19. $(-1, 9)$

$r = \sqrt{(-1-0)^2 + (9-0)^2} = \sqrt{1+81} = \sqrt{82}$

$x^2 + y^2 = 82$

20. $(6, -3)$

$r = \sqrt{(6-0)^2 + (-3-0)^2} = \sqrt{36+9} = \sqrt{45}$

$x^2 + y^2 = 45$

21. $(6, -6)$

$r = \sqrt{(6-0)^2 + (-6-0)^2} = \sqrt{36+36} = \sqrt{72}$

$x^2 + y^2 = 72$

22. $(-7, 8)$

$r = \sqrt{(-7-0)^2 + (8-0)^2} = \sqrt{49+64} = \sqrt{113}$

$x^2 + y^2 = 113$

23. $(35)^2 + (56)^2 \stackrel{?}{\leq} (65)^2$

$1225 + 3136 \stackrel{?}{\leq} 4225$

$4361 \not\leq 4225$ no $\sqrt{35^2 + 56^2} \approx 66$ miles

Technology Activity 10.3 (p. 608)

1. $x^2 + y^2 = 121$

$-18 \leq x \leq 18$

$-12 \leq y \leq 12$

2. $x^2 + y^2 = 50$

$-12 \leq x \leq 12$

$-8 \leq y \leq 8$

3. $x^2 + y^2 = 484$

$-36 \leq x \leq 36$

$-24 \leq y \leq 24$

4. $5x^2 + 5y^2 = 120$

$-9 \leq x \leq 9$

$-6 \leq y \leq 6$

5. $x^2 + y^2 = \frac{16}{9}$

$-3 \leq x \leq 3$

$-2 \leq y \leq 2$

6. $\frac{1}{2}x^2 + \frac{1}{2}y^2 = 72$

$-24 \leq x \leq 24$

$-16 \leq y \leq 16$

7. $\frac{4}{5}x^2 + \frac{4}{5}y^2 = 20$

$-9 \leq x \leq 9$

$-6 \leq y \leq 6$

8. $9x^2 + 9y^2 = 4$

$-1.5 \leq x \leq 1.5$

$-1 \leq y \leq 1$

9. $125x^2 + 125y^2 = 1000$

$-6 \leq x \leq 6$

$-4 \leq y \leq 4$

Lesson 10.4

10.4 Guided Practice (p. 612)

1. (a) vertices (b) co-vertices (c) foci (d) major axis

2. Write the equation in standard form; if the larger denominator is under x^2, the major axis is horizontal; if the larger denominator is under y^2, the major axis is vertical.

3. Find a and b, then use $c = \sqrt{a^2 - b^2}$. The foci are at $(\pm c, 0)$ for vertices at $(\pm a, 0)$, and at $(0, \pm c)$ for vertices at $(0, \pm a)$.

4. The student incorrectly identified a as 2 and b as 3. Since the major axis is horizontal, $a = 3$ and $b = 2$. The correct equation is $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

5. $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Vertex: $(0, 5)$

Co-vertex: $(-4, 0)$

6. $\frac{x^2}{81} + \frac{y^2}{4} = 1$

Vertex: $(9, 0)$

Co-vertex: $(0, 2)$

7. $\frac{x^2}{49} + \frac{y^2}{9} = 1$

Vertex: $(-7, 0)$

Focus: $\left(-2\sqrt{10}, 0\right)$

$b^2 = a^2 - c^2$

$b^2 = (-7)^2 - \left(-2\sqrt{10}\right)^2$

$b^2 = 49 - 40 = 9$

$b = \pm 3$

8. $\frac{x^2}{144} + \frac{y^2}{169} = 1$

Vertex: $(0, 13)$

Focus: $(0, -5)$

$b^2 = a^2 - c^2$

$b^2 = (13)^2 - (-5)^2$

$b^2 = 169 - 25 = 144$

$b = 12$

9. $\frac{x^2}{91} + \frac{y^2}{100} = 1$

Co-vertex: $\left(\sqrt{91}, 0\right)$

Focus: $(0, 3)$

$a^2 = c^2 + b^2$

$a^2 = (3)^2 + \left(\sqrt{91}\right)^2$

$a^2 = 9 + 91 = 100$

$a = 10$

10. $\frac{x^2}{49} + \frac{y^2}{33} = 1$

Co-vertex: $\left(0, \sqrt{33}\right)$

Focus: $(4, 0)$

$a^2 = c^2 + b^2$

$a^2 = (4)^2 + \left(\sqrt{33}\right)^2$

$a^2 = 16 + 33$

$a^2 = 49$

$a = 7$

11. $\frac{x^2}{49} + \frac{y^2}{25} = 1$

Vertex: $(-7, 0), (7, 0)$

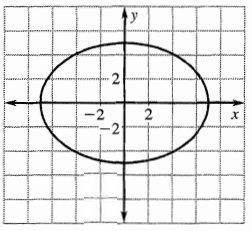
Co-vertices: $(0, -5), (0, 5)$

$c^2 = (7)^2 - (5)^2$

$c^2 = 49 - 25 = 24$

$c = \sqrt{24} \approx \pm 4.9$

Foci: $(-4.9, 0), (4.9, 0)$



12. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

Vertices: $(0, -4), (0, 4)$

Co-vertices: $(-3, 0), (3, 0)$

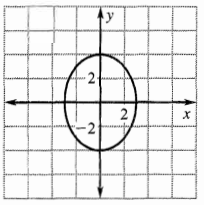
$c^2 = a^2 - b^2$

$c^2 = (4)^2 - (3)^2$

$c^2 = 16 - 9 = 7$

$c = \sqrt{7} \approx \pm 2.65$

Foci: $(0, -2.65), (0, 2.65)$



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13. $\frac{x^2}{30} + \frac{y^2}{4} = 1$

Vertices: $(-5.48, 0), (5.48, 0)$

Co-vertices: $(0, -2), (0, 2)$

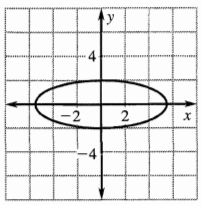
$c^2 = a^2 - b^2$

$c^2 = \left(\sqrt{30}\right)^2 - (2)^2$

$c^2 = 30 - 4 = 26$

$c = \sqrt{26} \approx \pm 5.1$

Foci: $(-5.1, 0), (5.1, 0)$



14. $\frac{x^2}{64} + \frac{y^2}{45} = 1$

Vertices: $(-8, 0), (8, 0)$

Co-vertices: $\left(0, -\sqrt{45}\right), \left(0, \sqrt{45}\right)$

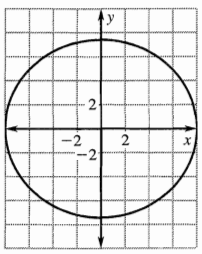
$c^2 = a^2 - b^2$

$c^2 = (8)^2 - \left(\sqrt{45}\right)^2$

$c^2 = 64 - 45 = 19$

$c = \sqrt{19} \approx \pm 4.39$

Foci: $(-4.36, 0), (4.36, 0)$



15. $75x^2 + 36y^2 = 2700$

$\frac{75x^2}{2700} + \frac{36y^2}{2700} = \frac{2700}{2700}$

$\frac{x^2}{36} + \frac{y^2}{75} = 1$

Vertices: $\left(0, -\sqrt{75}\right), \left(0, \sqrt{75}\right)$

Co-vertices: $(-6, 0), (6, 0)$

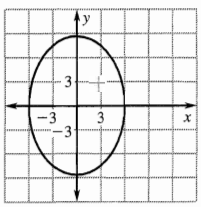
$c^2 = a^2 - b^2$

$c^2 = (8.66)^2 - (6)^2$

$c^2 = 75 - 36 = 39$

$c = \sqrt{39} \approx \pm 6.24$

Foci: $(0, -6.24), (0, 6.24)$



16. $81x^2 + 63y^2 = 5103$

$\frac{81x^2}{5103} + \frac{63y^2}{5103} = \frac{5103}{5103}$

$\frac{x^2}{63} + \frac{y^2}{81} = 1$

Vertices: $(0, -9), (0, 9)$

Co-vertices: $\left(-\sqrt{63}, 0\right), \left(\sqrt{63}, 0\right)$

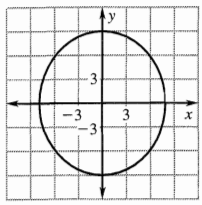
$c^2 = a^2 - b^2$

$c^2 = (9)^2 - (7.94)^2$

$c^2 = 81 - 63 = 18$

$c = \sqrt{18} \approx \pm 4.24$

Foci: $(0, -4.24), (0, 4.24)$



17. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Vertices: $(-5, 0), (5, 0)$

Co-vertices: $(0, -3), (0, 3)$

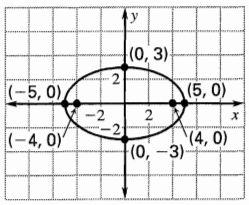
$c^2 = a^2 - b^2$

$c^2 = (5)^2 - (3)^2$

$c^2 = 25 - 9 = 16$

$c = \pm 4$

Foci: $(-4, 0), (4, 0)$



10.4 Practice and Applications (pp. 612–614)

18. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Vertices: $(-5, 0), (5, 0)$

Co-vertices: $(0, -4), (0, 4)$

$c^2 = a^2 - b^2 = (5)^2 - (4)^2 = 25 - 16 = 9$

$c = \sqrt{9} = \pm 3$

Foci: $(-3, 0), (3, 0)$

19. $\frac{x^2}{121} + \frac{y^2}{100} = 1$

Vertices: $(-11, 0), (11, 0)$

Co-vertices: $(0, -10), (0, 10)$

$c^2 = a^2 - b^2 = (11)^2 - (10)^2 = 121 - 100 = 21$

$c = \sqrt{21} \approx \pm 4.58$

Foci: $(-4.58, 0), (4.58, 0)$

20. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Vertices: $(0, -3), (0, 3)$

Co-vertices: $(-2, 0), (2, 0)$

$c^2 = a^2 - b^2 = (3)^2 - (2)^2 = 9 - 4 = 5$

$c = \sqrt{5} \approx \pm 2.24$

Foci: $(0, -2.24), (0, 2.24)$

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21. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Vertices: $(0, -5), (0, 5)$

Co-vertices: $(-3, 0), (3, 0)$

$c^2 = a^2 - b^2 = (5)^2 - (3)^2 = 25 - 9 = 16$

$c = \sqrt{16} = \pm 4$

Foci: $(0, -4), (0, 4)$

22. $\frac{x^2}{12} + \frac{y^2}{36} = 1$

Vertices: $(0, -6), (0, 6)$

Co-vertices: $\left(-\sqrt{12}, 0\right), \left(\sqrt{12}, 0\right)$

$c^2 = a^2 - b^2 = (6)^2 - \left(\sqrt{12}\right)^2 = 36 - 12 = 24$

$c = \sqrt{24} \approx \pm 4.9$

Foci: $(0, -4.9), (0, 4.9)$

23. $\frac{x^2}{28} + \frac{y^2}{20} = 1$

Vertices: $\left(-\sqrt{28}, 0\right), \left(\sqrt{28}, 0\right)$

Co-vertices: $\left(0, -\sqrt{20}\right), \left(0, \sqrt{20}\right)$

$c^2 = a^2 - b^2 = \left(\sqrt{28}\right)^2 - \left(\sqrt{20}\right)^2 = 28 - 20 = 8$

$c = \pm\sqrt{8} \approx \pm 2.83$

Foci: $(-2.83, 0), (2.83, 0)$

24. $16x^2 + y^2 = 16$

$\frac{16x^2}{16} + \frac{y^2}{16} = \frac{16}{16}$

$\frac{x^2}{1} + \frac{y^2}{16} = 1$

Vertices: $(0, -4), (0, 4)$

Co-vertices: $(-1, 0), (1, 0)$

$c^2 = a^2 - b^2 = (4)^2 - (1)^2 = 16 - 1 = 15$

$c = \pm\sqrt{15} \approx \pm 3.87$

Foci: $(0, -3.87), (0, 3.87)$

25. $49x^2 + 4y^2 = 196$

$\frac{49x^2}{196} + \frac{4y^2}{196} = \frac{196}{196}$

$\frac{x^2}{4} + \frac{y^2}{49} = 1$

Vertices: $(0, \pm 7)$

Co-vertices: $(\pm 2, 0)$

$c^2 = a^2 - b^2 = (7)^2 - (2)^2 = 49 - 4 = 45$

$c = \pm\sqrt{45} = \pm\sqrt{9 \cdot 5} = \pm 3\sqrt{5}$

Foci: $\left(0, \pm 3\sqrt{5}\right)$

26. $9x^2 + 100y^2 = 900$

$\frac{9x^2}{900} + \frac{100y^2}{900} = \frac{900}{900}$

$\frac{x^2}{100} + \frac{y^2}{9} = 1$

Vertices: $(\pm 10, 0)$

Co-vertices: $(0, \pm 3)$

$c^2 = a^2 - b^2 = (10)^2 - (3)^2 = 100 - 9 = 91$

$c = \pm\sqrt{91}$

Foci: $\left(\pm\sqrt{91}, 0\right)$

27. $x^2 + 10y^2 = 10$

$\frac{x^2}{10} + \frac{10y^2}{10} = \frac{10}{10}$

$\frac{x^2}{10} + y^2 = 1$

Vertices: $\left(\pm\sqrt{10}, 0\right)$

Co-vertices: $(0, \pm 1)$

$c^2 = a^2 - b^2 = \left(\sqrt{10}\right) - (1)^2 = 10 - 1 = 9$

$c = \pm 3$

Foci: $(\pm 3, 0)$

28. $10x^2 + 25y^2 = 250$

$\frac{10x^2}{250} + \frac{25y^2}{250} = \frac{250}{250}$

$\frac{x^2}{25} + \frac{y^2}{10} = 1$

Vertices: $(\pm 5, 0)$

Co-vertices: $\left(0, \pm\sqrt{10}\right)$

$c^2 = a^2 - b^2 = 25 - 10 = 15$

$c = \pm\sqrt{15}$

Foci: $\left(\pm\sqrt{15}, 0\right)$

29. $25x^2 + 15y^2 = 375$

$\frac{25x^2}{375} + \frac{15y^2}{375} = \frac{375}{375}$

$\frac{x^2}{15} + \frac{y^2}{25} = 1$

Vertices: $(0, \pm 5)$

Co-vertices: $\left(\pm\sqrt{15}, 0\right)$

$c^2 = a^2 - b^2$

$c^2 = 25 - 15 = 10$

$c = \pm\sqrt{10}$

Foci: $\left(0, \pm\sqrt{10}\right)$

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30. $\frac{x^2}{16} + \frac{y^2}{36} = 1$

Vertices: $(0, \pm 6)$

Co-vertices: $(\pm 4, 0)$

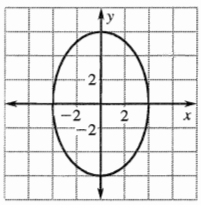
Foci: $(0, \pm 2\sqrt{5})$

$c^2 = a^2 - b^2$

$c^2 = 36 - 16 = 20$

$c = \pm\sqrt{20} = \pm\sqrt{4 \cdot 5}$

$c = \pm 2\sqrt{5}$



31. $\frac{x^2}{4} + \frac{y^2}{49} = 1$

Vertices: $(0, \pm 7)$

Co-vertices: $(\pm 2, 0)$

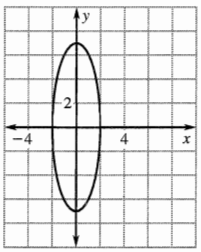
Foci: $(0, \pm 3\sqrt{5})$

$c^2 = a^2 - b^2$

$c^2 = 49 - 4 = 45$

$c = \pm\sqrt{45}$

$c = \pm 3\sqrt{5}$



32. $\frac{x^2}{36} + \frac{y^2}{64} = 1$

Vertices: $(0, \pm 8)$

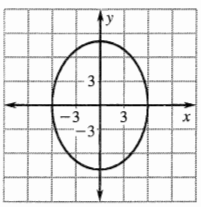
Co-vertices: $(\pm 6, 0)$

Foci: $(0, \pm 2\sqrt{7})$

$c^2 = a^2 - b^2$

$c^2 = 64 - 36 = 28$

$c = \pm\sqrt{28} = \pm 2\sqrt{7}$



33. $\frac{x^2}{49} + \frac{y^2}{144} = 1$

Vertices: $(0, \pm 12)$

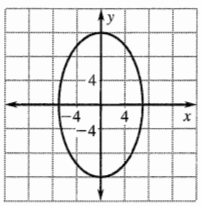
Co-vertices: $(\pm 7, 0)$

Foci: $(0, \pm\sqrt{95})$

$c^2 = a^2 - b^2$

$c^2 = 144 - 49 = 95$

$c = \pm\sqrt{95}$



34. $\frac{x^2}{196} + \frac{y^2}{100} = 1$

Vertices: $(\pm 14, 0)$

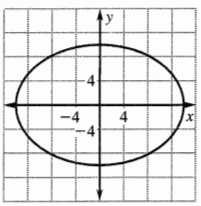
Co-vertices: $(0, \pm 10)$

Foci: $(\pm\sqrt{96}, 0)$

$c^2 = a^2 - b^2$

$c^2 = 196 - 100 = 96$

$c = \pm\sqrt{96}$



35. $\frac{x^2}{256} + \frac{y^2}{36} = 1$

Vertices: $(\pm 16, 0)$

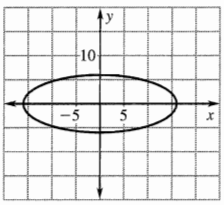
Co-vertices: $(0, \pm 6)$

Foci: $(\pm\sqrt{220}, 0)$

$c^2 = a^2 - b^2$

$c^2 = 256 - 36 = 220$

$c = \pm\sqrt{220}$



36. $\frac{x^2}{225} + \frac{y^2}{81} = 1$

Vertices: $(\pm 15, 0)$

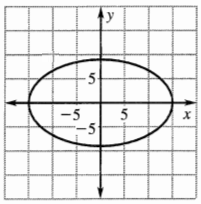
Co-vertices: $(0, \pm 9)$

Foci: $(\pm 12, 0)$

$c^2 = a^2 - b^2$

$= 225 - 81 = 144$

$c = \pm 12$



37. $\frac{x^2}{121} + \frac{y^2}{169} = 1$

Vertices: $(0, \pm 13)$

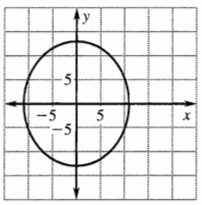
Co-vertices: $(\pm 11, 0)$

Foci: $(0, \pm 4\sqrt{3})$

$c^2 = a^2 - b^2$

$c^2 = 169 - 121 = 48$

$c = \pm\sqrt{48} = 4\sqrt{3}$



38. $\frac{x^2}{144} + \frac{y^2}{400} = 1$

Vertices: $(0, \pm 20)$

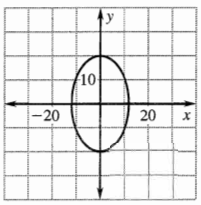
Co-vertices: $(\pm 12, 0)$

Foci: $(0, \pm 16)$

$c^2 = a^2 - b^2$

$c^2 = 400 - 144 = 256$

$c = \pm 16$



39. $\frac{x^2}{49} + \frac{y^2}{64} = 1$

Vertices: $(0, \pm 8)$

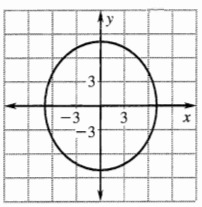
Co-vertices: $(\pm 7, 0)$

Foci: $(0, \pm\sqrt{15})$

$c^2 = a^2 - b^2$

$c^2 = 64 - 49 = 15$

$c = \pm\sqrt{15}$



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40. $\frac{x^2}{4} + \frac{y^2}{1} = 100$

$\frac{x^2}{400} + \frac{y^2}{100} = \frac{100}{100}$

$\frac{x^2}{400} + \frac{y^2}{100} = 1$

Vertices: $(\pm 20, 0)$

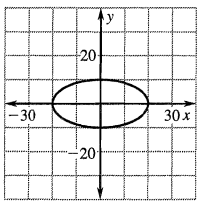
Co-vertices: $(0, \pm 10)$

Foci: $(\pm 10\sqrt{3}, 0)$

$c^2 = a^2 - b^2$

$c^2 = 400 - 100 = 300$

$c = \pm\sqrt{300} = \pm 10\sqrt{3}$



41. $\frac{x^2}{4} + \frac{y^2}{25} = \frac{1}{4}$

$4\left(\frac{x^2}{4} + \frac{y^2}{25}\right) = \frac{1}{4}(4)$

$\frac{x^2}{1} + \frac{y^2}{6.25} = 1$

Vertices: $(0, \pm 2.5)$

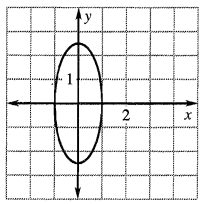
Co-vertices: $(\pm 1, 0)$

Foci: $(0, \pm 2.29)$

$c^2 = a^2 - b^2$

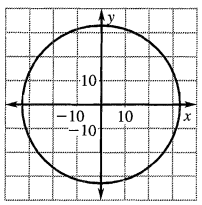
$c^2 = 6.25 - 1 = 5.25$

$c = \pm\sqrt{5.25} \approx \pm 2.29$



42. $x^2 + y^2 = 33^2$

$r = 33$



43. $64x^2 + 25y^2 = 1600$

$\frac{64x^2}{1600} + \frac{25y^2}{1600} = \frac{1600}{1600}$

$\frac{x^2}{25} + \frac{y^2}{64} = 1$

Vertices: $(0, \pm 8)$

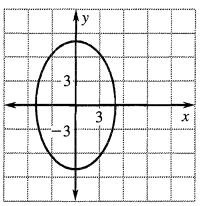
Co-vertices: $(\pm 5, 0)$

Foci: $(0, \pm\sqrt{39})$

$c^2 = a^2 - b^2$

$c^2 = 64 - 25 = 39$

$c = \pm\sqrt{39}$



44. $24y + x^2 = 0$

$x^2 = -24y$

$4p = -24$

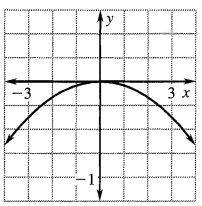
$p = -6$

Focus: $(0, -6)$

Directrix: $y = 6$

$p < 0$ parabola opens down

x	-1	-2	-3
y	± 4.9	± 6.93	± 8.49



45. $72x^2 = 144y$

$x^2 = 2y$

$4p = 2$

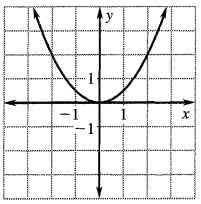
$p = \frac{1}{2}$

Focus: $(0, \frac{1}{2})$

Directrix: $y = -\frac{1}{2}$

$p > 0$ parabola opens up

x	1	2	3
y	± 1.41	± 2	± 2.45

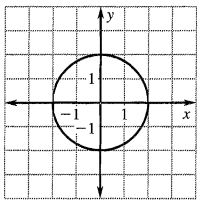


46. $24x^2 + 24y^2 = 96$

$\frac{24x^2}{24} + \frac{24y^2}{24} = \frac{96}{24}$

$x^2 + y^2 = 4$

$r = 2$



47. $\frac{x^2}{81} + \frac{4y}{9} = 1$

$x^2 + 36y = 81$

$36y = 81 - x^2$

$y = \frac{81}{36} - \frac{x^2}{36}$

$y = -\frac{1}{36}x^2 + \frac{9}{4}$

$\frac{-b}{2a} = \frac{0}{2\left(-\frac{1}{36}\right)} = 0$

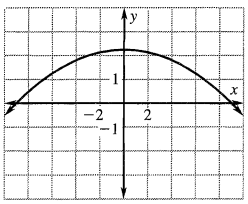
$y = -\frac{1}{36}(0) + 0(0) + \frac{9}{4}$

$y = \frac{9}{4}$

Vertex: $\left(0, \frac{9}{4}\right)$

Axis of symmetry $x = 0$

x	0	6	-6
y	$\frac{9}{4}$	$\frac{5}{4}$	$\frac{5}{4}$



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48. $\frac{3x^2}{12} + \frac{5y^2}{500} = 1$

$\frac{x^2}{4} + \frac{y^2}{100} = 1$

Vertices: $(0, \pm 10)$

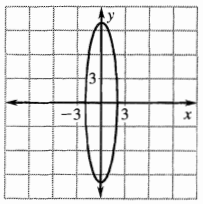
Co-vertices: $(\pm 2, 0)$

Foci: $(0, \pm\sqrt{96})$

$c^2 = a^2 - b^2$

$c^2 = 100 - 4 = 96$

$c = \pm\sqrt{96}$

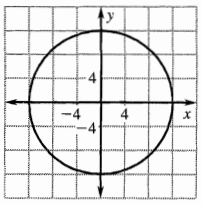


49. $\frac{x^2}{36} + \frac{y^2}{36} = 4$

$36\left(\frac{x^2}{36} + \frac{y^2}{36}\right) = 4(36)$

$x^2 + y^2 = 144$

$r = 12$



50. $5x^2 + 9y^2 = 45$

$\frac{5x^2}{45} + \frac{9y^2}{45} = \frac{45}{45}$

$\frac{x^2}{9} + \frac{y^2}{5} = 1$

Vertices: $(\pm 3, 0)$

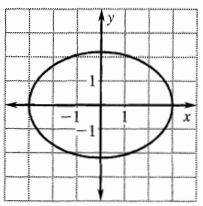
Co-vertices: $(0, \pm\sqrt{5})$

Foci: $(\pm 2, 0)$

$c^2 = a^2 - b^2$

$c^2 = 9 - 5 = 4$

$c = \pm 2$



51. Vertex: $(0, 6)$

Co-vertex: $(5, 0)$

$\frac{x^2}{25} + \frac{y^2}{36} = 1$

52. Vertex: $(0, 6)$

Co-vertex: $(-2, 0)$

$\frac{x^2}{4} + \frac{y^2}{36} = 1$

53. $\frac{x^2}{(-4)^2} + \frac{y^2}{(3)^2} = 1$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

54. $\frac{x^2}{(-1)^2} + \frac{y^2}{(-7)^2} = 1$

$x^2 + \frac{y^2}{49} = 1$

55. $\frac{x^2}{(9)^2} + \frac{y^2}{(-8)^2} = 1$

$\frac{x^2}{81} + \frac{y^2}{64} = 1$

56. $\frac{x^2}{(10)^2} + \frac{y^2}{(4)^2} = 1$

$\frac{x^2}{100} + \frac{y^2}{16} = 1$

57. $b^2 = a^2 - c^2$

$b^2 = (7)^2 - (3)^2$

$b^2 = 49 - 9 = 40$

$\frac{x^2}{40} + \frac{y^2}{49} = 1$

58. $b^2 = a^2 - c^2$

$b^2 = (-5)^2 - \left(2\sqrt{6}\right)^2$

$b^2 = 25 - 24 = 1$

$\frac{x^2}{25} + y^2 = 1$

59. $b^2 = a^2 - c^2$

$b^2 = (8)^2 - \left(-4\sqrt{3}\right)^2 = 64 - 48 = 16$

$\frac{x^2}{16} + \frac{y^2}{64} = 1$

60. $b^2 = a^2 - c^2$

$b^2 = (15)^2 - (12)^2$

$b^2 = 225 - 144 = 81$

$\frac{x^2}{225} + \frac{y^2}{81} = 1$

61. $b^2 = a^2 - c^2$

$b^2 = (5)^2 - (-3)^2$

$b^2 = 25 - 9 = 16$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

62. $b^2 = a^2 - c^2$

$b^2 = (-30)^2 - (20)^2$

$b^2 = 900 - 400 = 500$

$\frac{x^2}{500} + \frac{y^2}{900} = 1$

63. $a^2 = c^2 + b^2$

$a^2 = (-3)^2 + \left(\sqrt{55}\right)^2$

$a^2 = 9 + 55 = 64$

$\frac{x^2}{55} + \frac{y^2}{64} = 1$

64. $a^2 = c^2 + b^2$

$a^2 = (-1)^2 + \left(-\sqrt{3}\right)^2$

$a^2 = 1 + 3 = 4$

$\frac{x^2}{4} + \frac{y^2}{3} = 1$

65. $a^2 = c^2 + b^2$

$a^2 = (9)^2 + \left(-2\sqrt{10}\right)^2$

$a^2 = 81 + 40 = 121$

$\frac{x^2}{40} + \frac{y^2}{121} = 1$

66. $a^2 = c^2 + b^2$

$a^2 = (3)^2 + \left(-3\sqrt{3}\right)^2$

$a^2 = 9 + 27 = 36$

$\frac{x^2}{36} + \frac{y^2}{27} = 1$

67. $a^2 = c^2 + b^2$

$a^2 = (-7)^2 + \left(5\sqrt{11}\right)^2$

$a^2 = 49 + 275 = 324$

$\frac{x^2}{275} + \frac{y^2}{324} = 1$

68. $a^2 = c^2 + b^2$

$a^2 = (-2)^2 + \left(-\sqrt{77}\right)^2$

$a^2 = 4 + 77 = 81$

$\frac{x^2}{81} + \frac{y^2}{77} = 1$

69. $(23)^2 = 529$

$(48.5)^2 = 2352.25$

$\frac{x^2}{2352.25} + \frac{y^2}{529} = 1$

$c^2 = a^2 - b^2$

$c^2 = 2352.25 - 529$

$= 1823.25$

$c = \pm\sqrt{1823.25} \approx \pm 42.7$

70. $2(42.7) \approx 85.4$ ft.

about 85.4 ft

71. $A = \pi ab = \pi(48.5)(23)$

≈ 3504 ft^2

about 3500 ft^2

72. $a - c = 4132$

$a + c = 4583$

$2a = 8715$

$a = 4357.5$

$4357.5 + c = 4583$

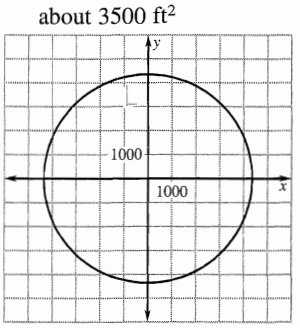
$c = 225.5$

$b^2 = a^2 - c^2$

$b^2 = (4357.5)^2 - (225.5)^2$

$b = 4351.7$

$\frac{x^2}{(4357.5)^2} + \frac{y^2}{(4351.7)^2} = 1$



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73. $\frac{185}{2} = 92.5$

$\frac{155}{2} = 77.5$

$\frac{x^2}{(92.5)^2} + \frac{y^2}{(77.5)^2} = 1$

74. $\frac{135}{2} = 67.5$

$\frac{110}{2} = 55$

$\frac{x^2}{(67.5)^2} + \frac{y^2}{(55)^2} = 1$

75. $A = \pi ab$

$\pi(67.5)(55) \le A \le \pi(92.5)(77.5)$

$3712.5\pi \le A \le 7168.75\pi$

76. a. The greatest distance the boat can travel between islands is 20 miles, going straight out to a point and then straight to the other island. This is the definition of an ellipse.

b. $(\pm 6, 0)$

c. 20 mi; $(\pm 10, 0)$

d. $b^2 = a^2 - c^2 = (10)^2 - (6)^2 = 100 - 36 = 64$

$\frac{x^2}{100} + \frac{y^2}{64} = 1$

77. The distance from one focus to a vertex is $a - c$. The distance from the vertex to the other focus is $a + c$. So the distance from focus to vertex to other focus is $(a + c) + (a - c)$, or $2a$. The distance from each focus to a co-vertex is $\sqrt{b^2 + c^2}$, so from focus to co-vertex to other focus is $2\sqrt{b^2 + c^2}$. By the definition of an ellipse, these distances must be equal, so $2a = 2\sqrt{b^2 + c^2}$, $a^2 = b^2 + c^2$, so $c^2 = a^2 - b^2$.

10.4 Mixed Review (p. 614)

78. $125^{2/3} = \sqrt[3]{(125)^2} = \sqrt[3]{(5^3)^2} = \sqrt[3]{5^6} = \sqrt[3]{(5^2)^3}$

$= 5^2 = 25$

79. $-8^{5/3} = \sqrt[3]{(-8)^5} = \sqrt[3]{(-8)^3(8)^2} = \sqrt[3]{(-8)^3(4)^3}$

$= -8(4) = -32$

80. $4^{5/2} = \sqrt{4^5} = \sqrt{4^2 \cdot 4^2 \cdot 4} = 4(4)\sqrt{4}$

$= 4(4)(2) = 32$

81. $27^{-2/6} = \frac{1}{(27)^{1/3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{\sqrt[3]{3^3}} = \frac{1}{3}$

82. $4^{7/2} = \sqrt{4^7} = \sqrt{(4)^2(4)^2(4)^2 4} = 4(4)(4)\sqrt{4}$

$= 4(4)(4)(2) = 128$

83. $81^{3/4} = \sqrt[4]{(81)^3} = \sqrt[4]{(9^2)^3}\ \sqrt[4]{(9)^6} = \sqrt[4]{(9)^4(9)^2} = 9\sqrt[4]{(3^2)^2}$

$= 9\sqrt[4]{(3)^4} = 9(3) = 27$

84. $64^{-2/3} = \frac{1}{(64)^{2/3}} = \frac{1}{\sqrt[3]{(64)^2}} = \frac{1}{\sqrt[3]{(8^2)^2}} = \frac{1}{\sqrt[3]{8^4}}$

$= \frac{1}{\sqrt[3]{(8)^3 \cdot 8}} = \frac{1}{8\sqrt[3]{(2)^3}} = \frac{1}{8(2)} = \frac{1}{16}$

85. $32^{4/5} = \sqrt[5]{(32)^4} = \sqrt[5]{(2^5)^4} = \sqrt[5]{(2)^{20}} = \sqrt[5]{(2^4)^5}$

$= (2)^4 = 16$

86. $x = 3, y = -2$

$y = -\frac{6}{x}$

87. $x = 4, y = 6$

$y = \frac{24}{x}$

88. $x = 5, y = 1$

$y = \frac{5}{x}$

89. $x = 8, y = 9$

$y = \frac{72}{x}$

90. $x = 9, y = 2$

$y = \frac{18}{x}$

91. $x = 0.5, y = 24$

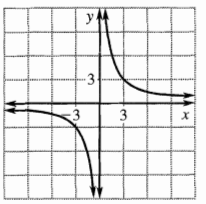
$y = \frac{12}{x}$

92. $f(x) = \frac{9}{x}$

Domain: $x \neq 0$

Range: $y \neq 0$

x	1	-1	2	-2
y	9	-9	$\frac{9}{2}$	$-\frac{9}{2}$

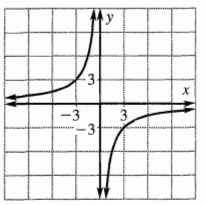


93. $f(x) = -\frac{9}{x}$

Domain: $x \neq 0$

Range: $y \neq 0$

x	1	-1	2	-2
y	9	-9	$\frac{9}{2}$	$-\frac{9}{2}$

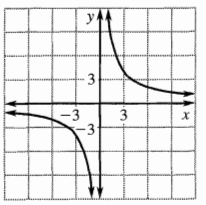


94. $f(x) = \frac{12}{x}$

Domain: $x \neq 0$

Range: $y \neq 0$

x	1	-1	2	-2
y	12	-12	6	-6

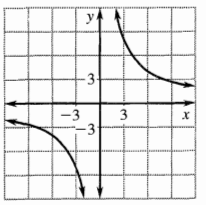


95. $f(x) = \frac{24}{x}$

Domain: $x \neq 0$

Range: $y \neq 0$

x	1	-1	2	-2
y	24	-24	12	-12



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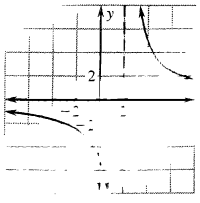
96. $f(x) = \frac{10}{x}$

D: $x \neq 0$

R: $y \neq 0$

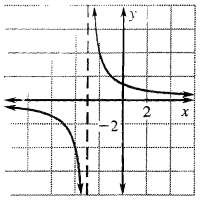


x	0	1	3	-1	-2
y	-5	-110	10	$-\frac{10}{3}$	$-\frac{5}{2}$

97. $f(x) = \frac{4}{x+3}$

Domain: $x \neq -3$

Range: $y = 0$



x	0	-2	2	-4	1	-1
y	$\frac{4}{3}$	4	$\frac{4}{5}$	-4	1	2

Lesson 10.5

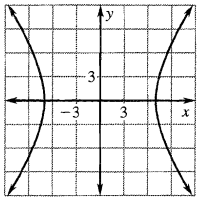
10.5 Guided Practice (p. 618)

1. Vertices; transverse axis

2. Both involve all points a certain distance from 2 foci; for an ellipse, the sum of distances is constant; for a hyperbola, the difference is constant.

3. Start at the vertices and draw curves that approach but don't meet the asymptotes.

4. $\frac{x^2}{49} - \frac{y^2}{81} = 1$



$a^2 = 49, b^2 = 81$

so $a = 7, b = 9, 2a = 14, 2b = 18$

Vertices $(\pm 7, 0)$

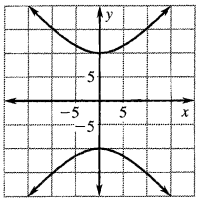
$c^2 = a^2 + b^2 = 49 + 81 = 130$

$c = \sqrt{130}$

Foci: $(\pm\sqrt{130}, 0)$

Asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{9}{7}x$

5. $\frac{y^2}{100} - \frac{x^2}{75} = 1$



$a^2 = 100, b^2 = 75$

so $a = 10, b = \sqrt{75}$

$2a = 10, 2b = 2\sqrt{75}$

Vertices: $(0, \pm 10)$

$c^2 = a^2 + b^2 = 100 + 75 = 175$

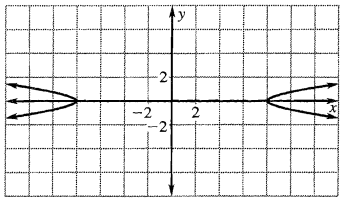
$c = \sqrt{175} = \pm 5\sqrt{7}$

Foci: $(0, \pm 5\sqrt{7})$

Asymptotes: $y = \pm\frac{a}{b}x = \pm\frac{10}{\sqrt{75}}$

$= \pm\frac{10}{5\sqrt{3}} = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$

6. $\frac{x^2}{64} - y^2 = 1$



$\frac{x^2}{64} - \frac{y^2}{1} = 1$

$a^2 = 64, b^2 = 1$

so $a = 8, b = 1$

$2a = 16, 2b = 2$

Vertices: $(\pm 8, 0)$

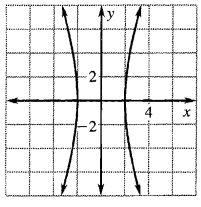
$c^2 = a^2 + b^2 = 64 + 1 = 65$

$c = \sqrt{65}$

Foci: $(\pm\sqrt{65}, 0)$

Asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{1}{8}x$

7. $36x^2 - 4y^2 = 144$



$\frac{36x^2}{144} - \frac{4y^2}{144} = \frac{144}{144}$

$\frac{x^2}{4} - \frac{y^2}{36} = 1$

$a^2 = 4, b^2 = 36$ so $a = 2, b = 6$

$2a = 4, 2b = 12$

Vertices: $(\pm 2, 0)$

$c^2 = a^2 + b^2 = 4 + 36 = 40$

$c = \sqrt{40} = \pm 2\sqrt{10}$

Foci: $(\pm 2\sqrt{10}, 0)$

Asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{6}{2}x$

$= \pm 3x$

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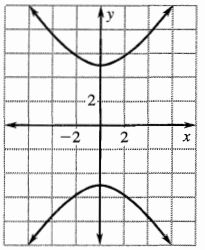
8. $12y^2 - 25x^2 = 300$

$\frac{12y^2}{300} - \frac{25x^2}{300} = \frac{300}{300}$

$\frac{y^2}{25} - \frac{x^2}{12} = 1$



$a^2 = 25, b^2 = 12, a = 5, b = \sqrt{12}$

$2a = 10, 2b = 2\sqrt{12}$

Vertices: $(0, \pm 5)$

$c^2 = a^2 + b^2 = 25 + 12 = 37, \ c = \sqrt{37}$

Foci: $\left(0, \pm\sqrt{37}\right)$

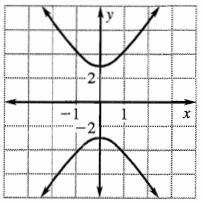
Asymptotes: $y = \pm\frac{a}{b}x = \pm\frac{5}{\sqrt{12}}x = \pm\frac{5}{2\sqrt{3}}x$

$= \pm\frac{10\sqrt{3}}{12} = \pm\frac{5\sqrt{3}}{6}$

9. $y^2 - 9x^2 = 9$

$\frac{y^2}{9} - \frac{9x^2}{9} = \frac{9}{9}$

$\frac{y^2}{9} - \frac{x^2}{1} = 1$



$a^2 = 9, b^2 = 1, a = 3, b = 1$

$2a = 18, 2b = 2$

Vertices: $(0, \pm 3)$

$c^2 = a^2 + b^2 = 9 + 1 = 10, \ c = \sqrt{10}$

Foci: $\left(0, \pm\sqrt{10}\right)$

Asymptotes: $y = \pm\frac{a}{b}x = \pm\frac{3}{1}x = \pm 3x$

10. Foci: $(0, -5), (0, 5)$ $c = 5$

Vertices: $(0, -3), (0, 3)$ $a = 3$

$b^2 = c^2 - a^2 = 25 - 9 = 16$

$\frac{y^2}{9} - \frac{x^2}{16} = 1$

11. Foci: $(-8, 0), (8, 0)$ $c = 8$

Vertices: $(-7, 0), (7, 0)$ $a = 7$

$b^2 = c^2 - a^2 = 64 - 49 = 15$

$\frac{x^2}{49} - \frac{y^2}{15} = 1$

12. Foci: $\left(-\sqrt{34}, 0\right), \left(\sqrt{34}, 0\right)$ $c = \sqrt{34}$

Vertices: $(-5, 0), (5, 0)$ $a = 5$

$b^2 = c^2 - a^2 = 34 - 25 = 9$

$\frac{x^2}{25} - \frac{y^2}{9} = 1$

13. Foci: $(0, -9), (0, 9)$ $c = 9$

Vertices: $\left(0, -3\sqrt{5}\right), \left(0, 3\sqrt{5}\right)$ $a = 3\sqrt{5}$

$b^2 = c^2 - a^2 = 81 - 45 = 36$

$\frac{y^2}{45} - \frac{x^2}{36} = 1$

14. $a^2 = 25, b^2 = 9, a = 5, b = 3$

$c^2 = a^2 + b^2 = 25 + 9 = 34, \ c = \sqrt{34}$

Vertices: $(\pm 5, 0)$

Foci: $\left(\pm\sqrt{34}, 0\right)$

lens at $\left(-\sqrt{34}, 0\right)$

Vertex: $(5, 0)$

distance between points is

$5 - \left(-\sqrt{34}\right) = 5 + \sqrt{34} \approx 10.8$ in.

Practice and Applications (pp. 618–620)

15. $\frac{x^2}{16} - \frac{y^2}{4} = 1$ C **16.** $\frac{y^2}{4} - \frac{x^2}{2} = 1$ A

17. $\frac{y^2}{16} - \frac{x^2}{4} = 1$ D **18.** $\frac{x^2}{4} - \frac{y^2}{2} = 1$ B

19. $36x^2 - 9y^2 = 324$

$\frac{36x^2}{324} - \frac{9y^2}{324} = \frac{324}{324}$

$\frac{x^2}{9} - \frac{y^2}{36} = 1$

20. $y^2 - 81x^2 = 81$

$\frac{y^2}{81} - \frac{81x^2}{81} = \frac{81}{81}$

$\frac{y^2}{81} - x^2 = 1$

21. $36y^2 - 4x^2 = 9$

$\frac{36y^2}{9} - \frac{4x^2}{9} = \frac{9}{9}$

$4y^2 - \frac{4}{9}x^2 = 1$

$\left(\frac{\frac{1}{4}}{\frac{1}{4}}\right)\left(4y^2 - \frac{4}{9}x^2\right) = 1\left(\frac{\frac{1}{4}}{\frac{1}{4}}\right)$

$\frac{y^2}{\left(\frac{1}{4}\right)} - \frac{x^2}{\left(\frac{9}{4}\right)} = 1$

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22. $16y^2 - 36x^2 + 9 = 0$

$$16y^2 - 36x^2 = -9$$

$$\frac{16y^2}{(-9)} - \frac{36x^2}{(-9)} = \left(\frac{-9}{-9}\right)$$

$$4x^2 - \frac{16y^2}{9} = 1$$

$$\left(\frac{\frac{1}{16}}{\frac{1}{16}}\right)\left(4x^2 - \frac{16}{9}y^2\right) = 1\left(\frac{\frac{1}{16}}{\frac{1}{16}}\right)$$

$$\frac{\frac{1}{4}x^2}{\frac{1}{16}} - \frac{\frac{y^2}{9}}{\frac{1}{16}} = 1$$

$$\left(\frac{4}{4}\right)\left(\frac{\frac{1}{4}x^2}{\frac{1}{16}} - \frac{\frac{y^2}{9}}{\frac{1}{16}}\right) = 1\left(\frac{4}{4}\right)$$

$$\frac{x^2}{\left(\frac{1}{4}\right)} - \frac{y^2}{\left(\frac{9}{16}\right)} = 1$$

23. $y^2 - \frac{x^2}{36} = 4$

$$\frac{y^2}{4} - \frac{x^2}{36 \cdot 4} = \frac{4}{4}$$

$$\frac{y^2}{4} - \frac{x^2}{144} = 1$$

24. $\frac{x^2}{9} - \frac{4y^2}{9} = 9$

$$\frac{x^2}{9(9)} - \frac{4y^2}{9(9)} = \frac{9}{9}$$

$$\frac{x^2}{81} - \frac{4y^2}{81} = 1$$

$$\left(\frac{\frac{1}{4}}{\frac{1}{4}}\right)\left(\frac{x^2}{81} - \frac{4y^2}{81}\right) = 1\left(\frac{\frac{1}{4}}{\frac{1}{4}}\right)$$

$$\frac{x^2}{81} - \frac{y^2}{\left(\frac{81}{4}\right)} = 1$$

25. $\frac{x^2}{9} - \frac{y^2}{64} = 1$

$a^2 = 9, a = 3$

Vertices: $(\pm 3, 0)$

$b^2 = 64, b = 8$

$c^2 = a^2 + b^2 = 9 + 64 = 73$

$c = \sqrt{73}$

Foci: $\left(\pm\sqrt{73}, 0\right)$

26. $\frac{y^2}{49} - x^2 = 1$

$a^2 = 49, a = 7$

Vertices: $(0, \pm 7)$

$b^2 = 1, b = 1$

$c^2 = a^2 + b^2 = 49 + 1 = 50$

$c = \sqrt{50}$

Foci: $\left(0, \pm\sqrt{50}\right)$

27. $\frac{x^2}{121} - \frac{y^2}{4} = 1$

$a^2 = 121, a = 11$

Vertices: $(\pm 11, 0)$

$b^2 = 4, b = 2$

$c^2 = a^2 + b^2 = 121 + 4 = 125$

$c = 5\sqrt{5}$

Foci: $\left(\pm 5\sqrt{5}, 0\right)$

28. $4y^2 - 81x^2 = 324$

$$\frac{4y^2}{324} - \frac{81x^2}{324} = \frac{324}{324}$$

$$\frac{y^2}{81} - \frac{x^2}{4} = 1$$

$a^2 = 81, a = 9$

Vertices: $(0, \pm 9)$

$b^2 = 4, b = 2$

$c^2 = a^2 + b^2 = 81 + 4 = 85$

$c = \sqrt{85}$

Foci: $\left(0, \pm\sqrt{85}\right)$

29. $25y^2 - 4x^2 = 100$

$$\frac{25y^2}{100} - \frac{4x^2}{100} = \frac{100}{100}$$

$$\frac{y^2}{4} - \frac{x^2}{25} = 1$$

$a^2 = 4, a = 2$

Vertices: $(0, \pm 2)$

$b^2 = 25, b = 5$

$c^2 + a^2 + b^2 = 4 + 25 = 29$

$c = \sqrt{29}$

Foci: $\left(0, \pm\sqrt{29}\right)$

30. $36x^2 - 10y^2 = 360$

$$\frac{36x^2}{360} - \frac{10y^2}{360} = \frac{360}{360}$$

$$\frac{x^2}{10} - \frac{y^2}{36} = 1$$

$a^2 = 10, a = \sqrt{10}$

Vertices: $\left(\pm\sqrt{10}, 0\right)$

—CONTINUED—

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30. —CONTINUED—

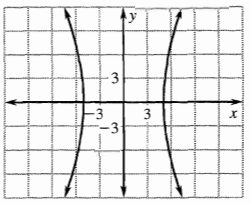
$b^2 = 36, b = 6$

$c^2 = a^2 + b^2 = 10 + 36 = 46$

$c = \sqrt{46}$

Foci: $(\pm\sqrt{46}, 0)$

31. $\frac{x^2}{25} - \frac{y^2}{121} = 1$



$a^2 = 25, b^2 = 121, a = 5, b = 11$

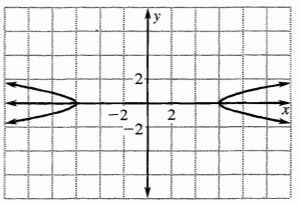
$c^2 = a^2 + b^2 = 25 + 121 = 146$

$c = \sqrt{146}$

Foci: $(\pm\sqrt{146}, 0)$

Asymptotes: $y = \pm\frac{11}{5}x$

32. $\frac{x^2}{36} - y^2 = 1$



$a^2 = 36, a = 6, b^2 = 1, b = 1$

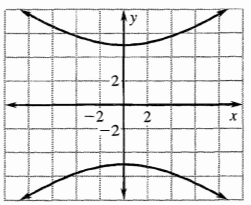
$c^2 = a^2 + b^2 = 36 + 1 = 37$

$c = \sqrt{37}$

Foci: $(\pm\sqrt{37}, 0)$

Asymptotes: $y = \pm\frac{1}{6}x$

33. $\frac{y^2}{25} - \frac{x^2}{49} = 1$



$a^2 = 25, a = 5, b^2 = 49, b = 7$

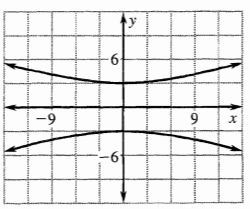
$c^2 = a^2 + b^2 = 25 + 49 = 74$

$c = \sqrt{74}$

Foci: $(0, \pm\sqrt{74})$

Asymptotes: $y = \pm\frac{5}{7}x$

34. $\frac{y^2}{9} - \frac{x^2}{100} = 1$



$a^2 = 9, a = 3, b^2 = 100, b = 10$

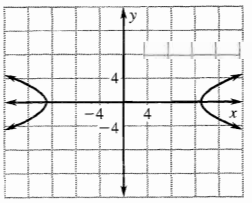
$c^2 = a^2 + b^2 = 9 + 100 = 109$

$c = \sqrt{109}$

Foci: $(0, \pm\sqrt{109})$

Asymptotes: $y = \pm\frac{3}{10}x$

35. $\frac{x^2}{169} - \frac{y^2}{16} = 1$



$a^2 = 169, a = 13, b^2 = 16, b = 4$

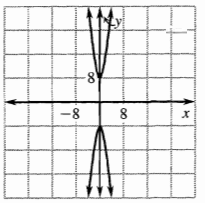
$c^2 = a^2 + b^2 = 169 + 16 = 185$

$c = \sqrt{185}$

Foci: $(\pm\sqrt{185}, 0)$

Asymptotes: $y = \pm\frac{4}{13}x$

36. $\frac{y^2}{64} - x^2 = 1$



$a^2 = 64, a = 8, b^2 = 1, b = 1$

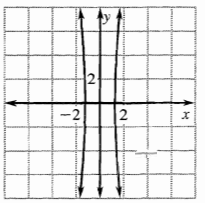
$c^2 = a^2 + b^2 = 64 + 1 = 65$

$c = \sqrt{65}$

Foci: $(0, \pm\sqrt{65})$

Asymptotes: $y = \pm\frac{8}{1}x = \pm 8x$

37. $\frac{16x^2}{25} - \frac{y^2}{81} = 1$



$\left(\frac{\frac{1}{16}}{\frac{1}{16}}\right)\left(\frac{16x^2}{25} - \frac{y^2}{81}\right) = 1\left(\frac{\frac{1}{16}}{\frac{1}{16}}\right)$

$\frac{x^2}{\left(\frac{25}{16}\right)} - \frac{y^2}{81} = 1$

$a^2 = \frac{25}{16}, a = \frac{5}{4}, b^2 = 81, b = 9$

$c^2 = a^2 + b^2 = \frac{25}{16} + 81 = \frac{25}{16} + \frac{1296}{16} = \frac{1321}{16}$

$c = \frac{\sqrt{1321}}{4}$

Foci: $\left(\pm\frac{\sqrt{1321}}{4}, 0\right)$

Asymptotes: $y = \pm\frac{9}{\frac{5}{4}}x$

$y = \frac{9}{1} \cdot \left(\frac{4}{5}\right)x$

$y = \pm\frac{36}{5}x$

38. $\frac{x^2}{144} - \frac{y^2}{121} = 1$

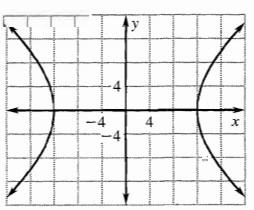
$a^2 = 144, a = 12, b^2 = 121, b = 11$

$c^2 = a^2 + b^2 = 144 + 121 = 265$

$c = \sqrt{265}$

Foci: $(\pm\sqrt{265}, 0)$

Asymptotes: $y = \pm\frac{11}{12}x$



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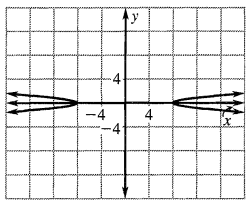
39. $\dfrac{x^2}{64} - \dfrac{9y^2}{4} = 1$



$$\left(\frac{\frac{1}{9}}{\frac{1}{9}}\right)\left(\frac{x^2}{64} - \frac{9y^2}{4}\right) = 1\left(\frac{\frac{1}{9}}{\frac{1}{9}}\right)$$

$$\frac{x^2}{64} - \frac{y^2}{\left(\frac{4}{9}\right)} = 1$$

$a^2 = 64, a = 8, b^2 = \dfrac{4}{9}, b = \dfrac{2}{3}$

$c^2 = a^2 + b^2 = 64 + \dfrac{4}{9} = \dfrac{576}{9} + \dfrac{4}{9} = \dfrac{580}{9}$

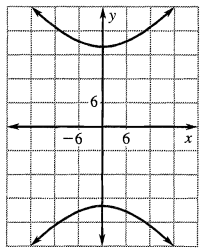
$c = \dfrac{\sqrt{580}}{3}$

Foci: $\left(\pm\dfrac{\sqrt{580}}{3}, 0\right)$

Asmyptotes: $y = \pm\dfrac{\frac{2}{3}}{8}x$

$y = \pm\dfrac{2}{3}\left(\dfrac{1}{8}\right)x = \pm\dfrac{1}{12}x$

40. $\dfrac{y^2}{25} - \dfrac{x^2}{16} = 16$



$\dfrac{y^2}{25(16)} - \dfrac{x^2}{16(16)} = \dfrac{16}{16}$

$\dfrac{y^2}{400} - \dfrac{x^2}{256} = 1$

$a^2 = 400, a = 20, b^2 = 256, b = 16$

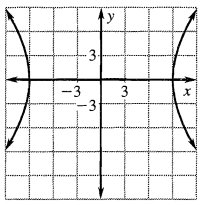
$c^2 = a^2 + b^2 = 400 + 256 = 656$

$c = \sqrt{656} = \sqrt{16 \cdot 41} = 4\sqrt{41}$

Foci: $\left(0, \pm 4\sqrt{41}\right)$

Asymptotes: $y = \pm\dfrac{20}{16}x = \pm\dfrac{5}{4}x$

41. $100x^2 - 81y^2 = 8100$



$\dfrac{100x^2}{8100} - \dfrac{81y^2}{8100} = \dfrac{8100}{8100}$

$\dfrac{x^2}{81} - \dfrac{y^2}{100} = 1$

$a^2 = 81, a = 9, b^2 = 100, b = 10$

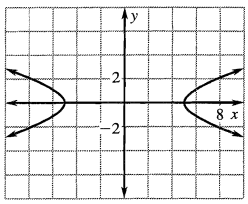
$c^2 = a^2 + b^2 = 81 + 100 = 181$

$c = \sqrt{181}$

Foci: $(\pm\sqrt{181}, 0)$

Asymptotes: $y = \pm\dfrac{10}{9}x$

42. $x^2 - 9y^2 = 25$



$\dfrac{x^2}{25} - \dfrac{9y^2}{25} = \dfrac{25}{25}$

$$\frac{\left(\frac{1}{9}\right)}{\left(\frac{1}{9}\right)}\left(\frac{x^2}{25} - \frac{9y^2}{25}\right) = 1\left(\frac{\frac{1}{9}}{\frac{1}{9}}\right)$$

$$\frac{x^2}{25} - \frac{y^2}{\left(\frac{25}{9}\right)} = 1$$

$a^2 = 25, a = 5, b^2 = \dfrac{25}{9}, b = \dfrac{5}{3}$

$c^2 = a^2 + b^2 = 25 + \dfrac{25}{9} = \dfrac{225}{9} + \dfrac{25}{9} = \dfrac{250}{9}$

$c = \dfrac{\sqrt{250}}{3} = \dfrac{5\sqrt{10}}{3}$

Foci: $\left(\pm\dfrac{5\sqrt{10}}{3}, 0\right)$

Asymptotes: $y = \pm\dfrac{\frac{5}{3}}{5}x = \pm\dfrac{5}{3}\left(\dfrac{1}{5}\right)x = \pm\dfrac{1}{3}x$

43. $\dfrac{y^2}{144} = \dfrac{x^2}{100} + 1$

$y^2 = \dfrac{144x^2}{100} + 144$

$y = \pm\sqrt{\dfrac{144x^2 + 14{,}400}{100}} = \pm\dfrac{12\sqrt{x^2 + 100}}{10}$

$= \pm\dfrac{6\sqrt{x^2 + 100}}{5}$

44. $\dfrac{y^2}{25} = \dfrac{x^2}{16} - 1$

$y^2 = \dfrac{25x^2}{16} - 25$

$y = \pm\sqrt{\dfrac{25x^2 - 400}{16}} = \pm\dfrac{5\sqrt{x^2 - 16}}{4}$

45. $\dfrac{y^2}{72.25} = \dfrac{x^2}{42.25} - 1$

$y^2 = \dfrac{72.25x^2}{42.25} - 72.25$

$y = \pm\sqrt{\dfrac{72.25x^2 - 3052.5625}{42.25}}$

$= \pm\dfrac{8.5\sqrt{x^2 - 42.25}}{6.5}$

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46. $\frac{y^2}{2.73} = \frac{x^2}{3.58} + 1$

$y^2 = \frac{2.73x^2}{3.58} + 2.73$

$y = \pm\sqrt{\frac{2.73(x^2 + 3.58)}{3.58}}$

47. $\frac{y^2}{22.3} = \frac{x^2}{10.1} - 1$

$y^2 = \frac{22.3x^2}{10.1} - 22.3$

$y = \pm\sqrt{\frac{22.3(x^2 - 10.1)}{10.1}}$

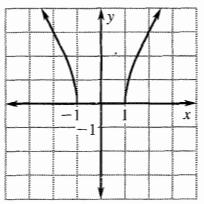
48. $12x^2 - 85y^2 = 46$

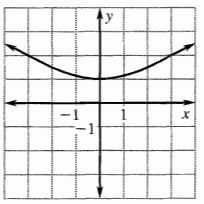
$85y^2 = 12x^2 - 46$

$y^2 = \frac{12x^2 - 46}{85}$

$y = \pm\sqrt{\frac{12x^2 - 46}{85}}$

49.





50. $\frac{x^2}{169} - \frac{y^2}{25} = 1$

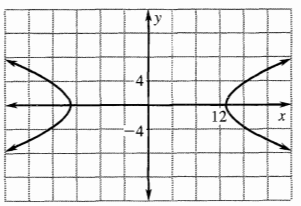
$a^2 = 169, a = 13, b^2 = 25, b = 5$

$c^2 = a^2 + b^2 = 169 + 25 = 194$

$c = \sqrt{194}$

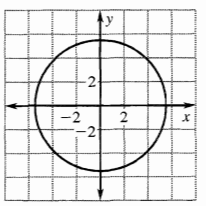
Foci: $(\pm\sqrt{194}, 0)$

Asymptotes: $y = \pm\frac{5}{13}x$



51. $x^2 + y^2 = 30$

$r = \sqrt{30} \approx 5.48$



52. $\frac{y^2}{9} - \frac{x^2}{64} = 1$

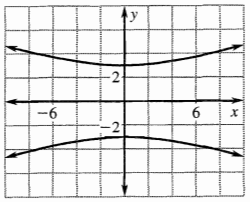
$a^2 = 9, a = 3, b^2 = 64, b = 8$

$c^2 = a^2 + b^2 = 9 + 64 = 73$

$c = \sqrt{73}$

Foci: $(0, \pm\sqrt{73})$

Asymptotes: $y = \pm\frac{3}{8}x$



53. $x^2 = 15y$

$4p = 15$

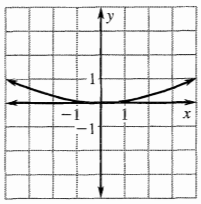
$p = \frac{15}{4}$

Focus: $(0, \frac{15}{4})$

Directrix: $y = -\frac{15}{4}$

$p > 0$ parabola opens up

x	1	2	3
y	±3.9	±5.5	±6.7



54. $\frac{x^2}{196} + \frac{y^2}{256} = 1$

$c^2 = a^2 - b^2$

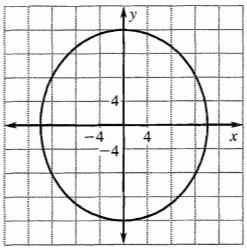
$c^2 = 16 - 14 = 2$

$c = \sqrt{2}$

Vertices: $(0, \pm 16)$

Co-vertices: $(\pm 14, 0)$

Foci: $(0, \pm\sqrt{2})$



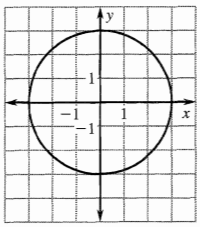
55. $14x^2 + 14y^2 = 126$

$\frac{14x^2}{126} + \frac{14y^2}{126} = \frac{126}{126}$

$\frac{x^2}{9} + \frac{y^2}{9} = 1$

$x^2 + y^2 = 9$

$r = 3$



56. Foci: $(0, -13), (0, 13)$ $c = 13$

Vertices: $(0, -5), (0, 5)$ $a = 5$

$b^2 = c^2 - a^2 = 169 - 25 = 144$

$\frac{y^2}{25} - \frac{x^2}{144} = 1$

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57. Foci: $(-8, 0), (8, 0)$ $c = 8$

Vertices: $(-6, 0), (6, 0)$ $a = 6$

$b^2 = c^2 - a^2 = 64 - 36 = 28,$

$\frac{x^2}{36} - \frac{y^2}{28} = 1$

58. Foci: $(-4, 0), (4, 0)$ $c = 4$

Vertices: $(-1, 0), (1, 0)$ $a = 1$

$b^2 = c^2 - a^2 = 16 - 1 = 15,$

$x^2 - \frac{y^2}{15} = 1$

59. Foci: $(-6, 0), (6, 0)$ $c = 6$

Vertices: $(-5, 0), (5, 0)$ $a = 5$

$b^2 = c^2 - a^2 = 36 - 25 = 11,$

$\frac{x^2}{25} - \frac{y^2}{11} = 1$

60. Foci: $(0, -7), (0, 7)$ $c = 7$

Vertices: $(0, -3), (0, 3),$ $a = 3$

$b^2 = c^2 - a^2 = 49 - 9 = 40,$

$\frac{y^2}{9} - \frac{x^2}{40} = 1$

61. Foci: $(0, -9), (0, 9)$ $c = 9$

Vertices: $(0, -8), (0, 8),$ $a = 8$

$b^2 = c^2 - a^2 = 81 - 64 = 17$

$\frac{y^2}{64} - \frac{x^2}{17} = 1$

62. Foci: $(-8, 0), (8, 0)$ $c = 8$

Vertices: $(-4\sqrt{3}, 0), (4\sqrt{3}, 0)$ $a = 4\sqrt{3}$

$b^2 = c^2 - a^2 = 64 - 48 = 16$

$\frac{x^2}{48} - \frac{y^2}{16} = 1$

63. Foci: $(0, -5\sqrt{6}), (0, 5\sqrt{6})$ $c = 5\sqrt{6}$

Vertices: $(0, -4), (0, 4)$ $a = 4$

$b^2 = c^2 - a^2 = 150 - 16 - 134$

$\frac{y^2}{16} - \frac{x^2}{134} = 1$

64. $355 - 266 = 89,$ $a = 89,$ $355 - 207 = 148,$ $c = 148$

$b^2 = c^2 - a^2 = (148)^2 - (89)^2 = 21{,}904 - 7921$

$= 13{,}983$

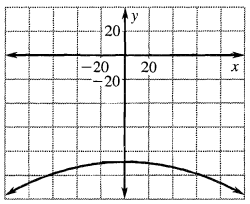
$\frac{y^2}{7921} - \frac{x^2}{13{,}983} = 1$

65. $419 - 355 = 64,$ $c = 64,$ $387 - 355 = 32,$ $a = 32$

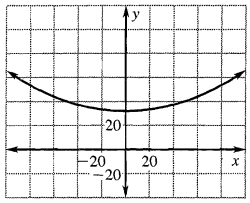
$b^2 = c^2 - a^2 = (64)^2 - (32)^2 = 4096 - 1024 = 3072$

$\frac{y^2}{1024} - \frac{x^2}{3072} = 1$

66. lower branch summer solstice



upper branch winter solstice



67. $\frac{x^2}{100} - \frac{y^2}{4} = 1,$ $a = 10,$ $b = 2,$ $a^2 = 100,$ $b^2 = 4$

Because the x^2-term is positive, the transverse axis is horizontal and the vertices are at $(-10, 0), (10, 0)$ so the shortest horizontal distance is 10 miles.

68. **a.** Area of circle $= \pi r^2$

Area of inner circle $= \pi x^2$

Area of outer circle $= \pi y^2$

Ring with area 2in.^{2}: $\pi y^2 - \pi x^2 = 4$

b. $(1.65, 2),$ $y - x = 2 - 1.65 = 0.35$ in.

$\pi(2)^2 - \pi(1.65)^2 = 4$

$\pi(4) - \pi(2.7225) = 4$

$12.56 - 8.56 = 4$

$(2.23, 2.5),$ $y - x = 2.5 - 2.23 = 0.27$ in.

$\pi(2.5)^2 - \pi(2.23)^2 = 4$

$\pi(6.25) - \pi(4.9729) = 4$

$19.63 - 15.63 = 4$

$(1.34, 1.75),$ $y - x = 1.75 - 1.34 = 0.41$ in.

$\pi(1.75)^2 - \pi(1.34)^2 = 4$

$\pi(3.0625) - \pi(1.7956) = 4$

$9.62 - 5.64 = 3.98 \approx 4$

c. It decreases; the larger the outer circle of the ring, the more narrow the ring must become to keep the same area. If the inner circle is pushed out, so is the outer circle.

69. $d_2 - d_1$ is a constant by the definition of a hyperbola. Find $d_2 - d_1$ at $(a, 0)$; $d_2 - d_1 = (c + a) - (c - a) = c + a - c + a = 2a$

70. No; You could create a branch of a hyperbola such that microphone A is a constant 2200 feet farther than microphone B. The sound from any point on that branch would reach microphone A 2 seconds later. However, you would need more information to decide which point on the branch was the source of the sound.

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10.5 Mixed Review (p. 621)

71. $y = 2|x + 4| + 1$

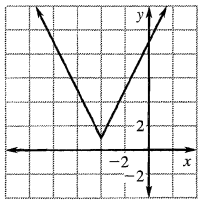
$x + 4 = 0$

$x = -4$

$y = 2|-4 + 4| + 1$

$y = 1$

Vertex: $(-4, 1)$



x	-7	-6	-5	-4	-3	-2	-1
y	7	5	3	1	3	5	7

72. $y = |x - 4| + 5$

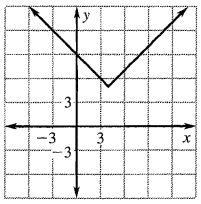
$x - 4 = 0$

$x = 4$

$y = |4 - 4| + 5$

$y = 5$

Vertex: $(4, 5)$



x	1	2	3	4	5	6	7
y	8	7	6	5	6	7	8

73. $y = -|x - 6| - 8$

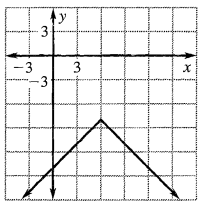
$x - 6 = 0$

$x = 6$

$y = -|6 - 6| - 8$

$y = -8$

Vertex: $(6, -8)$



x	3	4	5	6	7	8	9
y	-11	-10	-9	-8	-9	-10	-11

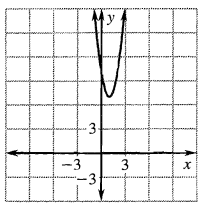
74. $y = 3(x - 1)^2 + 7$

$a = 3, a > 0$

curve opens up

Vertex $= (1, 7)$

Axis of symmetry: $x = 1$



x	2	0	3	-1
y	10	10	19	19

75. $y = -2(x - 3)^2 - 6$

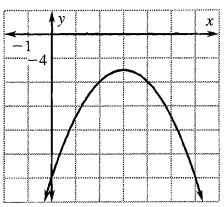
$a = -2$

$a < 0$

curve opens down

Vertex $= (3, -6)$

Axis os symmetry: $x = 3$



x	2	4	1	5
y	-8	-8	-14	-14

76. $y = \frac{1}{2}(x + 4)^2 + 5$

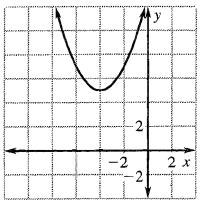
$a = \frac{1}{2}$

$a > 0$

Vertex $= (-4, 5)$

curve opens up

Axis of symmetry: $x = -4$



x	-2	-3	-5	-6
y	7	5.5	5.5	7

77. 3, 1, 2; $f(x) = x^3 - 6x^2 + 11x - 6$

$c = 6$ or -6

78. $-7, -1, 3$; $f(x) = x^3 + 5x^2 - 17x - 21$

$c = 21$ or -21

79. $6, -2, 2$; $f(x) = x^3 - 6x^2 - 4x + 24$

$c = 24$ or -24

80. $-6, 4, 2$; $f(x) = x^3 - 28x + 48$

$c = 48$ or -48

81. $5, i, -i$; $f(x) = x^3 - 5x^2 + x - 5$

$c = 5$ or -5

82. $3, -3, 2i$; $f(x) = x^4 - 5x^2 - 36$

$c = 36$ or -36

83. $\log 10{,}000$; $10^x = 10{,}000$ so $10^4 = 10{,}000$; $x = 4$

84. $\log_3 27$; $3^x = 27$ so $x = 3$

85. $\log_5 625$; $5^x = 625$ so $x = 4$

86. $\log_2 128$; $2^x = 128$ so $x = 7$

87. $\log_4 64$; $4^x = 64$ so $x = 3$

88. $\log_3 243$; $3^x = 243$ so $x = 5$

89. $\log_6 216$; $6^x = 216$ so $x = 3$

90. $\log_{100} 100{,}000{,}000$; $100^x = 100{,}000{,}000$ so $x = 4$

91. 63, 67, 72, 75, 77, 78, 81, 81, 85, 86, 89, 91, 92, 99

Mean $= 81.67$; Median $= 81$; Modes: 81, 89

Range: $99 - 63 = 36$

Quiz 2 (p. 621)

1. Vertex: $(0, 7)$

Co-vertex: $(-3, 0)$

$\frac{x^2}{9} + \frac{y^2}{49} = 1$

2. Vertex: $(-6, 0)$

Co-vertex: $(0, -1)$

$\frac{x^2}{36} + \frac{y^2}{1} = 1$

3. Vertex: $(-10, 0)$ $a = -10$

Focus: $(6, 0)$ $c = 6$

$b^2 = a^2 - c^2 = 100 - 36 = 64$

$b = 8$

$\frac{x^2}{100} + \frac{y^2}{64} = 1$

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4. Vertex: $(0, 5)$ $a = 5$

Focus: $(0, \sqrt{17})$ $c = \sqrt{17}$

$b^2 = a^2 - c^2 = 25 - 17 = 8$

$\frac{x^2}{8} - \frac{y^2}{25} = 1$

5. Co-vertex: $(0, 2\sqrt{3})$ $b = 2\sqrt{3}$

Focus: $(-\sqrt{3}, 0)$ $c = -\sqrt{3}$

$a^2 = c^2 + b^2 = 3 + 12 = 15$

$\frac{x^2}{15} + \frac{y^2}{12} = 1$

6. Co-vertex: $(-9, 0)$ $b = 9$

Focus: $(0, 4)$ $c = 4$

$a^2 = c^2 + b^2 = 16 + 81 = 97$

$\frac{x^2}{81} + \frac{y^2}{97} = 1$

7. $\frac{x^2}{-} + \frac{y^2}{49} = 1$

$^2 = a^2 - b^2$

$^2 = 49 - 4$

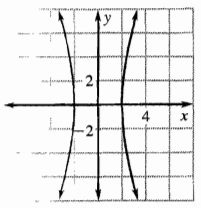
$^2 = 45$

$= 3\sqrt{5}$

Vertices: $(0, \pm 7)$

Co-vertices: $(\pm 2, 0)$

Foci: $(0, \pm 3\sqrt{5})$



8. $\frac{x^2}{6} + y^2 = 1$

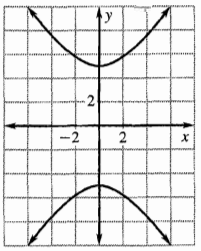
$c^2 = a^2 - b^2 = 6 - 1 = 5$

$c = \sqrt{5}$

Vertices: $(\pm\sqrt{6}, 0)$

Co-vertices: $(0, \pm 1)$

Foci: $(\pm\sqrt{5}, 0)$



$- - 9y^2 = 36$

$\frac{}{36} - \frac{9y^2}{36} = \frac{36}{36}$

$\frac{}{36} - \frac{y^2}{4} = 1$

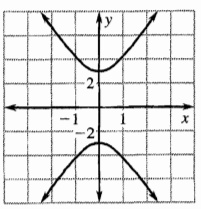
$- = a^2 - b^2 = 36 - 4 = 32$

$= \sqrt{32}$

Vertices: $(\pm 6, 0)$

Co-vertices: $(0, \pm 2)$

Foci: $(\pm\sqrt{32}, 0)$



10. Foci: $(0, -8), (0, 8)$ $c = 8$

Vertices: $(0, 5), (0, 5)$ $a = 5$

$b^2 = c^2 - a^2 = 64 - 25 = 39$

$\frac{y^2}{25} - \frac{x^2}{39} = 1$

11. Foci: $(-3, 0), (3, 0)$ $c = 3$

Vertices: $(-1, 0), (1, 0)$ $a = 1$

$b^2 = c^2 - a^2 = 9 - 1 = 8$

$x^2 - \frac{y^2}{8} = 1$

12. Foci: $(-6, 0), (6, 0)$ $c = 6$

$b^2 = c^2 - a^2 = 36 - 16 = 20$

Vertices: $(-4, 0), (4, 0)$ $a = 4$

$\frac{x^2}{16} - \frac{y^2}{20} = 1$

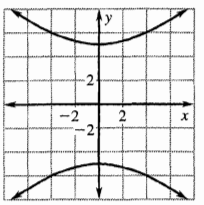
13. Foci: $(0, -2\sqrt{5}), (0, 2\sqrt{5})$ $c = 2\sqrt{5}$

Vertices: $(0, -4), (0, 4)$

$b^2 = c^2 - a^2 = 20 - 16 = 4$ $a = 4$

$\frac{y^2}{16} - \frac{x^2}{4} = 1$

14. $\frac{y^2}{25} - \frac{x^2}{36} = 1$



$a^2 = 25, a = 5, b^2 = 36, b = 6$

$c^2 = a^2 + b^2 = 25 + 36 = 61$

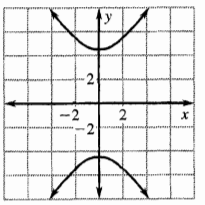
$c = \sqrt{61}$

Vertices: $(0, \pm 5)$

Foci: $(0, \pm\sqrt{61})$

Asymptotes: $y = \pm\frac{a}{b}x = \pm\frac{5}{6}x$

15. $8y^2 - 20x^2 = 160$



$\frac{8y^2}{160} - \frac{20x^2}{160} = \frac{160}{160}$

$\frac{y^2}{20} - \frac{x^2}{8} = 1$

$a^2 = 20, a = \sqrt{20}, b^2 = 8,$

$b = \sqrt{8}$

$c^2 = a^2 + b^2 = 20 + 8 = 28;$ $c = \sqrt{28}$

Vertices: $(0, \pm 2\sqrt{5})$

Foci: $(0, \pm 2\sqrt{7})$

Asymptotes: $y = \pm\frac{\sqrt{20}}{\sqrt{8}} = \pm\frac{\sqrt{160}}{8} = \pm\frac{4\sqrt{10}}{8}$

$= \pm\frac{\sqrt{10}}{2}$

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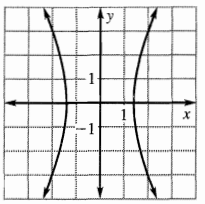
16. $18x^2 - 4y^2 = 36$



$\frac{18x^2}{36} - \frac{4y^2}{36} = \frac{36}{36}$

$\frac{x^2}{2} - \frac{y^2}{9} = 1$

$a^2 = 2, a = \sqrt{2}$

$b^2 = 9, b = 3$

$c^2 = a^2 + b^2 = 2 + 9 = 11, c = \sqrt{11}$

Vertices: $(\pm\sqrt{2}, 0)$

Foci: $(\pm\sqrt{11}, 0)$

Asymptotes: $y = \pm\frac{3}{\sqrt{2}}x = \pm\frac{3\sqrt{2}}{2}x$

17. $a - c = 4150$

$a + c = 4600$

$2a = 8750$

$a = 4375$

$a + c = 4600$

$4375 + c = 4600$

$c = 225$

$c^2 = a^2 - b^2$

$b = \sqrt{a^2 - c^2}$

$b = \sqrt{(4375)^2 - (225)^2}$

$b = \sqrt{19{,}140{,}625 - 50{,}625}$

$b = \sqrt{19{,}090{,}000}$

$b \approx 4369$

$\frac{x^2}{4375^2} + \frac{y^2}{4369^2} = 1$ where x and y are in miles

Lesson 10.6

Developing Concepts Activity 10.6 (p. 622)

1. Answers will vary; expect different equations, different size flashlight beams may be used and/or people may shine the flashlights at different angles to the graph paper.
2. Answers will vary; expect different equations.

10.6 Guided Practice (p. 628)

1. All are formed by intersecting a plane and a double-napped cone.
2. They are each circles of radius 5; the first is centered at $(0, 0)$ and the second at $(1, -2)$.
3. If the discriminant is greater than 0, it is a hyperbola. If it is 0, it is a parabola. If it less than 0, it is a circle or an ellipse; a circle if $A = C$, an ellipse if $A \neq C$.
4. $(x - 4)^2 + (y + 1)^2 = 49$
5. $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

$(h, k) = \left(\frac{8 + (-1)}{2}, \frac{-4 + (-4)}{2}\right)$

$(h, k) = (3.5, -4)$

$a = \sqrt{(-1 - 3.5)^2 + (-4 + 4)^2} = \sqrt{20.25 + 0} = 4.5$

$a^2 = 20.25$

$c = \sqrt{(2 - 3.5)^2 + (-4 + 4)^2} = \sqrt{2.25 + 0} = 1.5$

$c^2 = 2.25$

$b^2 = a^2 - c^2 = 20.25 - 2.25 = 18$

$b = \sqrt{18} = \sqrt{2 \cdot 9} = 3\sqrt{2}$

$\frac{(x - 3.5)^2}{20.25} + \frac{(y + 4)^2}{18} = 1$

6. $(x - h)^2 = 4p(y - k)$

where $p < 0$ vertex at $(3, -2)$, so $h = 3$, and $k = -2$

$|p| = \sqrt{(3 - 3)^2 + [-4 - (-2)]^2}$

$|p| = \sqrt{0 + 4} = 2$

Since $p < 0$, $p = -2$.

$(x - 3)^2 = -8(y + 2)$

7. $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$

$(h, k) = \left(\frac{5 + 5}{2}, \frac{0 + (-4)}{2}\right)$

$(h, k) = (5, -2)$

$a = \sqrt{(5 - 5)^2 + (0 + 2)^2}$

$a = \sqrt{4} = 2$

$c = \sqrt{(5 - 5)^2 + (2 + 2)^2}$

$c = \sqrt{16} = 4$

$b^2 = c^2 - a^2$

$b^2 = 16 - 4 = 12$

$b = 2\sqrt{3}$

$\frac{(y + 2)^2}{4} - \frac{(x - 5)^2}{12} = 1$

8. $A = 1, B = 0, C = 0$

$B^2 - 4AC = 0^2 - 4(1)(0) = 0$

parabola

9. $A = 3, B = 0, C = -5$

$B^2 - 4AC = 0^2 - 4(3)(-5) = 60$

$B^2 - 4AC > 0$

hyperbola

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