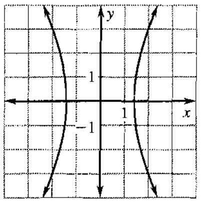
16. $18x^2 - 4y^2 = 36$



$$\frac{18x^2}{36} - \frac{4y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{2} - \frac{y^2}{9} = 1$$

$a^2 = 2, a = \sqrt{2}$

$b^2 = 9, b = 3$

$c^2 = a^2 + b^2 = 2 + 9 = 11, c = \sqrt{11}$

Vertices: $(\pm\sqrt{2}, 0)$

Foci: $(\pm\sqrt{11}, 0)$

Asymptotes: $y = \pm\frac{3}{\sqrt{2}}x = \pm\frac{3\sqrt{2}}{2}x$

17. $a - c = 4150$

$a + c = 4600$

$2a = 8750$

$a = 4375$

$a + c = 4600$

$4375 + c = 4600$

$c = 225$

$c^2 = a^2 - b^2$

$b = \sqrt{a^2 - c^2}$

$b = \sqrt{(4375)^2 - (225)^2}$

$b = \sqrt{19{,}140{,}625 - 50{,}625}$

$b = \sqrt{19{,}090{,}000}$

$b \approx 4369$

$\frac{x^2}{4375^2} + \frac{y^2}{4369^2} = 1$ where x and y are in miles

Lesson 10.6

Developing Concepts Activity 10.6 (p. 622)

1. Answers will vary; expect different equations, different size flashlight beams may be used and/or people may shine the flashlights at different angles to the graph paper.
2. Answers will vary; expect different equations.

10.6 Guided Practice (p. 628)

1. All are formed by intersecting a plane and a double-napped cone.
2. They are each circles of radius 5; the first is centered at $(0, 0)$ and the second at $(1, -2)$.
3. If the discriminant is greater than 0, it is a hyperbola. If it is 0, it is a parabola. If it less than 0, it is a circle or an ellipse; a circle if $A = C$, an ellipse if $A \neq C$.
4. $(x - 4)^2 + (y + 1)^2 = 49$

5. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$(h, k) = \left(\frac{8 + (-1)}{2}, \frac{-4 + (-4)}{2}\right)$

$(h, k) = (3.5, -4)$

$a = \sqrt{(-1 - 3.5)^2 + (-4 + 4)^2} = \sqrt{20.25 + 0} = 4.5$

$a^2 = 20.25$

$c = \sqrt{(2 - 3.5)^2 + (-4 + 4)^2} = \sqrt{2.25 + 0} = 1.5$

$c^2 = 2.25$

$b^2 = a^2 - c^2 = 20.25 - 2.25 = 18$

$b = \sqrt{18} = \sqrt{2 \cdot 9} = 3\sqrt{2}$

$\frac{(x - 3.5)^2}{20.25} + \frac{(y + 4)^2}{18} = 1$

6. $(x - h)^2 = 4p(y - k)$

where $p < 0$ vertex at $(3, -2)$, so $h = 3$, and $k = -2$

$|p| = \sqrt{(3 - 3)^2 + [-4 - (-2)]^2}$

$|p| = \sqrt{0 + 4} = 2$

Since $p < 0$, $p = -2$.

$(x - 3)^2 = -8(y + 2)$

7. $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$(h, k) = \left(\frac{5 + 5}{2}, \frac{0 + (-4)}{2}\right)$

$(h, k) = (5, -2)$

$a = \sqrt{(5 - 5)^2 + (0 + 2)^2}$

$a = \sqrt{4} = 2$

$c = \sqrt{(5 - 5)^2 + (2 + 2)^2}$

$c = \sqrt{16} = 4$

$b^2 = c^2 - a^2$

$b^2 = 16 - 4 = 12$

$b = 2\sqrt{3}$

$\frac{(y + 2)^2}{4} - \frac{(x - 5)^2}{12} = 1$

8. $A = 1, B = 0, C = 0$

$B^2 - 4AC = 0^2 - 4(1)(0) = 0$

parabola

9. $A = 3, B = 0, C = -5$

$B^2 - 4AC = 0^2 - 4(3)(-5) = 60$

$B^2 - 4AC > 0$

hyperbola

10. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0$

$B = 0,\ A = C$

circle

11. $A = -5, B = 0, C = -2$

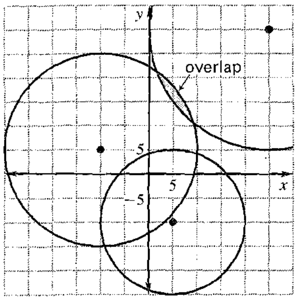
$B^2 - 4AC = 0^2 - 4(-5)(-2) = -40$

$B^2 - 4AC < 0$

$B = 0, A \neq C$

ellipse

12. yes;



10.6 Practice and Applications (pp. 628–630)

13. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 9)^2 + (y - 3)^2 = 16$

14. $(x - h)^2 + (y - k)^2 = r^2$

$(x + 4)^2 + (y - 2)^2 = 9$

15. parabola opens up

$(x - h)^2 = 4p(y - k)$

Vertex: $(1, -2)$ so $h = 1, k = -2$

$|p| = \sqrt{(1 - 1)^2 + (1 + 2)^2} = \sqrt{9} = \pm 3$

$p > 0, p = 3$

$(x - 1)^2 = 4(3)(y + 2)$

$(x - 1)^2 = 12(y + 2)$

16. $|p| = \sqrt{(-8 + 3)^2 + (1 - 1)^2} = \sqrt{25} = 5$

$p > 0, p = 5$

horizontal axis of symmetry $y = 1$

vertex $(-3, 1)$ so $h = -3, k = 1$

$(y - k)^2 = 4p(x - h)$

$(y - 1)^2 = 4(5)(x + 3)$

$(y - 1)^2 = 20(x + 3)$

17. $\dfrac{(x - h)^2}{b^2} + \dfrac{(y - k)^2}{a^2} = 1$

$(h, k) = \left(\dfrac{2 + 2}{2}, \dfrac{-3 + 6}{2}\right)$

$(h, k) = (2, 1.5)$

$a = \sqrt{(2 - 2)^2 + (-3 - 1.5)^2}$

$a = \sqrt{20.25} = 4.5$

$c = \sqrt{(2 - 2)^2 + (0 - 1.5)^2}$

$c = \sqrt{2.25} = 1.5$

$b^2 = a^2 - c^2 = 20.25 - 2.25 = 18$

$\dfrac{(x - 2)^2}{18} + \dfrac{(y - 1.5)^2}{20.25} = 1$

18. $\dfrac{(x - h)^2}{a^2} + \dfrac{(y - k)^2}{b^2} = 1$

$(h, k) = \left(\dfrac{-2 + 4}{2}, \dfrac{2 + 2}{2}\right)$

$(h, k) = (1, 2)$

$a = \sqrt{(-2 - 1)^2 + (2 - 2)^2}$

$a = \sqrt{9} = 3$

$b = \sqrt{(1 - 1)^2 + (1 - 2)^2} = \sqrt{1} = 1$

$\dfrac{(x - 1)^2}{9} + (y - 2)^2 = 1$

19. $\dfrac{(y - k)^2}{a^2} - \dfrac{(x - h)^2}{b^2} = 1$

$(h, k) = \left(\dfrac{5 + 5}{2}, \dfrac{-4 + 4}{2}\right)$

$(h, k) = (5, 0)$

$a = \sqrt{(5 - 5)^2 + (-4 - 0)^2} = \sqrt{16} = 4$

$c = \sqrt{(5 - 5)^2 + (-6 - 0)^2} = \sqrt{36} = 6$

$b^2 = c^2 - a^2 = 36 - 16 = 20$

$b = \sqrt{20} = 2\sqrt{5}$

$\dfrac{(y - 0)^2}{16} - \dfrac{(x - 5)^2}{20} = 1$

$\dfrac{y^2}{16} - \dfrac{(x - 5)^2}{20} = 1$

20. $\dfrac{(x - h)^2}{a^2} - \dfrac{(y - k)^2}{b^2} = 1$

$(h, k) = \left(\dfrac{-4 + 1}{2}, \dfrac{2 + 2}{2}\right)$

$(h, k) = (-1.5, 2)$

$a = \sqrt{(-4 + 1.5)^2 + (2 - 2)^2} = \sqrt{6.25} = 2.5$

$c = \sqrt{(-7 + 1.5)^2 + (2 - 2)^2} = \sqrt{30.25} = 5.5$

$b^2 = c^2 - a^2 = 30.25 - 6.25 = 24$

$b = \sqrt{24} = 2\sqrt{6}$

$\dfrac{(x + 1.5)^2}{6.25} - \dfrac{(y - 2)^2}{24} = 1$

21. $r = 2$;

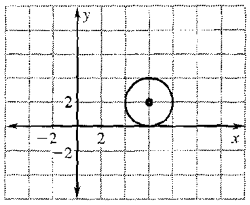
center: $(6, 2)$; points:

$(6 + 2, 2) = (8, 2)$,

$(6 - 2, 2) = (4, 2)$,

$(6, 2 + 2) = (6, 4)$,

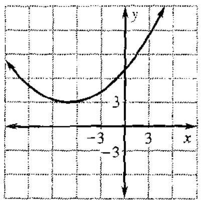
$(6, 2 - 2) = (6, 0)$



22. vertical axis

$(x - h)^2 = 4p(y - k)$, $4p = 12$ so $p = 3$, parabola opens up

vertex: $(-7, 3)$; focus: $(-7, 6)$; directrix: $y = 0$



23. vertical axis

$a = 4, b = 2,$

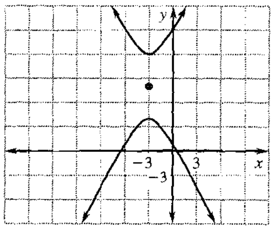
$c^2 = a^2 + b^2$

$c^2 = (4)^2 + (2)^2 = 16 + 4 = 20$

$c = \sqrt{20} = 2\sqrt{5}$

center: $(-3, 8)$, vertices: $(-3, 4)$, $(-3, 12)$;

foci: $\left(-3, 8 \pm 2\sqrt{5}\right)$



24. Vertical axis

$a = 7, b = 5$

$b = 5$

$c^2 = a^2 - b^2$

$c^2 = 49 - 25$

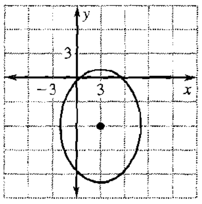
$c^2 = 24$

$c = \pm 2\sqrt{6}$

center: $(3, -6)$;

vertices: $(3, -13)$, $(3, 1)$;

co-vertices: $(-2, -6)$, $(8, -6)$; foci: $(3, -6 \pm 2\sqrt{6})$



25. horizontal axis

$a = 4, b = 3$

$c^2 = a^2 - b^2$

$c^2 = 16 - 9$

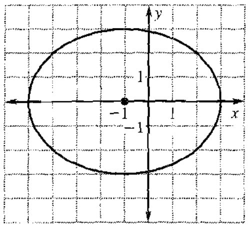
$c^2 = 7$

$c = \pm\sqrt{7}$

center: $(-1, 0)$

vertices: $(-5, 0)$, $(3, 0)$;

co-vertices: $(-1, 3)$, $(-1, 3)$; foci $(-1 \pm \sqrt{7}, 0)$



26. horizontal axis

$a = 4, b = 1$

$c^2 = a^2 + b^2$

$c^2 = 16 + 1$

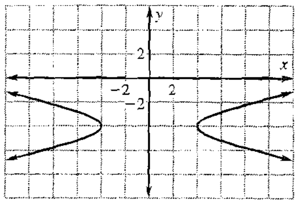
$c^2 = 17$

$c = \pm\sqrt{17}$

center: $(0, -4)$;

vertices: $(4, -4)$, $(-4, -4)$;

foci: $(\pm\sqrt{17}, -4)$



27. $r = 1$;

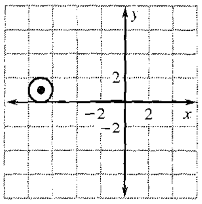
center: $(-7, 1)$; points:

$(-7 - 1, 1) = (-8, 1)$,

$(-7 + 1, 1) = (-6, 1)$,

$(-7, 1 + 1) = (-7, 2)$,

$(-7, 1 - 1) = (-7, 0)$



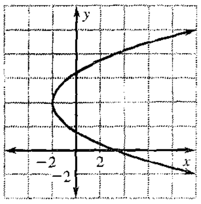
28. $(y - k)^2 = 4p(x - h)$,

$4p = 3$ so $p = \frac{3}{4}$, parabola opens right

vertex: $(-2, 4)$;

focus: $\left(-\frac{5}{4}, 4\right)$;

directrix: $x = -\frac{11}{4}$



29. $A = 9, B = 0, C = 4,$

$B^2 - 4AC = 0^2 - 4(9)(4) = -144$

$B^2 - 4AC < 0$

$B = 0, A \neq C$

ellipse

30. $A = 1, B = 0, C = -4$

$B^2 - 4AC = 0^2 - 4(1)(-4) = 16$

$B^2 - 4AC > 0$

hyperbola

31. $A = 4, B = 0, C = -9$

$B^2 - 4AC = 0^2 - 4(4)(-9) = 144$

$B^2 - 4AC > 0$

hyperbola

32. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0,$

$B = 0, A = C$

circle

33. $A = 36, B = 0, C = 16$

$B^2 - 4AC = 0^2 - 4(36)(16) = -2304$

$B^2 - 4AC < 0$

$B = 0, A \neq C$

ellipse

34. $A = 4, B = 0, C = 4$

$B^2 - 4AC = 0^2 - 4(4)(4) = -64$

$B^2 - 4AC < 0$

$B = 0, A = C$

circle

35. $A = -1, B = 0, C = 9$

$B^2 - 4AC = 0^2 - 4(-1)(9) = 36$

$B^2 - 4AC > 0$

hyperbola

36. $A = 16, B = 0, C = 25$

$B^2 - 4AC = 0^2 - 4(16)(25) = -1600$

$B^2 - 4AC < 0$

$B = 0, A \neq C$

ellipse

37. $A = 1, B = 0, C = 0$

$B^2 - 4AC = 0^2 - 4(1)(0)$

$B^2 - 4AC = 0$

parabola

38. $A = 0, B = 0, C = 2$

$B^2 - 4AC = 0^2 - 4(0)(2) = 0$

$B^2 - 4AC = 0$

parabola

39. $A = 12, B = 0, C = 20$

$B^2 = 4AC = 0^2 - 4(12)(20) = -960$

$B^2 - 4AC < 0$

$B = 0, A \neq C$

ellipse

40. $A = 9, B = 0, C = -1$

$B^2 - 4AC = 0^2 - 4(9)(-1) = 36$

$B^2 - 4AC > 0$

hyperbola

41. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0$

$A = C, B = 0$

circle

42. $A = 16, B = 0, C = 9$

$B^2 - 4AC = 0^2 - 4(16)(9) = -576$

$B^2 - 4AC < 0$

$B = 0, A \neq C$

ellipse

43. $A = -1, B = 0, C = 16$

$B^2 - 4AC = 0^2 - 4(-1)(16) = 64$

$B^2 - 4AC > 0$

hyperbola

44. $A = 1, B = 0, C = 0$

$B^2 - 4AC = 0^2 - 4(1)(0) = 0$

$B^2 - 4AC = 0$

parabola

45. $A = 9, B = 0, C = -4$

$B^2 - 4AC = 0^2 - 4(9)(-4) = 144$

$B^2 - 4AC > 0$; hyperbola

Positive x^2 term means a horizontal transverse axis. E

46. $A = 0, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(0)(1) = 0$

$B^2 - 4AC = 0$; parabola

A

47. $A = 9, B = 0, C = 4$

$B^2 - 4AC = 0^2 - 4(9)(4) = -144$

$B^2 - 4AC < 0,\ B = 0, A \neq C$; ellipse

$$9x^2 + 4y^2 + 36x + 24y + 36 = 0$$
$$(9x^2 + 36x) + (4y^2 + 24y) = -36$$
$$9(x^2 + 4x) + 4(y^2 + 6y) = -36$$
$$9(x^2 + 4x + 4) + 4(y^2 + 6y + 9) = -36 + 9(4) + 4(9)$$
$$9(x + 2)^2 + 4(y + 3)^2 = 36$$
$$\frac{9(x + 2)^2}{36} + \frac{4(y + 3)^2}{36} = \frac{36}{36}$$
$$\frac{(x + 2)^2}{4} + \frac{(y + 3)^2}{9} = 1$$
$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

$h = -2, k = -3, a = 3, b = 2$

D

48. $A = -1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(-1)(1) = 4$

$B^2 - 4AC > 0$; hyperbola

Negative x^2 term means a vertical transverse axis. F

49. $A = 4, B = 0, C = 9$

$B^2 - 4AC = 0^2 - 4(4)(9) = -144$

$B^2 - 4AC < 0, B = 0, A \neq C$, ellipse

$$4x^2 + 9y^2 - 16x + 54y + 61 = 0$$
$$(4x^2 - 16x) + (9y^2 + 54y) = -61$$
$$4(x^2 - 4x) + 9(y^2 + 6y) = -61$$
$$4(x^2 - 4x + 4) + 9(y^2 + 6y + 9) = -61 + 4(4) + 9(9)$$
$$4(x - 2)^2 + 9(y + 3)^2 = 36$$
$$\frac{4(x^2 - 2)}{36} + \frac{9(y + 3)^2}{36} = \frac{36}{36}$$
$$\frac{(x - 2)^2}{9} + \frac{(y + 3)^2}{4} = 1$$
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$h = 2, k = -3, a = 3, b = 2$

B

50. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0, B = 0, A = C$; circle

C

51. $A = 0, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(0)(1) = 0$

$B^2 - 4AC = 0$; parabola

$$y^2 - 12y + 4x + 4 = 0$$
$$(y^2 - 12y) = -4x - 4$$
$$(y^2 - 12y + 36) = -4x - 4 + 36$$
$$(y - 6)^2 = -4x + 32$$
$$(y - 6)^2 = -4(x - 8)$$

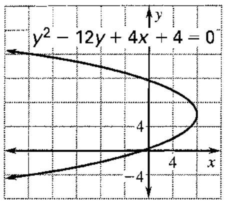
$(h, k) = \text{vertex} = (8, 6)$

horizontal axis

$(y - k)^2 = 4p(x - h)$

$4p = -4, p = -1$

$p < 0$, parabola opens left



52. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0, B = 0, A = C$

Circle

$$x^2 + y^2 - 6x - 8y + 24 = 0$$
$$(x^2 - 6x) + (y^2 - 8y) = -24$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = -24 + 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 1$$

$(h, k) = \text{center} = (3, 4)$

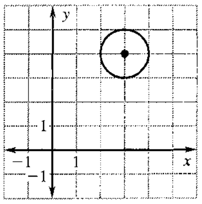
$r = 1$; points:

$(3 - 1, 4) = (2, 4),$

$(3 + 1, 4) = (4, 4),$

$(3, 4 + 1) = (3, 5),$

$(3, 4 - 1) = (3, 3)$



53. $A = 9, B = 0, C = -1$

$B^2 - 4AC = 0^2 - 4(9)(-1) = 36$

$B^2 - 4AC > 0$; hyperbola

$$9x^2 - y^2 - 72x + 8y + 119 = 0$$
$$(9x^2 - 72x) - (y^2 - 8y) = -119$$
$$9(x^2 - 8x) - (y^2 - 8y) = -119$$
$$9(x^2 - 8x + 16) - (y^2 - 8y + 16) = -119 + 9(16) - 16$$
$$9(x - 4)^2 - (y - 4)^2 = 9$$
$$\frac{1}{9}[9(x - 4)^2 - (y - 4)^2] = 9\left(\frac{1}{9}\right)$$
$$(x - 4)^2 - \frac{(y - 4)^2}{9} = 1$$
$$\frac{(x - h)^2}{a^2} - \frac{(y - k)}{b^2} = 1$$

—CONTINUED—

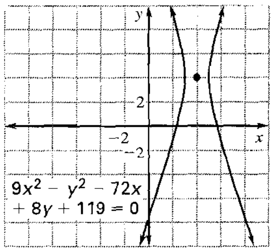
—CONTINUED—

= center = $(4, 4)$

$= 1, b = 3$

…ices:

…, $(5, 4)$



$= 4, B = 0, C = 1$

$- 4AC = 0^2 - 4(4)(1) = -16$

$- 4AC < 0, B = 0,\ A \neq C$; ellipse

$$4x^2 + y^2 - 48x - 4y + 48 = 0$$

$$(4x^2 - 48x) + (y^2 - 4y) = -48$$

$$4(x^2 - 12x) + (y^2 - 4y) = -48$$

$$\ldots - 12x + 36) + (y^2 - 4y + 4) = -48 + 4(36) + 4$$

$$4(x - 6)^2 + (y - 2)^2 = 100$$

$$\frac{4(x - 6)^2}{100} + \frac{(y - 2)^2}{100} = \frac{100}{100}$$

$$\frac{(x - 6)^2}{25} + \frac{(y - 2)^2}{100} = 1$$

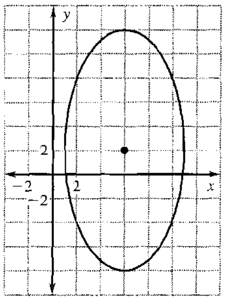
$$\frac{\ldots - h)^2}{\ldots} + \frac{(y - k)^2}{a^2} = 1$$

$)$ = center = $(6, 2)$

$= 10, b = 5$

…ices: $(6, 12), (6, -8)$;

…-vertices: $(11, 2), (1, 2)$



$= 1, B = 0, C = 4$

$- 4AC = 0^2 - 4(1)(4) = -16$

$- 4AC < 0,\ B = 0, A \neq C$; ellipse

$$x^2 + 4y^2 - 2x - 8y + 1 = 0$$

$$(x^2 - 2x) + (4y^2 - 8y) = -1$$

$$(x^2 - 2x) + 4(y^2 - 2y) = -1$$

$$\ldots - 2x + 1) + 4(y^2 - 2y + 1) = -1 + 1 + 4$$

$$(x - 1)^2 + 4(y - 1)^2 = 4$$

$$\frac{(x - 1)^2}{4} + (y - 1)^2 = 1$$

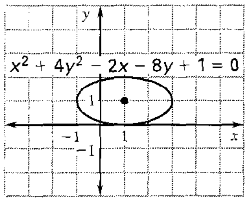
$$\frac{\ldots - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$)$ = center = $(1, 1)$

$= 2, b = 1$

…ices: $(3, 1), (-1, 1)$

…-vertices: $(1, 2), (1, 0)$



56. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0,\ B = 0, A = C$; circle

$$x^2 + y^2 - 12x - 24y + 36 = 0$$

$$(x^2 - 12x) + (y^2 - 24y) = -36$$

$$(x^2 - 12x + 36) + (y^2 - 24y + 144) = -36 + 36 + 144$$

$$(x - 6)^2 + (y - 12)^2 = 144$$

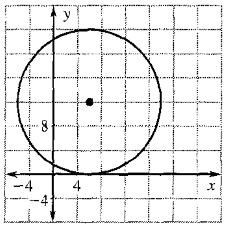
(h, k) = center = $(6, 12)$; $r = 12$; points:

$(6 + 12, 12) = (18, 12)$,

$(6 - 12, 12) = (-6, 12)$,

$(6, 12 + 12) = (6, 24)$,

$(6, 12 - 12) = (6, 0)$



57. $A = 16, B = 0, C = -1$

$B^2 - 4AC = 0^2 - 4(16)(-1) = 64$

$B^2 - 4AC > 0$; hyperbola

$$16x^2 - y^2 + 16y - 128 = 0$$

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

$$16x^2 - (y^2 - 16y) = 128$$

$$16x^2 - (y^2 - 16y + 64) = 128 - 64$$

$$16x^2 - (y - 8)^2 = 64$$

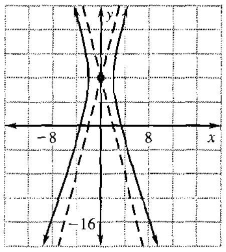
$$\frac{16x^2}{64} - \frac{(y - 8)^2}{64} = 1$$

$$\frac{x^2}{4} - \frac{(y - 8)^2}{64} = 1$$

(h, k) = center = $(0, 8)$

$a = 2, b = 8$

vertices: $(2, 8), (-2, 8)$



58. $A = 1, B = 0, C = 9$

$B^2 - 4AC = 0^2 - 4(1)(9) = -36$

$B^2 - 4AC < 0, B = 0, A \neq C$; ellipse

$$x^2 + 9y^2 + 8x + 4y + 7 = 0$$

$$(x^2 + 8x) + (9y^2 + 4y) = -7$$

$$(x^2 + 8x) + 9\left(y^2 + \frac{4}{9}y\right) = -7$$

$$(x^2 + 8x + 16) + 9\left(y^2 + \frac{4}{9}y + \frac{4}{81}\right) = -7 + 16 + \frac{4}{9}$$

$$(x + 4)^2 + 9\left(y + \frac{2}{9}\right)^2 = -\frac{63}{9} + \frac{144}{9} + \frac{4}{9}$$

$$(x + 4)^2 + 9\left(y + \frac{2}{9}\right)^2 = \frac{85}{9}$$

$$\frac{9(x + 4)^2}{85} + \frac{81\left(y + \frac{2}{9}\right)^2}{85} = 1$$

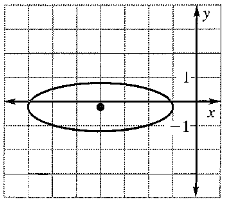
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$(h, k) = \text{center} = \left(-4, -\frac{2}{9}\right)$

$a = \frac{\sqrt{85}}{3}, b = \frac{\sqrt{85}}{9}$

vertices: $\left(-4 \pm \frac{\sqrt{85}}{3}, -\frac{2}{9}\right)$

co-vertices: $\left(-4, -\frac{2}{9} \pm \frac{\sqrt{85}}{9}\right)$



59. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0; B = 0, A = C$; circle

$$x^2 + y^2 - 12x - 12y + 36 = 0$$

$$(x^2 - 12x) + (y^2 - 12y) = -36$$

$$(x^2 - 12x + 36) + (y^2 - 12y + 36) = -36 + 36 + 36$$

$$(x - 6)^2 + (y - 6)^2 = 36$$

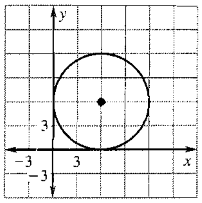
$(h, k) = \text{center} = (6, 6)$; $r = 6$; points:

$(6 + 6, 6) = (12, 6)$,

$(6 - 6, 6) = (0, 6)$,

$(6, 6 + 6) = (6, 12)$,

$(6, 6 - 6) = (6, 0)$



60. $A = 0, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(0)(1) = 0$

$B^2 - 4AC = 0$; parabola

$y^2 - 2x - 20y + 94 = 0$

$$y^2 - 20y = 2x - 94$$

$$y^2 - 20y + 100 = 2x - 94 + 100$$

$$(y - 10)^2 = 2x + 6$$

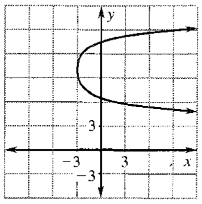
$$(y - 10)^2 = 2(x + 3)$$

$$(y - k)^2 = 4p(x - h)^2$$

$(h, k) = \text{vertex} = (-3, 10)$

$4p = 2, p = \frac{1}{2}, p > 0$

parabola opens right



61. $A = 1, B = 0, C = 0$

$B^2 - 4AC = 0^2 - 4(1)(0) = 0$

$B^2 - 4AC = 0$; parabola

$x^2 + 4x - 8y + 12 = 0$

$$(x^2 + 4x) = 8y - 12$$

$$(x^2 + 4x + 4) = 8y - 12 + 4$$

$$(x + 2)^2 = 8y - 8$$

$$(x + 2)^2 = 8(y - 1)$$

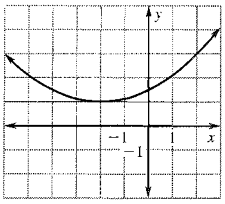
$$(x - h)^2 = 4p(y - k)$$

$(h, k) = \text{vertex} = (-2, 1)$

$4p = 8$

$p = 2, p > 0$

parabola opens up



62. $A = -9, B = 0, C = 4$

$B^2 - 4AC = 0^2 - 4(-9)(4) = 144$

$B^2 - 4AC > 0$; hyperbola

$$-9x^2 + 4y^2 - 36x - 16y - 164 = 0$$

$$-9(x^2 + 4x) + 4(y^2 - 4y) = 164$$

$$-9(x^2 + 4x + 4) + 4(y^2 - 4y + 4) = 164 - 9(4) + 4(4)$$

$$-9(x + 2)^2 + 4(y - 2)^2 = 144$$

$$\frac{-9(x + 2)^2}{144} + \frac{4(y - 2)^2}{144} = \frac{144}{144}$$

$$\frac{-(x + 2)^2}{16} + \frac{(y - 2)^2}{36} = 1$$

$$\frac{(y - 2)^2}{36} - \frac{(x + 2)^2}{16} = 1$$

—CONTINUED—

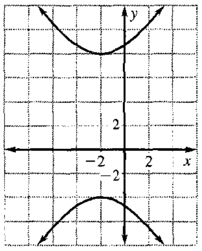
—CONTINUED—

$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$

(h, k) = center = $(-2, 2)$

$a = 6, b = 4$

Vertices: $(-2, -4), (-2, 8)$;



Vertex: $(0, 0)$

$y^2 = 4px$

Focus: $(p, 0) = (3, 0)$

$y^2 = 4(3)x$

$y^2 = 12x$

Vertex: $(50, 0)$

Focus: $(47, 0)$

$(y - k)^2 = 4p(x - h)$

$h = 50, k = 0$

$|p| = \sqrt{(47 - 50)^2 + (0 - 0)^2}$

$|p| = \sqrt{9} = 3,\ p < 0,\ p = -3$

$(y - 0)^2 = 4(-3)(x - 50)$

$y^2 = -12(x - 50)$ $(x, y$ in ft$)$

Point: $(3, 0)$

Origin: $(0, 0)$

$r = \frac{d}{2} = \frac{6}{2} = 3$

$x^2 + y^2 = r^2$

$x^2 + y^2 = (3)^2$

$x^2 + y^2 = 9$

(h, k) = center = $(0, 6)$; $r = 3$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - 6)^2 = (3)^2$

$x^2 + (y - 6)^2 = 9$ $(x, y$ in ft$)$

ellipse **66.** hyperbola branch

$A = 405, B = 0, C = 729$

$B^2 - 4AC = 0^2 - 4(405)(729) = -1{,}180{,}980$

$B^2 - 4AC < 0, B = 0,\ A \neq C$

ellipse

$A = 0, B = 0, C = -120$

$B^2 - 4AC = 0^2 - 4(0)(-120) = 0$

$B^2 - 4AC = 0$

parabola

The first is elliptical, the second parabolic.

68. $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$

$(h, k) = \left(\frac{3 + 3}{2}, \frac{5 + (-1)}{2}\right) = (3, 2)$

$a = \sqrt{(3 - 3)^2 + (5 - 2)^2}$

$a = \sqrt{9} = 3$

$c = \sqrt{(3 - 3)^2 + (7 - 2)^2}$

$c = \sqrt{25} = 5$

$b^2 = c^2 - a^2 = 25 - 9 = 16, b = 4$

$\frac{(y - 2)^2}{9} - \frac{(x - 3)^2}{16} = 1$

E

69. $A = 25, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(25)(1) = -100$

$B^2 - 4AC < 0, B = 0,\ A \neq C$ ellipse

C

70. **a.** Where the cones meet, the intersection is a point.

b. When it passes through the point where the cones meet, the intersection is an X (2 intersecting lines).

c. When it passes through the point where the cones meet, the intersection is a line.

10.6 Mixed Review (p. 631)

71.

$$x - y = 10 \qquad\qquad -3x + 3y = -30$$
$$3x - 2y = 25 \qquad\qquad \underline{3x - 2y = \ \ 25}$$
$$y = -5$$

$x - (-5) = 10$

$x = 10 - 5$

$x = 5$

$(5, -5)$

72.

$4x + 3y = 1$

$-3x - 6y = 3$

$$3(4x + 3y) = 1(3) \rightarrow \qquad 12x + 9y = 3$$
$$4(-3x - 6y) = 3(4) \rightarrow \qquad \underline{-12x - 24y = 12}$$
$$-15y = 15$$
$$y = -1$$

$4x + 3(-1) = 1$

$4x - 3 = 1$

$4x = 4$

$x = 1$

$(1, -1)$

73. $4x + y = 2$

$6x + 3y = 0$

$-12x - 3y = -6$

$\underline{6x + 3y = 0}$

$-6x = -6$

$x = 1$

$4(1) + y = 2$

$4 + y = 2$

$y = -2$

$(1, -2)$

74. $2x - 3y = 0$

$x + 6y = 14$

$2x - 3y = 0$

$\underline{-2x - 12y = -28}$

$-15y = -28$

$y = \frac{28}{15}$

$2x - 3\left(\frac{28}{15}\right) = 0$

$2x - \frac{28}{5} = 0$

$2x = \frac{28}{5}$

$x = \frac{28}{10} = \frac{14}{5}$

$\left(\frac{14}{5}, \frac{28}{15}\right)$

75. $23x = 68$

$x + 3y = 19$

$23x = 68$

$x = \frac{68}{23}$

$x + 3y = 19$

$\left(\frac{68}{23}\right) + 3y = 19$

$3y = 19 - \frac{68}{23}$

$3y = \frac{437}{23} - \frac{68}{23}$

$3y = \frac{369}{23}$

$y = \frac{369}{23} \cdot \frac{1}{3} = \frac{123}{23}$

$\left(\frac{68}{23}, \frac{123}{23}\right)$

76. $x = y$

$123x - 18y = 17$

$123y - 18y = 17$

$105y = 17$

$y = \frac{17}{105}$

$x = y = \frac{17}{105}$

$\left(\frac{17}{105}, \frac{17}{105}\right)$

77. $\log_7 7^5 = 5$

78. $\log_4 64, 4^x = 64, 4^3 = 64, x = 3$

79. $\log_5 1, 5^x = 1, 5^0 = 1, x = 0$

80. $\log_{1/3} 9, \frac{1}{3}^x = 9, \frac{1}{3}^{(-2)} = 3^2 = 9, x = -2$

81. $\log_{25} 625, 25^x = 625, 25^2 = 625, x = 2$

82. $\log 0.0001, 10^x = 0.0001, 10^{-4} = 0.0001 = \dfrac{1}{10{,}000}$

$x = -4$

83. $\dfrac{40}{1 + 6e^{-4x}} = 20$

$40 = 20(1 + 6e^{-4x})$

$2 = 1 + 6e^{-4x}$

$1 = 6e^{-4x}$

$\dfrac{1}{6} = e^{-4x}$

$\ln\dfrac{1}{6} = -4x$

$\dfrac{\ln\frac{1}{6}}{-4} = x$

$x \approx 0.45$

84. $\dfrac{10}{1 + 9e^{-2x}} = 1$

$10 = 1 + 9e^{-2x}$

$9 = 9e^{-2x}$

$1 = e^{-2x}$

$\ln 1 = -2x$

$\dfrac{\ln 1}{-2} = x$

$0 = x$

85. $\dfrac{8}{1 + 8e^{-x}} = 7$

$8 = 7(1 + 8e^{-x})$

$\dfrac{8}{7} = 1 + 8e^{-x}$

$\dfrac{1}{7} = 8e^{-x}$

$\dfrac{1}{56} = e^{-x}$

$\ln\dfrac{1}{56} = -x$

$x = -\left(\ln\dfrac{1}{56}\right)$

$x \approx 4.03$

86. $\dfrac{15}{1 + 3e^{-6x}} = 3$

$15 = 3(1 + 3e^{-6x})$

$5 = 1 + 3e^{-6x}$

$4 = 3e^{-6x}$

$\dfrac{4}{3} = e^{-6x}$

$\ln\dfrac{4}{3} = -6x$

$\dfrac{\ln\frac{4}{3}}{-6} = x$

$x \approx -0.048$

87. $\dfrac{24}{1 + 5e^{-4x}} = 9$

$24 = 9(1 + 5e^{-4x})$

$\dfrac{24}{9} = 1 + 5e^{-4x}$

$\dfrac{8}{3} - \dfrac{3}{3} = 5e^{-4x}$

$\dfrac{5}{3} = 5e^{-4x}$

$\dfrac{1}{3} = e^{-4x}$

$\ln\dfrac{1}{3} = -4x$

$\dfrac{\ln\frac{1}{3}}{-4} = x$

$x \approx 0.27$

88. $\dfrac{9}{1 + 2e^{-3x}} = 7$

$9 = 7(1 + 2e^{-3x})$

$\dfrac{9}{7} = 1 + 2e^{-3x}$

$\dfrac{2}{7} = 2e^{-3x}$

$\dfrac{1}{7} = e^{-3x}$

$\ln\dfrac{1}{7} = -3x$

$\dfrac{\ln\frac{1}{7}}{-3} = x$

$x \approx 0.65$

Math and History (p. 631)

1. $A = 3550, B = 0, C = 0$

   $B^2 - 4AC = 0^2 - 4(3550)(0) = 0$

   $B^2 - 4AC = 0$

   parabolic

2. $A = 2200, B = 0, C = 4600$

   $B^2 - 4AC = 0^2 - 4(2200)(4600) = -40{,}480{,}000$

   $B^2 - 4AC < 0, B = 0, A \neq C$

   Elliptical; will pass by the sun more than once.

3. $A = 5000, B = 0, C = -6500$

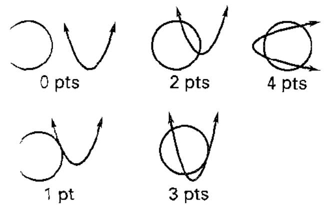
   $B^2 - 4AC = 0^2 - 4(5000)(-6500) = 130{,}000{,}000$

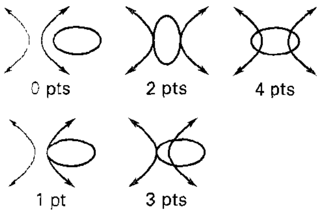
   $B^2 - 4AC > 0$

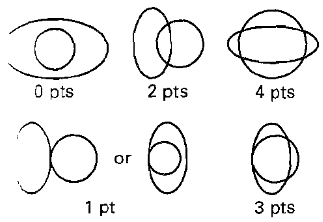
   hyperbolic

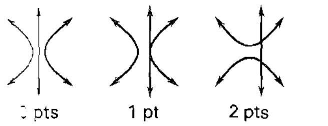
Lesson 10.7

Activity (p. 632)

1.


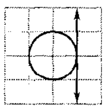
2.


3.


4.


10.7 Guided Practice (page 635)

1. quadratic

2. *Sample answer:* 

3. *Sample answer:* Linear combination since the y^2 terms can be eliminated.

4. $x^2 + y^2 = 17$

   $y = x + 3$

   $x^2 + (x + 3)^2 = 17$

   $x^2 + x^2 + 6x + 9 = 17$

   $2x^2 + 6x + 9 = 17$

   $2x^2 + 6x - 8 = 0$

   $2(x^2 + 3x - 4) = 0$

   $2(x + 4)(x - 1) = 0$

   $x = -4, x = 1$

   $y = -4 + 3 = -1 \quad (-4, -1)$

   $y = 1 + 3 = 4 \quad (1, 4)$

5. $$\begin{array}{r} x^2 + y^2 + 8x - 20y + 7 = 0 \\ \underline{-(x^2 + 9y^2 + 8x + 4y + 7) = 0} \\ -8y^2 - 24y = 0 \end{array}$$

   $-8y(y + 3) = 0$

   $y = 0$ or $y = -3$

   $x^2 + (0)^2 + 8x - 20(0) + 7 = 0$

   $x^2 + 8x + 7 = 0$

   $(x + 1)(x + 7) = 0$

   $x = -1$ or $x = -7$

   $x^2 + (-3)^2 + 8x - 20(-3) + 7 = 0$

   $x^2 + 9 + 8x + 60 + 7 = 0$

   $x^2 + 8x + 76 = 0$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4AC}}{2A}$

   $x = \dfrac{-8 \pm \sqrt{64 - 4(1)(76)}}{2(1)}$

   $x = \dfrac{-8 \pm \sqrt{64 - 304}}{2}$

   $x = \dfrac{-8 \pm \sqrt{-240}}{2}$, no real roots

   $y = 0, x = -1$ or $x = -7$

   $(-1, 0), (-7, 0)$

6. $$\begin{array}{r} x^2 + y^2 - 3x = 8 \\ \underline{2x^2 - y^2 = 10} \\ 3x^2 - 3x = 18 \end{array}$$

   $3x^2 - 3x - 18 = 0$

   $3(x^2 - x - 6) = 0$

   $3(x - 3)(x + 2) = 0$

   $x = 3$ or $x = -2$

   $2(-2)^2 - y^2 = 10$

   $8 - y^2 = 10$

   $-y^2 = 10 - 8$

   $-y^2 = 2$

   $y^2 = -2$

   $y = \pm\sqrt{-2}$

   no real roots

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