87. $x = 0$: $y = -\frac{1}{2}(0 - 10) + 14 = 5 + 14 = 19$. y-intercept: $(0, 19)$

$y = 0$: $0 = -\frac{1}{2}(x - 10) + 14 = -\frac{1}{2}x + 19 \implies x = 38$. x-intercept: $(38, 0)$

89. $x = 0$: $|2(0) - 9| - 5 = 9 - 5 = 4$. y-intercept: $(0, 4)$

$y = 0$: $|2x - 9| = 5 \implies x = 7, 2$. x-intercepts: $(2, 0), (7, 0)$

91. $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ because $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

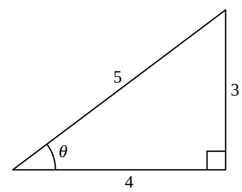
93. $\arcsin 1 = \frac{\pi}{2}$ because $\sin \frac{\pi}{2} = 1$.

Section 5.5 Multiple-Angle and Product-Sum Formulas

- You should know the following double-angle formulas.
  (a) $\sin 2u = 2 \sin u \cos u$
  (b) $\cos 2u = \cos^2 u - \sin^2 u$
  $= 2\cos^2 u - 1$
  $= 1 - 2\sin^2 u$
  (c) $\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$
- You should be able to reduce the power of a trigonometric function.
  (a) $\sin^2 u = \dfrac{1 - \cos 2u}{2}$ (b) $\cos^2 u = \dfrac{1 + \cos 2u}{2}$ (c) $\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$
- You should be able to use the half-angle formulas.
  (a) $\sin \dfrac{u}{2} = \pm\sqrt{\dfrac{1 - \cos u}{2}}$ (b) $\cos \dfrac{u}{2} = \pm\sqrt{\dfrac{1 + \cos u}{2}}$ (c) $\tan \dfrac{u}{2} = \dfrac{1 - \cos u}{\sin u} = \dfrac{\sin u}{1 + \cos u}$
- You should be able to use the product-sum formulas.
  (a) $\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$ (b) $\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$
  (c) $\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$ (d) $\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$
- You should be able to use the sum-product formulas.
  (a) $\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$ (b) $\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$
  (c) $\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$ (d) $\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$

Solutions to Odd-Numbered Exercises

Figure for Exercises 1–7



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

1. $\sin\theta = \frac{3}{5}$

3. $$\begin{aligned}\cos 2\theta &= 2\cos^2\theta - 1\\ &= 2\left(\tfrac{4}{5}\right)^2 - 1\\ &= \tfrac{32}{25} - \tfrac{25}{25}\\ &= \tfrac{7}{25}\end{aligned}$$

5. $$\begin{aligned}\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}\\ &= \frac{2(3/4)}{1-(3/4)^2}\\ &= \frac{3/2}{1-(9/16)}\\ &= \frac{3}{2}\cdot\frac{16}{7}\\ &= \frac{24}{7}\end{aligned}$$

7. $$\begin{aligned}\csc 2\theta &= \frac{1}{\sin 2\theta}\\ &= \frac{1}{2\sin\theta\cos\theta}\\ &= \frac{1}{2(3/5)(4/5)}\\ &= \frac{25}{24}\end{aligned}$$

9. Solutions: 0, 1.047, 3.142, 5.236

$$\sin 2x - \sin x = 0$$
$$2\sin x\cos x - \sin x = 0$$
$$\sin x(2\cos x - 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$
$$x = 0, \pi \qquad \cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$
$$x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

11. Solutions: 0.1263, 1.4445, 3.2679, 4.5860

13. Solutions: 1.047, 3.142, 5.236

$$\cos 2x = -\cos x$$
$$2\cos^2 x - 1 = -\cos x$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0$$
$$2\cos x = 1 \quad \text{or} \quad \cos x = -1$$
$$\cos x = \frac{1}{2} \qquad x = \pi$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

15. Solutions: 0, 1.571, 3.142, 4.712

$$\sin 4x = -2\sin 2x$$
$$\sin 4x + 2\sin 2x = 0$$
$$2\sin 2x\cos 2x + 2\sin 2x = 0$$
$$2\sin 2x(\cos 2x + 1) = 0$$
$$2\sin 2x = 0 \quad \text{or} \quad \cos 2x + 1 = 0$$
$$\sin 2x = 0 \qquad \cos 2x = -1$$
$$2x = n\pi \qquad 2x = \pi + 2n\pi$$
$$x = \frac{n}{2}\pi \qquad x = \frac{\pi}{2} + n\pi$$
$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

17. $8\sin x\cos x = 4(2\sin x\cos x) = 4\sin 2x$

19. $5 - 10\sin^2 x = 5(1 - 2\sin^2 x) = 5\cos 2x$

21. $\sin u = \frac{3}{5},\ 0 < u < \frac{\pi}{2} \implies \cos u = \frac{4}{5}$

$$\sin 2u = 2\sin u\cos u = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$
$$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$
$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2(3/4)}{1 - (9/16)} = \frac{24}{7}$$

23. $\tan u = \frac{1}{2},\ \pi < u < \frac{3\pi}{2} \implies \sin u = -\frac{1}{\sqrt{5}}$ and

$$\cos u = -\frac{2}{\sqrt{5}}$$
$$\sin 2u = 2\sin u\cos u = 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$$
$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$$
$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2(1/2)}{1 - (1/4)} = \frac{4}{3}$$

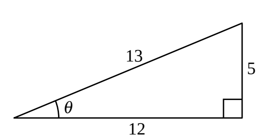
25. $\cos^4 x = (\cos^2 x)(\cos^2 x) = \left(\frac{1 + \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right) = \frac{1 + 2\cos 2x + \cos^2 2x}{4}$

$$= \frac{1 + 2\cos 2x + (1 + \cos 4x)/2}{4}$$
$$= \frac{2 + 4\cos 2x + 1 + \cos 4x}{8}$$
$$= \frac{3 + 4\cos 2x + \cos 4x}{8}$$
$$= \frac{1}{8}(3 + 4\cos 2x + \cos 4x)$$

27. $$(\sin^2 x)(\cos^2 x) = \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$$
$$= \frac{1 - \cos^2 2x}{4}$$
$$= \frac{1}{4}\left(1 - \frac{1 + \cos 4x}{2}\right)$$
$$= \frac{1}{8}(2 - 1 - \cos 4x)$$
$$= \frac{1}{8}(1 - \cos 4x)$$

29. $$\sin^2 x \cos^4 x = \sin^2 x \cos^2 x \cos^2 x = \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$$
$$= \frac{1}{8}(1 - \cos 2x)(1 + \cos 2x)(1 + \cos 2x)$$
$$= \frac{1}{8}(1 - \cos^2 2x)(1 + \cos 2x)$$
$$= \frac{1}{8}(1 + \cos 2x - \cos^2 2x - \cos^3 2x)$$
$$= \frac{1}{8}\left[1 + \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) - \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]$$
$$= \frac{1}{16}[2 + 2\cos 2x - 1 - \cos 4x - \cos 2x - \cos 2x \cos 4x]$$
$$= \frac{1}{16}\left[1 + \cos 2x - \cos 4x - \left(\frac{1}{2}\cos 2x + \frac{1}{2}\cos 6x\right)\right]$$
$$= \frac{1}{32}(2 + 2\cos 2x - 2\cos 4x - \cos 2x - \cos 6x)$$
$$= \frac{1}{32}(2 + \cos 2x - 2\cos 4x - \cos 6x)$$

Figure for Exercises 31 – 35



$\sin \theta = \frac{5}{13}$

$\cos \theta = \frac{12}{13}$

31. $$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + (12/13)}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$

33. $$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta} = \frac{5/13}{1 + (12/13)} = \frac{5}{25} = \frac{1}{5}$$

35. $$\csc \frac{\theta}{2} = \frac{1}{\sin(\theta/2)} = \frac{1}{\sqrt{(1 - \cos \theta)/2}} = \frac{1}{\sqrt{\left(1 - \frac{12}{13}\right)/2}} = \frac{1}{\sqrt{1/26}} = \sqrt{26}$$

37. $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin\theta = \frac{5}{13}$

39. $\sin 15° = \sin\left(\frac{1}{2}\cdot 30°\right) = \sqrt{\frac{1-\cos 30°}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}} = \frac{1}{2}\sqrt{2-\sqrt{3}}$

$\cos 15° = \cos\left(\frac{1}{2}\cdot 30°\right) = \sqrt{\frac{1+\cos 30°}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \frac{1}{2}\sqrt{2+\sqrt{3}}$

$\tan 15° \tan\left(\frac{1}{2}\cdot 30°\right) = \frac{\sin 30°}{1+\cos 30°} = \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$

41. $\sin 112° 30' = \sin\left(\frac{1}{2}\cdot 225°\right) = \sqrt{\frac{1-\cos 225°}{2}} = \sqrt{\frac{1+(\sqrt{2}/2)}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\cos 112° 30' = \cos\left(\frac{1}{2}\cdot 225°\right) = -\sqrt{\frac{1+\cos 225°}{2}} = -\sqrt{\frac{1-(\sqrt{2}/2)}{2}} = -\frac{1}{2}\sqrt{2-\sqrt{2}}$

$\tan 112° 30' = \tan\left(\frac{1}{2}\cdot 225°\right) = \frac{\sin 225°}{1+\cos 225°} = \frac{-\sqrt{2}/2}{1-(\sqrt{2}/2)} = -1-\sqrt{2}$

43. $\sin\frac{\pi}{8} = \sin\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1-\cos(\pi/4)}{2}} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

$\cos\frac{\pi}{8} = \cos\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1+\cos(\pi/4)}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\tan\frac{\pi}{8} = \tan\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \frac{\sin(\pi/4)}{1+\cos(\pi/4)} = \frac{\sqrt{2}/2}{1+(\sqrt{2}/2)} = \sqrt{2}-1$

45. $\sin\frac{3\pi}{8} = \sin\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \sqrt{\frac{1-\cos(3\pi/4)}{2}} = \sqrt{\frac{1+\sqrt{2}/2}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\cos\frac{3\pi}{8} = \cos\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \sqrt{\frac{1+\cos 3\pi/4}{2}} = \sqrt{\frac{1-\sqrt{2}/2}{2}} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

$\tan\frac{3\pi}{8} = \tan\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \frac{\sin\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2-\sqrt{2}} = \sqrt{2}+1$

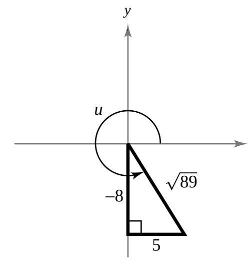
47. $\sin u = \frac{5}{13}, \frac{\pi}{2} < u < \pi \implies \cos u = -\frac{12}{13}$

$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+(12/13)}{2}} = \frac{5\sqrt{26}}{26}$

$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1-(12/13)}{2}} = \frac{\sqrt{26}}{26}$

$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u} = \frac{5/13}{1-(12/13)} = \frac{5}{1} = 5$

49. $\tan u = -\frac{8}{5}, \frac{3\pi}{2} < u < 2\pi$. Quadrant IV



$\sin u = -\frac{8}{\sqrt{89}}, \cos u = \frac{5}{\sqrt{89}}$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - (5/\sqrt{89})}{2}}$$

$$= \sqrt{\frac{\sqrt{89} - 5}{2\sqrt{89}}} = \sqrt{\frac{89 - 5\sqrt{89}}{178}}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 + (5/\sqrt{89})}{2}} = -\sqrt{\frac{\sqrt{89} + 5}{2\sqrt{89}}} = -\sqrt{\frac{89 + 5\sqrt{89}}{178}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{5}{\sqrt{89}}}{\frac{-8}{\sqrt{89}}} = \frac{5 - \sqrt{89}}{8}$$

51. $\sqrt{\frac{1 - \cos 6x}{2}} = |\sin 3x|$

53. $-\sqrt{\frac{1 - \cos 8x}{1 + \cos 8x}} = -\frac{\sqrt{(1 - \cos 8x)/2}}{\sqrt{(1 + \cos 8x)/2}}$

$$= -\left|\frac{\sin 4x}{\cos 4x}\right|$$

$$= -|\tan 4x|$$

55. $\sin \frac{x}{2} - \cos x = 0$

$$\pm\sqrt{\frac{1 - \cos x}{2}} = \cos x$$

$$\frac{1 - \cos x}{2} = \cos^2 x$$

$$0 = 2\cos^2 x + \cos x - 1$$

$$= (2\cos x - 1)(\cos x + 1)$$

$\cos x = \frac{1}{2}$ or $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad x = \pi$

By checking these values in the original equations, we see that $x = \pi/3$ and $x = 5\pi/3$ are the only solutions. $x = \pi$ is extraneous.

57. $\cos \frac{x}{2} - \sin x = 0$

$$\pm\sqrt{\frac{1 + \cos x}{2}} = \sin x$$

$$\frac{1 + \cos x}{2} = \sin^2 x$$

$$1 + \cos x = 2\sin^2 x$$

$$1 + \cos x = 2 - 2\cos^2 x$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad x = \pi$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

$\pi/3$, π, and $5\pi/3$ are all solutions to the equation.

59. $6 \sin \frac{\pi}{3} \cos \frac{\pi}{3} = 6 \cdot \frac{1}{2}\left[\sin\left(\frac{\pi}{3} + \frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3} - \frac{\pi}{3}\right)\right]$

$$= 3\left[\sin \frac{2\pi}{3} + \sin 0\right] = 3 \sin \frac{2\pi}{3}$$

61. $\sin 5\theta \cos 3\theta = \frac{1}{2}[\sin(5\theta + 3\theta) + \sin(5\theta - 3\theta) = \frac{1}{2}(\sin 8\theta + \sin 2\theta)$

63. $5\cos(-5\beta)\cos 3\beta = 5 \cdot \frac{1}{2}[\cos(-5\beta - 3\beta) + \cos(-5\beta + 3\beta)] = \frac{5}{2}[\cos(-8\beta) + \cos(-2\beta)]$
$= \frac{5}{2}(\cos 8\beta + \cos 2\beta)$

65. $\sin 60° + \sin 30° = 2\sin\left(\frac{60° + 30°}{2}\right)\cos\left(\frac{60° - 30°}{2}\right) = 2\sin 45° \cos 15°$

67. $\sin 5\theta - \sin\theta = 2\cos\left(\frac{5\theta + \theta}{2}\right)\sin\left(\frac{5\theta - \theta}{2}\right) = 2\cos 3\theta \cdot \sin 2\theta$

69. $\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2\cos\left(\frac{\alpha + \beta + \alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta - \alpha + \beta}{2}\right) = 2\cos\alpha\sin\beta$

71. $\cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right) = -2\sin\left(\frac{\theta + (\pi/2) + \theta - (\pi/2)}{2}\right)\sin\left(\frac{\theta + (\pi/2) - \theta + (\pi/2)}{2}\right)$
$= -2\sin\theta\sin\frac{\pi}{2} = -2\sin\theta$

73. $\sin 6x + \sin 2x = 0$

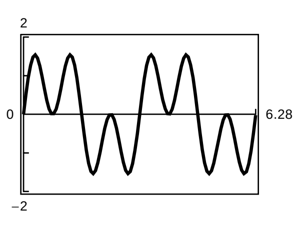
$2\sin\left(\frac{6x + 2x}{2}\right)\cos\left(\frac{6x - 2x}{2}\right) = 0$

$\sin 4x \cos 2x = 0$

$\sin 4x = 0$ or $\cos 2x = 0$

$4x = n\pi$ $\qquad 2x = \frac{\pi}{2} + n\pi$

$x = \frac{n\pi}{4}$ $\qquad x = \frac{\pi}{4} + \frac{n\pi}{2}$



In the interval we have

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}.$

75. $\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$

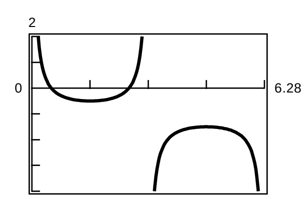
$\frac{\cos 2x}{\sin 3x - \sin x} = 1$

$\frac{\cos 2x}{2\cos 2x \sin x} = 1$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$



77. $\sin^2\alpha = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$

$\sin^2\alpha = 1 - \cos^2\alpha = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}$

79. $\sin\alpha\cos\beta = \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{4}{13}$

$$\sin\alpha\cos\beta = \cos\left(\frac{\pi}{2} - \alpha\right)\sin\left(\frac{\pi}{2} - \beta\right) = \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{4}{13}$$

81. $$\csc 2\theta = \frac{1}{\sin 2\theta} = \frac{1}{2\sin\theta\cos\theta} = \frac{1}{\sin\theta}\cdot\frac{1}{2\cos\theta} = \frac{\csc\theta}{2\cos\theta}$$

83. $$\cos^2 2\alpha - \sin^2 2\alpha = \cos[2(2\alpha)] = \cos 4\alpha$$

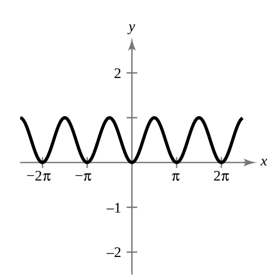
85. $$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x = (\sin^2 x + \cos^2 x) + 2\sin x\cos x = 1 + \sin 2x$$

87. $$\sec\frac{u}{2} = \pm\frac{1}{\cos(u/2)} = \pm\sqrt{\frac{2}{1+\cos u}} = \pm\sqrt{\frac{2\sin u}{\sin u(1+\cos u)}} = \pm\sqrt{\frac{2\sin u}{\sin u + \sin u\cos u}} = \pm\sqrt{\frac{(2\sin u)/(\cos u)}{(\sin u)/(\cos u) + (\sin u\cos u)/(\cos u)}} = \pm\sqrt{\frac{2\tan u}{\tan u + \sin u}}$$

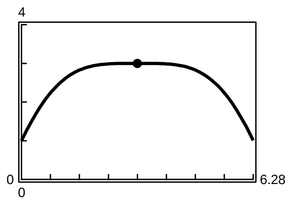
89. $$\cos 3\beta = \cos(2\beta + \beta) = \cos 2\beta\cos\beta - \sin 2\beta\sin\beta = (\cos^2\beta - \sin^2\beta)\cos\beta - 2\sin\beta\cos\beta\sin\beta = \cos^3\beta - \sin^2\beta\cos\beta - 2\sin^2\beta\cos\beta = \cos^3\beta - 3\sin^2\beta\cos\beta$$

91. $$\frac{\cos 4x - \cos 2x}{2\sin 3x} = \frac{-2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)}{2\sin 3x} = \frac{-2\sin 3x\sin x}{2\sin 3x} = -\sin x$$

93. $$\sin^2 x = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{\cos 2x}{2}$$



95. (a) $y = 4\sin\frac{x}{2} + \cos x$



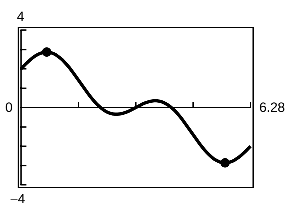
Maximum: $(\pi, 3)$

(b) $2\cos\frac{x}{2} - \sin x = 0$

$$2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right) = \sin x$$
$$4\left(\frac{1+\cos x}{2}\right) = \sin^2 x$$
$$2(1+\cos x) = 1 - \cos^2 x$$

$$\cos^2 x + 2\cos x + 1 = 0$$
$$(\cos x + 1)^2 = 0$$
$$\cos x = -1$$
$$x = \pi$$

97. (a) $y = \cos\frac{x}{2} + \sin 2x$

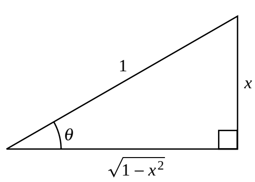


Maximum: $(0.699, 2.864)$

Minimum: $(5.584, -2.864)$

(b) $2\cos 2x - \sin\frac{x}{2} = 0$ has 4 zeros on $[0, 2\pi)$. Two of the zeros are $x = 0.699$ and $x = 5.584$. (The other two are 2.608, 3.675)

99. $\sin(2\arcsin x) = 2\sin(\arcsin x)\cos(\arcsin x) = 2x\sqrt{1-x^2}$



101. $\cos(2\arcsin x) = 1 - 2\sin^2(\arcsin x)$

$$= 1 - 2x^2.$$

103. $r = \frac{1}{32}v_0\sin 2\theta$

$$= \frac{1}{32}v_0(2\sin\theta\cos\theta)$$
$$= \frac{1}{16}v_0\sin\theta\cos\theta$$

105. $\sin\frac{\theta}{2} = \frac{1}{M}$

$$\sin\frac{\theta}{2} = \frac{1}{4.5}$$
$$\frac{\theta}{2} = \arcsin\frac{1}{4.5} \approx 0.2241$$
$$\theta \approx 0.4482 \text{ or } 25.68^\circ$$

107. False. If $x = \pi$, $\sin\frac{x}{2} = \sin\frac{\pi}{2} = 1$, whereas

$$-\sqrt{\frac{1-\cos\pi}{2}} = -1$$