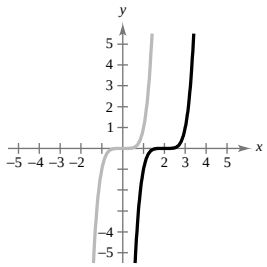
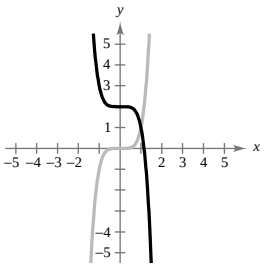
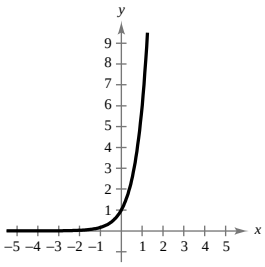
108.



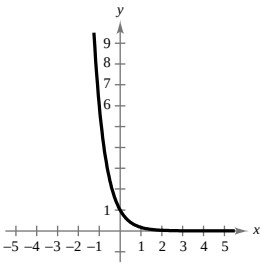
110.



112.



114.



Section 4.2 Trigonometric Functions: The Unit Circle

Solutions to Even-Numbered Exercises

2. $(x, y) = \left(\frac{12}{13}, \frac{5}{13}\right)$

$\sin t = y = \frac{5}{13}$

$\cos t = x = \frac{12}{13}$

$\tan t = \frac{y}{x} = \frac{5/13}{12/13} = \frac{5}{12}$

$\csc t = \frac{1}{y} = \frac{1}{5/13} = \frac{13}{5}$

$\sec t = \frac{1}{x} = \frac{1}{12/13} = \frac{13}{12}$

$\cot t = \frac{x}{y} = \frac{12/13}{5/13} = \frac{12}{5}$

4. $(x, y) = \left(-\frac{4}{5}, -\frac{3}{5}\right)$

$\sin t = y = -\frac{3}{5}$

$\cos t = x = -\frac{4}{5}$

$\tan t = \frac{y}{x} = \frac{-3/5}{-4/5} = \frac{3}{4}$

$\csc t = \frac{1}{y} = \frac{1}{-3/5} = -\frac{5}{3}$

$\sec t = \frac{1}{x} = \frac{1}{-4/5} = -\frac{5}{4}$

$\cot t = \frac{x}{y} = \frac{-4/5}{-3/5} = \frac{4}{3}$

6. $t = \frac{\pi}{3} \Rightarrow \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

8. $t = \frac{5\pi}{4} \Rightarrow \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

10. $t = \frac{5\pi}{3}$ corresponds to $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

12. $t = \pi \Rightarrow (-1, 0)$

14. $t = \dfrac{\pi}{3}$ corresponds to the point:

$$(x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\sin\frac{\pi}{3} = y = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{3} = x = \frac{1}{2}$$

$$\tan\frac{\pi}{3} = \frac{y}{x} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

16. $t = -\dfrac{\pi}{4}$ corresponds to the point:

$$(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$\sin\left(-\frac{\pi}{4}\right) = y = -\frac{\sqrt{2}}{2}$$

$$\cos\left(-\frac{\pi}{4}\right) = x = \frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{\pi}{4}\right) = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

18. $t = -\dfrac{4\pi}{3}$ corresponds to $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

$$\sin t = y = \frac{\sqrt{3}}{2}$$

$$\cos t = x = -\frac{1}{2}$$

$$\tan t = \frac{y}{x} = -\sqrt{3}$$

20. $t = \dfrac{5\pi}{3}$ corresponds to $\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

$$\sin t = y = -\frac{\sqrt{3}}{2}$$

$$\cos t = x = \frac{1}{2}$$

$$\tan t = \frac{y}{x} = -\sqrt{3}$$

22. $t = -2\pi$ corresponds to the point: $(x, y) = (1, 0)$

$$\sin(-2\pi) = y = 0$$

$$\cos(-2\pi) = x = 1$$

$$\tan(-2\pi) = \frac{y}{x} = \frac{0}{1} = 0$$

24. $t = \dfrac{5\pi}{6}$ corresponds to $\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$$\sin t = y = \frac{1}{2}$$

$$\cos t = x = -\frac{\sqrt{3}}{2}$$

$$\tan t = \frac{y}{x} = -\frac{1}{\sqrt{3}}$$

$$\csc t = \frac{1}{y} = 2$$

$$\sec t = \frac{1}{x} = -\frac{2}{\sqrt{3}}$$

$$\cot t = \frac{x}{y} = -\sqrt{3}$$

26. $t = \dfrac{3\pi}{2}$ corresponds to the point: $(x, y) = (0, -1)$

$$\sin\frac{3\pi}{2} = y = -1$$

$$\cos\frac{3\pi}{2} = x = 0$$

$$\tan\frac{3\pi}{2} = \frac{y}{x} = \frac{-1}{0} \implies \text{undefined}$$

$$\csc\frac{3\pi}{2} = \frac{1}{y} = \frac{1}{-1} = -1$$

$$\sec\frac{3\pi}{2} = \frac{1}{x} = \frac{1}{0} \implies \text{undefined}$$

$$\cot\frac{3\pi}{2} = \frac{x}{y} = \frac{0}{-1} = 0$$

28. $t = -\dfrac{3\pi}{2}$ corresponds to $(0, 1)$

$$\sin t = y = 1$$

$$\cos t = x = 0$$

$$\tan t = \frac{y}{x} \text{ undefined}$$

$$\csc t = \frac{1}{y} = 1$$

$$\sec t = \frac{1}{x} \text{ undefined}$$

$$\cot t = \frac{x}{y} = 0$$

30. Because $5\pi = 4\pi + \pi$:

$$\cos 5\pi = \cos(4\pi + \pi) = \cos \pi = -1$$

32. Because $\frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$:

$$\sin \frac{9\pi}{4} = \sin\left(2\pi + \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

34. $\sin(-3\pi) = \sin \pi = 0$

36. Because $-\frac{8\pi}{3} = -4\pi + \frac{4\pi}{3}$:

$$\cos\left(-\frac{8\pi}{3}\right) = \cos\left(-4\pi + \frac{4\pi}{3}\right) = \cos \frac{4\pi}{3} = -\frac{1}{2}$$

38. $\sin(-t) = \frac{3}{8}$

(a) $\sin t = -\sin(-t) = -\frac{3}{8}$

(b) $\csc t = \frac{1}{\sin(t)} = \frac{1}{-\sin(-t)} = -\frac{8}{3}$

40. $\cos t = -\frac{3}{4}$

(a) $\cos(-t) = \cos t = -\frac{3}{4}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = \frac{1}{\cos t} = -\frac{4}{3}$

42. $\cos t = \frac{4}{5}$

(a) $\cos(\pi - t) = -\cos t = -\frac{4}{5}$

(b) $\cos(t + \pi) = -\cos t = -\frac{4}{5}$

44. $\tan \frac{\pi}{3} \approx 1.7321$

46. $\cot 1 = \frac{1}{\tan 1} \approx 0.6421$

48. $\cos(-2.5) \approx -0.8011$

50. $\sec 1.8 = \frac{1}{\cos 1.8} \approx -4.4014$

52. $\sin(-0.9) \approx -0.7833$

54. (a) $\sin 0.75 = y \approx 0.7$

(b) $\cos 2.5 = x \approx -0.8$

56. (a) $\sin t = -0.75$

$t \approx 4.0$ or $t \approx 5.4$

(b) $\cos t = 0.75$

$t \approx 0.72$ or $t \approx 5.56$

58. At $t = 1.4$, $I \approx 5e^{-2(1.4)} \sin 1.4 \approx 0.2996$ amperes

60. $y(t) = \frac{1}{4}e^{-t} \cos 6t$

(a) When $t = 0$: $y(0) = \frac{1}{4}e^{-0} \cos 0 = 0.2500$ foot

(b) When $t = \frac{1}{4}$: $y\left(\frac{1}{4}\right) = \frac{1}{4}e^{-1/4} \cos\left(6 \cdot \frac{1}{4}\right) \approx 0.0138$ foot

(c) When $t = \frac{1}{2}$: $y\left(\frac{1}{2}\right) = \frac{1}{2}e^{-1/2} \cos\left(6 \cdot \frac{1}{2}\right) \approx -0.1501$ foot

62. $\sin(0.25) + \sin(0.75) \approx 0.2474 + 0.6816 = 0.9290$

$\sin 1 \approx 0.8415$

Therefore, $\sin t_1 + \sin t_2 \neq \sin(t_1 + t_2)$

64. True

66. (a) The points (x_1, y_1) and (x_2, y_2) are symmetric about the origin.

(b) Because of the symmetry of the points, you can make the conjecture that $\sin(t_1 + \pi) = -\sin t_1$.

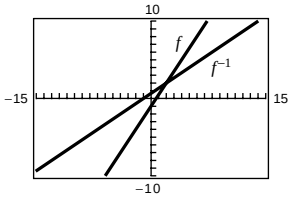
(c) Because of the symmetry of the points, you can make the conjecture that $\cos(t_1 + \pi) = -\cos t_1$.

68. Let $h(t) = f(t)g(t)$
$= \sin t \cos t.$

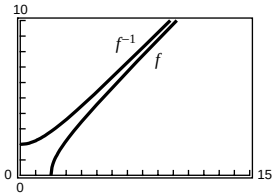
Then, $h(-t) = \sin(-t)\cos(-t)$
$= -\sin t \cos t$
$= -h(t).$

Thus, $h(t)$ is odd.

70.
$$f(x) = \tfrac{1}{2}(3x - 2)$$
$$y = \tfrac{1}{2}(3x - 2)$$
$$x = \tfrac{1}{2}(3y - 2)$$
$$2x = 3y - 2$$
$$2x + 2 = 3y$$
$$\tfrac{2}{3}(x + 1) = y$$
$$f^{-1}(x) = \tfrac{2}{3}(x + 1)$$

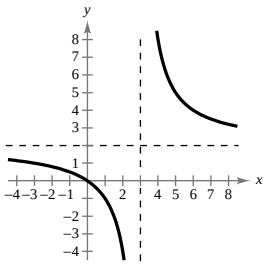


72.
$$f(x) = \sqrt{x^2 - 4},\ x \geq 2,\ y \geq 0$$
$$y = \sqrt{x^2 - 4}$$
$$x = \sqrt{y^2 - 4}$$
$$x^2 = y^2 - 4$$
$$x^2 + 4 = y^2$$
$$\sqrt{x^2 + 4} = y,\ x \geq 0$$
$$f^{-1}(x) = \sqrt{x^2 + 4},\ x \geq 0$$



74. $f(x) = \dfrac{2x}{x - 3}$

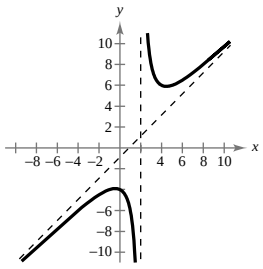
Asymptotes: $x = 3, y = 1$



76. $f(x) = \dfrac{x^2 - 3x + 8}{x - 2} = x - 1 + \dfrac{6}{x - 2}$

Slant asymptote: $y = x - 1$

Vertical asymptote: $x = 2$



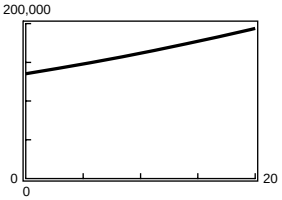
78. $C(10{,}000) = 50$ dollars/pound

$C(100{,}000) = 9.5$ dollars/pound

$C(1{,}000{,}000) = 5.45$ dollars/pound

As $x \to \infty$, $C \to 5$ dollars/pound.

80.



$P = 190{,}000$ when $t \approx 18.9$ or late 2008