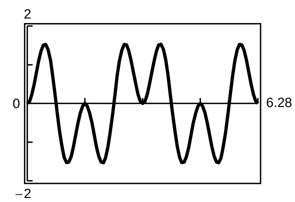
$2\sin 4x\sin 2x = 0$

$\sin 4x = 0$ or $\sin 2x = 0$

$4x = n\pi$ | $2x = n\pi$

$x = \frac{n\pi}{4}$ | $x = \frac{n\pi}{2}$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$ | $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$



76. $f(x) = \sin^2 3x - \sin^2 x$

$\sin^2 3x - \sin^2 x = 0$

$(\sin 3x + \sin x)(\sin 3x - \sin x) = 0$

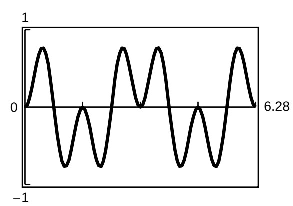
$(2\sin 2x\cos x)(2\cos 2x\sin x) = 0$

$\sin 2x = 0 \implies x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ or

$\cos x = 0 \implies x = \frac{\pi}{2}, \frac{3\pi}{2}$ or

$\cos 2x = 0 \implies x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ or

$\sin x = 0 \implies x = 0, \pi$



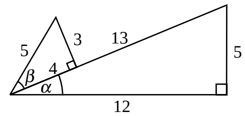


Figure for Exercises 78 and 80

78. $\cos^2 \alpha = (\cos \alpha)^2 = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

80. $\cos \alpha \sin \beta = \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{36}{65}$

$$\cos \alpha \sin \beta = \sin\left(\frac{\pi}{2} - \alpha\right)\cos\left(\frac{\pi}{2} - \beta\right) = \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{36}{65}$$

82. $$\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1}{\cos^2 \theta - \sin^2 \theta} = \frac{1/\cos^2 \theta}{1 - (\sin^2 \theta/\cos^2 \theta)} = \frac{\sec^2 \theta}{1 - \tan^2 \theta} = \frac{\sec^2 \theta}{1 - (\sec^2 \theta - 1)} = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$$

84. $$\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = (\cos 2x)(1) = \cos 2x$$

86. $$1 + \cos 10y = 1 + \cos^2 5y - \sin^2 5y = 1 + \cos^2 5y - (1 - \cos^2 5y) = 2 \cos^2 5y$$

88. $\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1}{\sin u} - \frac{\cos u}{\sin u} = \csc u - \cot u$

90. $$\sin 4\beta = 2 \sin 2\beta \cos 2\beta = 2[2 \sin \beta \cos \beta(\cos^2 \beta - \sin^2 \beta)] = 2[2 \sin \beta \cos \beta(1 - \sin^2 \beta - \sin^2 \beta)] = 4 \sin \beta \cos \beta(1 - 2 \sin^2 \beta)$$

Graph: $y_1 = \sin 4\beta$
$y_2 = 4 \sin \beta \cos \beta(1 - \sin^2 \beta)$

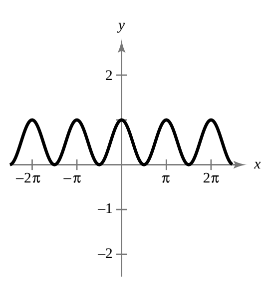
92. $$\frac{\cos 3x - \cos x}{\sin 3x - \sin x} = \frac{-2 \sin\left(\frac{3x + x}{2}\right) \sin\left(\frac{3x - x}{2}\right)}{2 \cos\left(\frac{3x + x}{2}\right) \sin\left(\frac{3x - x}{2}\right)} = \frac{-2 \sin 2x \sin x}{2 \cos 2x \sin x} = -\tan 2x$$

94. $f(x) = \cos^2 x = \frac{1 + \cos 2x}{2} = \frac{1}{2} + \frac{\cos 2x}{2}$

Shifted upward by $\frac{1}{2}$ unit.

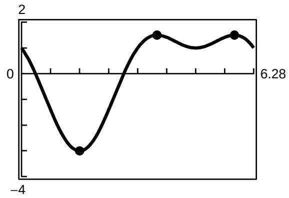
Amplitude: $|a| = \frac{1}{2}$

Period: $\frac{2\pi}{2} = \pi$



96. $f(x) = \cos 2x - 2\sin x$

(a)



Maximum points: $(3.6652, 1.5), (5.7596, 1.5)$

Minimum points: $(1.5708, -3)$

(b) $-2\cos x(2\sin x + 1) = 0$

$-2\cos x = 0$ or $2\sin x + 1 = 0$

$\cos x = 0$ $\qquad$ $\sin x = -\dfrac{1}{2}$

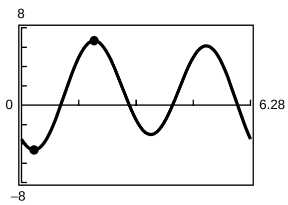
$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\qquad$ $x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\dfrac{\pi}{2} \approx 1.5708$ $\qquad$ $\dfrac{7\pi}{6} \approx 3.6652$

$\dfrac{3\pi}{2} \approx 4.7124$ $\qquad$ $\dfrac{11\pi}{6} \approx 5.7596$

98. $f(x) = 2\sin\dfrac{x}{2} - 5\cos\left(2x - \dfrac{\pi}{4}\right)$

(a)



Maximum point: $(1.9907, 6.6705)$

Minimum point: $(0.3434, -4.6340)$

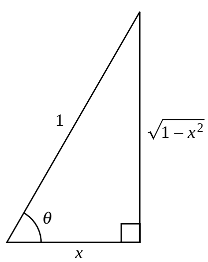
(b) $10\sin\left(2x - \dfrac{\pi}{4}\right) + \cos\dfrac{x}{2} = 0$

$x \approx 0.343, 1.991, 3.544, 5.064$

The first and second solutions correspond to the maximum and minimum points in part (a).

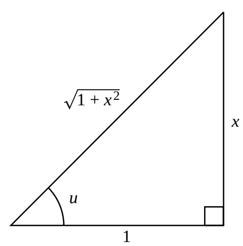
100. Let $u = \arccos x$.

$$\cos(2\arccos x) = \cos^2(\arccos x) - \sin^2(\arccos x)$$
$$= x^2 - (1 - x^2) = 2x^2 - 1$$



102. Let $u = \arctan x$.

$$\sin(2\arctan x) = 2\sin(\arctan x)\cos(\arctan x)$$
$$= 2\,\frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}}$$
$$= \frac{2x}{1+x^2}$$



104. (a) $\sin\left(\dfrac{\theta}{2}\right) = \dfrac{b/2}{10} \implies b = 20\sin\dfrac{\theta}{2}$

$\cos\left(\dfrac{\theta}{2}\right) = \dfrac{h}{10} \implies h = 10\cos\dfrac{\theta}{2}$

$$A = \frac{1}{2}bh = \frac{1}{2}\left(20\sin\frac{\theta}{2}\right)\left(10\cos\frac{\theta}{2}\right)$$
$$= 100\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

(b) $A = 50\left(2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right) = 50\sin\theta$

(c) The area is maximum when $\theta = \dfrac{\pi}{2}, A = 50$.