Math and History (p. 631)

1. $A = 3550, B = 0, C = 0$

$B^2 - 4AC = 0^2 - 4(3550)(0) = 0$

$B^2 - 4AC = 0$

parabolic

2. $A = 2200, B = 0, C = 4600$

$B^2 - 4AC = 0^2 - 4(2200)(4600) = -40,480,000$

$B^2 - 4AC < 0, B = 0, A \neq C$

Elliptical; will pass by the sun more than once.

3. $A = 5000, B = 0, C = -6500$

$B^2 - 4AC = 0^2 - 4(5000)(-6500) = 130,000,000$
$B^2 - 4AC > 0$

hyperbolic

Lesson 10.7

Activity (p. 632)

a.

0 pts   2 pts   4 pts

1 pt   3 pts

b.

0 pts   2 pts   4 pts

1 pt   3 pts

c.

0 pts   2 pts   4 pts

or

1 pt   3 pts

d.

0 pts   1 pt   2 pts
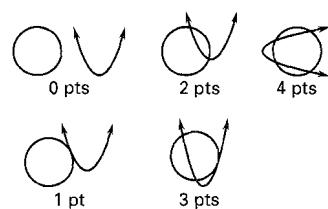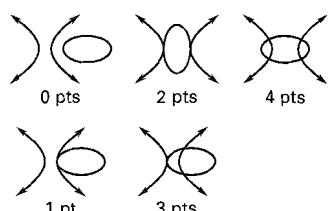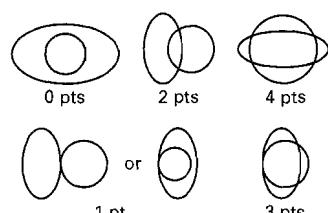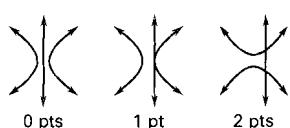
10.7 Guided Practice (page 635)

1. quadratic

2. *Sample answer:*
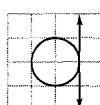
3. *Sample answer:* Linear combination since the y terms can be eliminated

4. $x^2 + y^2 = 17$

$y = x + 3$

$x^2 + (x + 3)^2 = 17$

$x^2 + x^2 + 6x + 9 = 17$

$2x^2 + 6x + 9 = 17$

$2x^2 + 6x - 8 = 0$

$2(x^2 + 3x - 4) = 0$

$2(x + 4)(x - 1) = 0$

$x = -4, x = 1$

$y = -4 + 3 = -1 \ (-4, -1)$

$y = 1 + 3 = 4 \ (1, 4)$

5. $x^2 + y^2 + 8x - 20y + 7 = 0$

$\underline{-(x^2 + 9y^2 + 8x + 4y + 7) = 0}$

$-8y^2 - \qquad 24y \qquad = 0$

$-8y(y + 3) = 0$

$y = 0 \text{ or } y = -3$

$x^2 + (0)^2 + 8x - 20(0) + 7 = 0$

$x^2 + 8x + 7 = 0$

$(x + 1)(x + 7) = 0$

$x = -1 \text{ or } x = -7$

$x^2 + (-3)^2 + 8x - 20(-3) + 7 = 0$

$x^2 + 9 + 8x + 60 + 7 = 0$

$x^2 + 8x + 76 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4AC}}{2A}$

$x = \dfrac{-8 \pm \sqrt{64 - 4(1)(76)}}{2(1)}$

$x = \dfrac{-8 \pm \sqrt{64 - 304}}{2}$

$x = \dfrac{-8 \pm \sqrt{-240}}{2}$, no real roots

$y = 0, x = -1 \text{ or } x = -7$

$(-1, 0), (-7, 0)$

6. $x^2 + y^2 - 3x = 8$

$\underline{2x^2 - y^2 \qquad = 10}$

$3x^2 \qquad - 3x = 18$

$3x^2 - 3x - 18 = 0$

$3(x^2 - x - 6) = 0$

$3(x - 3)(x + 2) = 0$

$x = 3 \text{ or } x = -2$

$2(-2)^2 - y^2 = 10$

$8 - y^2 = 10$

$-y^2 = 10 - 8$

$-y^2 = 2$

$y^2 = -2$

$y = \pm\sqrt{-2}$

no real roots

—CONTINUED—

Chapter 10 *continued*

6. —CONTINUED—

$2(3)^2 - y^2 = 10$

$18 - y^2 = 10$

$-y^2 = -8$

$y^2 = 8$

$y = \sqrt{8} = \pm 2\sqrt{2}$

$(3, \pm 2\sqrt{2})$

7. $x^2 - 2x + 2y + 2 = 0$

$\underline{-x^2 + 2x - y + 3 = 0}$

$y + 5 = 0$

$y = -5$

$x^2 - 2x + 2(-5) + 2 = 0$

$x^2 - 2x - 10 + 2 = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4 \text{ or } x = -2$

$(4)^2 - 2(4) + 2y + 2 = 0$

$16 - 8 + 2y + 2 = 0$

$2y + 10 = 0$

$2y = -10$

$y = -5$

$(-2)^2 - 2(-2) + 2y + 2 = 0$

$4 + 4 + 2y + 2 = 0$

$2y + 10 = 0$

$2y = -10$

$y = -5$

$(4, -5), (-2, -5)$

8. Because 2 circles can intersect in 2 points.

10.7 Practice and Applications (pp. 635–637)

9. $x^2 + y^2 = 25$

$y = -3$

$4 \neq -3$

No

10. $x^2 + y^2 = 41$

$y = -x - 1$

$4^2 + (-5)^2 \overset{?}{=} 41 \qquad -5 \overset{?}{=} -4 - 1$

$16 + 25 \overset{?}{=} 41 \qquad -5 = -5$

$41 = 41$

yes

11. $x^2 + 4x - 4y - 16 = 0$

$-2x + y + 1 = 0$

$6^2 + 4(6) - 4(11) - 16 \overset{?}{=} 0 \qquad -2(6) + 11 + 1 \overset{?}{=} 0$

$36 + 24 - 44 - 16 \overset{?}{=} 0 \qquad -12 + 12 = 0$

$0 = 0 \qquad\qquad 0 = 0$

yes

12. $3x^2 - 5y^2 + 2y = 45$

$y = 2x + 10$

$3(-3)^2 - 5(4)^2 + 2(4) \overset{?}{=} 45$

$27 - 80 + 8 \overset{?}{=} 45$

$-45 \neq 45$

no

13. $2x^2 - 4y = 22$

$y = -2x + 3$

$2(-5)^2 - 4(7) \overset{?}{=} 22 \qquad 7 \overset{?}{=} -2(-5) + 3$

$50 - 28 \overset{?}{=} 22 \qquad 7 \overset{?}{=} 10 + 3$

$22 = 22 \qquad 7 \neq 13$

no

14. $6x^2 - 5x + 8y^2 + y = 23$

$y = x - 1$

$6(2)^2 - 5(2) + 8(1)^2 + 1 \overset{?}{=} 23 \qquad 1 = 2 - 1$

$24 - 10 + 8 + 1 = 23 \qquad 1 = 1$

$23 = 23$

yes

15. $x^2 - y = 5$

$-3x + y = -7$

$y = 3x - 7$

$x^2 - (3x - 7) = 5$

$x^2 - 3x + 7 - 5 = 0$

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 2 \text{ or } x = 1$

$-3(2) + y = -7$

$-6 + y = -7$

$y = -7 + 6$

$y = -1$

$-3(1) + y = -7$

$-3 + y = -7$

$y = -7 + 3$

$y = -4$

$(2, -1), (1, -4)$

16. $x^2 + y^2 = 18$

$x - y = 0 \rightarrow x = y$

$x^2 + x^2 = 18$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

$3 = y$

$-3 = y$

$(3, 3), (-3, -3)$

Chapter 10 *continued*

17. $-3x^2 + y^2 = 9$

$\quad -2x + y = 0 \rightarrow y = 2x$

$\quad -3x^2 + (2x)^2 = 9$

$\quad -3x^2 - 4x^2 = 9$

$\quad\quad\quad\quad x^2 = 9$

$\quad\quad\quad\quad x = \pm 3$

$\quad y = 2(3) = 6$

$\quad y = 2(-3) = -6$

$\quad (3, 6), (-3, -6)$

18. $9x^2 + 4y^2 = 36$

$\quad -x + y = -4 \rightarrow y = -4 + x$

$\quad 9x^2 + 4(-4 + x)^2 - 36 = 0$

$\quad 9x^2 + 4(16 - 8x + x^2) - 36 = 0$

$\quad 9x^2 + 64 - 32x + 4x^2 - 36 = 0$

$\quad\quad\quad 13x^2 - 32x + 36 = 0$

$\quad x = \dfrac{32 \pm \sqrt{1024 - 4(13)(36)}}{2(13)}$

$\quad x = \dfrac{32 \pm \sqrt{1024 - 1872}}{26}$

$\quad x = \dfrac{32 \pm \sqrt{-848}}{26}$, no real roots

$\quad$ none

19. $x^2 + y^2 = 5$

$\quad x^2 + (-2x)^2 = 5 \quad\quad y = -2x$

$\quad\quad x^2 + 4x^2 = 5 \quad\quad y = -2(1) = -2$

$\quad\quad\quad\quad 5x^2 = 5 \quad\quad y = -2(-1) = 2$

$\quad\quad\quad\quad x^2 = 1$

$\quad\quad\quad\quad x = \pm 1$

$\quad (1, -2), (-1, 2)$

20. $\quad\quad x + 2y^2 = -6$

$\quad\quad x + 8y = 0 \rightarrow x = -8y \quad\quad x = -8y$

$\quad\quad -8y + 2y^2 = -6 \quad\quad\quad\quad x = -8(3)$

$\quad\quad 2y^2 - 8y + 6 = 0 \quad\quad\quad\quad x = -24$

$\quad\quad 2(y^2 - 4y + 3) = 0$

$\quad\quad 2(y - 3)(y - 1) = 0 \quad\quad\quad x = -8(1)$

$\quad\quad y = 3, y = 1 \quad\quad\quad\quad\quad x = -8$

$\quad\quad (-24, 3), (-8, 1)$

21. $5x^2 + 3y^2 = 17$

$\quad -x + y = -1 \rightarrow y = x - 1$

$\quad\quad 5x^2 + 3(x - 1)^2 = 17$

$\quad\quad 5x^2 + 3(x^2 - 2x + 1) = 17$

$\quad 5x^2 + 3x^2 - 6x + 3 - 17 = 0$

$\quad\quad\quad 8x^2 - 6x - 14 = 0$

$\quad\quad\quad 2(4x^2 - 3x - 7) = 0$

$\quad\quad\quad 2(4x - 7)(x + 1) = 0$

$\quad x = \dfrac{7}{4} \text{ or } x = -1$

$\quad y = x - 1$

$\quad y = \dfrac{7}{4} - 1 = \dfrac{3}{4}$

$\quad y = -1 - 1 = -2$

$\quad \left(\dfrac{7}{4}, \dfrac{3}{4}\right), (-1, -2)$

22. $4x^2 - 5y^2 = 16$

$\quad 3x + y = 6 \rightarrow y = -3x + 6$

$\quad\quad 4x^2 - 5(-3x + 6)^2 = 16$

$\quad\quad 4x^2 - 5(9x^2 - 36x + 36) = 16$

$\quad 4x^2 - 45x^2 + 180x - 180 - 16 = 0$

$\quad\quad\quad -41x^2 + 180x - 196 = 0$

$\quad\quad 41x^2 - 180x + 196 = 0$

$\quad\quad (41x - 98)(x - 2) = 0$

$\quad x = \dfrac{98}{41} \quad \text{or} \quad x = 2$

$\quad y = -3\left(\dfrac{98}{41}\right) + 6$

$\quad y = -\dfrac{294}{41} + 6$

$\quad y = -\dfrac{48}{41}$

$\quad y = -3(2) + 6$

$\quad y = 0$

$\quad \left(\dfrac{98}{41}, -\dfrac{48}{41}\right), (2, 0)$

23. $2x^2 + 2y^2 = 15$

$\cdot x + 2y = 6 \rightarrow x = -2y + 6$

$2(-2y + 6)^2 + 2y^2 - 15 = 0$

$2(4y^2 - 24y + 36) + 2y^2 - 15 = 0$

$8y^2 - 48y + 72 + 2y^2 - 15 = 0$

$10y^2 - 48y + 57 = 0$

$y = \dfrac{48 \pm \sqrt{2304 - 4(10)(57)}}{20}$

$y = \dfrac{48 \pm \sqrt{2304 - 2280}}{20}$

$y = \dfrac{48 \pm \sqrt{24}}{20}$

$y = \dfrac{48 \pm 2\sqrt{6}}{20}$

$y = \dfrac{24 \pm \sqrt{6}}{10}$

$x = -2\left(\dfrac{24 + \sqrt{6}}{10}\right) + 6 \qquad x = -2\left(\dfrac{24 - \sqrt{6}}{10}\right) + 6$

$x = \dfrac{6 - \sqrt{6}}{5} \qquad\qquad x = \dfrac{6 + \sqrt{6}}{5}$

$\left(\dfrac{6 - \sqrt{6}}{5}, \dfrac{24 + \sqrt{6}}{10}\right), \left(\dfrac{6 + \sqrt{6}}{5}, \dfrac{24 - \sqrt{6}}{10}\right)$

24. $x^2 + y^2 = 1$

$x + y = -1 \rightarrow y = -x - 1$

$x^2 + (-x - 1)^2 = 1$

$x^2 + x^2 + 2x + 1 - 1 = 0$

$2x^2 + 2x = 0$

$2x(x + 1) = 0$

$x = 0 \text{ or } x = -1$

$y = -0 - 1 = -1$

$y = -(-1) - 1 = 0$

$(0, -1), (-1, 0)$

25. $x^2 + y^2 = 20$

$y = x - 4$

$x^2 + (x - 4)^2 = 20$

$x^2 + x^2 - 8x + 16 - 20 = 0$

$2x^2 - 8x - 4 = 0$

$2(x^2 - 4x - 2) = 0$

$x = \dfrac{4 \pm \sqrt{16 - 4(1)(-2)}}{2(1)}$

$x = \dfrac{4 \pm \sqrt{24}}{2}$

$x = \dfrac{4 \pm 2\sqrt{6}}{2}$

$x = 2 \pm \sqrt{6}$

$y = 2 + \sqrt{6} - 4$

$y = -2 + \sqrt{6}$

$y = 2 - \sqrt{6} - 4$

$y = -2 - \sqrt{6}$

$\left(2 + \sqrt{6}, -2 + \sqrt{6}\right), \left(2 - \sqrt{6}, -2 - \sqrt{6}\right)$

26. $x^2 + y^2 = 5$

$y = 3x + 5$

$x^2 + (3x + 5)^2 = 5$

$x^2 + 9x^2 + 30x + 25 - 5 = 0$

$10x^2 + 30x + 20 = 0$

$10(x^2 + 3x + 2) = 0$

$10(x + 1)(x + 2) = 0$

$x = -1 \text{ or } x = -2$

$y = 3(-1) + 5 = 2$

$y = 3(-2) + 5 = -1$

$(-1, 2), (-2, -1)$

27. $x^2 = 6y \qquad\qquad x = 0 \text{ or } x = -6$

$y = -x \qquad\qquad\quad y = 0$

$x^2 = 6(-x) \qquad\quad y = -(-6) = 6$

$x^2 = -6x \qquad\qquad (0, 0), (-6, 6)$

$x^2 + 6x = 0$

$x(x + 6) = 0$

28. $x^2 + y^2 = 9$

$x - 3y = 3 \rightarrow x = 3y + 3$

$(3y + 3)^2 + y^2 = 9$

$9y^2 + 18y + 9 + y^2 - 9 = 0$

$10y^2 + 18y = 0$

$2y(5y + 9) = 0$

$y = 0 \text{ or } y = -\dfrac{9}{5}$

$x = 3(0) + 3$

$x = 3$

$x = 3\left(-\dfrac{9}{5}\right) + 3$

$x = -\dfrac{27}{5} + \dfrac{15}{5}$

$x = -\dfrac{12}{5}$

$(3, 0), \left(-\dfrac{12}{5}, -\dfrac{9}{5}\right)$

Chapter 10 *continued*

29. $x^2 + y^2 = 7$

$\qquad y = x - 7$

$\qquad\qquad x^2 + (x - 7)^2 = 7$

$x^2 + x^2 - 14x + 49 - 7 = 0$

$\qquad\qquad 2x^2 - 14x + 42 = 0$

$\qquad\qquad 2(x^2 - 7x + 21) = 0$

$x = \dfrac{7 \pm \sqrt{49 - 4(1)(21)}}{2(1)}$

$x = \dfrac{7 \pm \sqrt{-35}}{2}$, no real roots

30. $y^2 - 2x^2 = 6 \qquad\qquad y = -2x$

$\quad (-2x)^2 - 2x^2 - 6 = 0 \qquad y = -2(\sqrt{3})$

$\qquad\quad 4x^2 - 2x^2 - 6 = 0 \qquad y = -2\sqrt{3}$

$\qquad\qquad\quad 2x^2 - 6 = 0 \qquad y = -2(-\sqrt{3})$

$\qquad\qquad 2(x^2 - 3) = 0 \qquad y = 2\sqrt{3}$

$\qquad\qquad\qquad\quad x^2 = 3$

$\qquad\qquad\qquad\quad x = \pm\sqrt{3}$

$(\sqrt{3}, -2\sqrt{3}), (-\sqrt{3}, 2\sqrt{3})$

31. $6x^2 + 3y^2 = 12$

$\qquad\qquad 6x^2 + 3(-x + 2)^2 = 12$

$\quad 6x^2 + 3(x^2 - 4x + 4) - 12 = 0$

$\quad 6x^2 + 3x^2 - 12x + 12 - 12 = 0$

$\qquad\qquad\qquad 9x^2 - 12x = 0$

$\qquad\qquad\qquad 3x(3x - 4) = 0$

$x = 0 \text{ or } x = \frac{4}{3}$

$y = -x + 2$

$y = -\frac{4}{3} + 2$

$y = -\frac{4}{3} + \frac{6}{3}$

$y = \frac{2}{3}$

$y = 0 + 2 = 2$

$(0, 2), \left(\frac{4}{3}, \frac{2}{3}\right)$

32. $3x^2 - y^2 = -6$

$y = 2x + 1$

$\qquad\qquad 3x^2 - (2x + 1)^2 = -6$

$3x^2 - (4x^2 + 4x + 1) + 6 = 0$

$\quad 3x^2 - 4x^2 - 4x - 1 + 6 = 0$

$\qquad\qquad\quad -x^2 - 4x + 5 = 0$

$\qquad\qquad\qquad x^2 + 4x - 5 = 0$

$\qquad\qquad (x + 5)(x - 1) = 0$

$x = -5 \text{ or } x = 1$

$y = 2(-5) + 1$

$y = -10 + 1 = -9$

$y = 2(1) + 1 = 3$

$(-5, -9), (1, 3)$

33. $x^2 + y^2 = 16$

$x^2 - 5y = 5$

$\qquad x^2 + y^2 = 16$

$\qquad \underline{-x^2 + 5y = -5}$

$\qquad\qquad y^2 + 5y = 11$

$y^2 + 5y - 11 = 0$

$y = \dfrac{-5 \pm \sqrt{25 - 4(1)(-11)}}{2(1)}$

$y = \dfrac{-5 \pm \sqrt{69}}{2}$

$x^2 = 5\left(\dfrac{-5 - \sqrt{69}}{2}\right) + 5$

$x^2 = \dfrac{-25 - 5\sqrt{69} + 10}{2}$

$x^2 = \dfrac{-5\sqrt{69} - 15}{2}$

$x = \pm\sqrt{\dfrac{-5\sqrt{69} - 15}{2}}$, no real roots

$x^2 = 5\left(\dfrac{-5 + \sqrt{69}}{2}\right) + 5$

$x^2 = \dfrac{-25 + 5\sqrt{69} + 10}{2}$

$x^2 = \dfrac{5\sqrt{69} - 15}{2}$

$x = \pm\sqrt{\dfrac{5\sqrt{69} - 15}{2}}$

$\left(\pm\sqrt{\dfrac{5\sqrt{69} - 15}{2}}, \dfrac{-5 \pm \sqrt{69}}{2}\right)$

Algebra 2
Chapter 10 Worked-out Solution Key

34. $-3x^2 + y^2 - 3x = 0$

$\underline{x^2 - y^2 + 27 = 0}$

$-2x^2 - 3x + 27 = 0$

$2x^2 + 3x - 27 = 0$

$(2x + 9)(x - 3) = 0$

$x = -\dfrac{9}{2}$ or $x = 3$

$-3(3)^2 + y^2 - 3(3) = 0$

$-27 + y^2 - 9 = 0$

$y^2 = 36$

$y = \pm 6$

$-3\left(-\dfrac{9}{2}\right)^2 + y^2 - 3\left(-\dfrac{9}{2}\right) = 0$

$-\dfrac{243}{4} + y^2 + \dfrac{27}{2} = 0$

$y^2 = \dfrac{243}{4} - \dfrac{54}{4} = \dfrac{189}{4}$

$y = \pm\dfrac{\sqrt{189}}{2} = \pm\dfrac{3\sqrt{21}}{2}$

$(3, \pm 6), \left(-\dfrac{9}{2}, \pm\dfrac{3\sqrt{21}}{2}\right)$

35. $-x^2 + y^2 + 10 = 0$

$-3y^2 + x + 1 = 0$

$-3x^2 + 3y^2 + 30 = 0$

$\underline{-3y^2 + x + 1 = 0}$

$-3x^2 + x + 31 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4(-3)(31)}}{2(-3)}$

$x = \dfrac{-1 \pm \sqrt{1 + 372}}{-6}$

$x = \dfrac{1 \pm \sqrt{373}}{6}$

$-\left(\dfrac{1 + \sqrt{373}}{6}\right)^2 + y^2 + 10 = 0$

$y^2 = \left(\dfrac{(1 + \sqrt{373})(1 + \sqrt{373})}{36}\right) - \dfrac{360}{36}$

$y^2 = \dfrac{1 \div 2\sqrt{373} + 373}{36} - \dfrac{360}{36}$

$y^2 = \dfrac{374 - 360}{36} + \dfrac{2\sqrt{373}}{36}$

$y^2 = \dfrac{7 + \sqrt{373}}{18}$

$y = \pm\sqrt{\dfrac{7 + \sqrt{373}}{18}}$

$-\left(\dfrac{(1 - \sqrt{373})}{6}\right)^2 + y^2 + 10 = 0$

$y^2 = \left(\dfrac{(1 - \sqrt{373})(1 - \sqrt{373})}{36}\right) - \dfrac{360}{36}$

$y^2 = \dfrac{1 - 2\sqrt{373} + 373}{36} - \dfrac{360}{36}$

$y^2 = \dfrac{374 - 2\sqrt{373}}{36} - \dfrac{360}{36}$

$y^2 = \dfrac{374 - 360}{36} + -\dfrac{2\sqrt{373}}{36}$

$y^2 = \dfrac{7 + \sqrt{373}}{18}$

$y = \pm\sqrt{\dfrac{7 - \sqrt{373}}{18}}$; no real roots

$\left(\dfrac{1 + \sqrt{373}}{6}, \pm\sqrt{\dfrac{7 + \sqrt{373}}{18}}\right)$

36. $x^2 + 2y^2 - 10 = 0$

$4y^2 + x + 4 = 0$

$-2x^2 - 4y^2 + 20 = 0$

$\underline{4y^2 + x + 4 = 0}$

$-2x^2 + x + 24 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4(-2)(24)}}{2(-2)}$

$x = \dfrac{-1 \pm \sqrt{1 \div 192}}{-4}$

$x = \dfrac{1 \pm \sqrt{193}}{4}$

$\left(\dfrac{1 - \sqrt{193}}{4}\right)^2 + 2y^2 - 10 = 0$

$2y^2 = -\left(\dfrac{1 - 2\sqrt{193} + 193}{16}\right) - \dfrac{160}{16}$

$2y^2 = -\left(\dfrac{194 - 2\sqrt{193}}{16}\right) - \dfrac{160}{16}$

$2y^2 = \dfrac{-194 - 160 + 2\sqrt{193}}{16}$

$2y^2 = \dfrac{-354 + 2\sqrt{193}}{16}$

$y^2 = \dfrac{-177 + \sqrt{193}}{8}$

$y = \pm\sqrt{\dfrac{-177 + \sqrt{193}}{8}}$, no real roots

$\left(\dfrac{1 + \sqrt{193}}{4}\right)^2 + 2y^2 - 10 = 0$

$\dfrac{(1 + \sqrt{193})(1 + \sqrt{193})}{16} + 2y^2 - 10 = 0$

—CONTINUED—

Chapter 10 *continued*

36. —CONTINUED—

$$2y^2 = -\left(\frac{1 + 2\sqrt{193} + 193}{16}\right) - \frac{160}{16}$$

$$2y^2 = -\left(\frac{194 + 2\sqrt{193}}{16}\right) - \frac{160}{16}$$

$$2y^2 = \frac{-194 - 2\sqrt{193}}{16} - \frac{160}{16}$$

$$2y^2 = \frac{-194 - 160 - 2\sqrt{193}}{16}$$

$$y^2 = \frac{-177 - \sqrt{193}}{8}$$

$$y = \sqrt{\frac{-177 - \sqrt{193}}{8}}, \text{ no real roots}$$

37. $y^2 = 16x$

$4x - y = -24$

$-y = -4x - 24 \rightarrow y = 4x + 24$

$y^2 = 16x$

$$(4x + 24)^2 = 16x$$

$$16x^2 + 192x + 576 = 16x$$

$$16x^2 + 192x - 16x + 576 = 0$$

$$16x^2 + 176x + 576 = 0$$

$$16(x^2 + 11x + 36) = 0$$

$$x = \frac{-11 \pm \sqrt{121 - 4(1)(36)}}{2(1)}$$

$$x = \frac{-11 \pm \sqrt{121 - 144}}{2}$$

$$x = \frac{-11 \pm \sqrt{-23}}{2}, \text{ no real roots}$$

38. $10y = x^2$

$x^2 - 6 = -2y$

$10y - 6 = -2y$

$12y = 6$

$y = \frac{1}{2}$

$x^2 = 10\left(\frac{1}{2}\right)$

$x^2 = 5$

$x = \pm\sqrt{5}$

$\left(\pm\sqrt{5}, \frac{1}{2}\right)$

39. $y^2 + x = 2$

$3x + y = 8 \rightarrow y = -3x + 8$

$$(-3x + 8)^2 + x = 2$$

$$9x^2 - 48x + 64 + x - 2 = 0$$

$$9x^2 - 47x + 62 = 0$$

$$x = \frac{47 \pm \sqrt{2209 - 4(9)(62)}}{2(9)}$$

$$x = \frac{47 \pm \sqrt{2209 - 2232}}{18}$$

$$x = \frac{47 \pm \sqrt{-23}}{18}, \text{ no real roots}$$

40. $x^2 - 16y^2 = 16$

$x^2 + y^2 = 9$

$x^2 = 16y^2 + 16$

$$(16y^2 + 16) + y^2 - 9 = 0$$

$17y^2 + 7 = 0$

$17y^2 = -7$

$y^2 = -\frac{7}{17}$

$y = \pm\sqrt{-\frac{7}{17}}, \text{ no real roots}$

41. $x^2 + y^2 = 81$

$x + y = 0$

$y = -x$

$x^2 + (-x)^2 = 81$

$2x^2 = 81$

$x^2 = \frac{81}{2}$

$$x = \frac{\pm 9}{\sqrt{2}} = \frac{\pm 9\sqrt{2}}{2}$$

$-\left(\frac{9\sqrt{2}}{2}\right) = y$

$y = \frac{9\sqrt{2}}{2}$

$-\left(\frac{-9\sqrt{2}}{2}\right) = y$

$y = \frac{9\sqrt{2}}{2}$

$\left(\frac{9\sqrt{2}}{2}, -\frac{9\sqrt{2}}{2}\right), \left(-\frac{9\sqrt{2}}{2}, \frac{9\sqrt{2}}{2}\right)$

Algebra 2
Chapter 10 Worked-out Solution Key

Chapter 10 *continued*

42. $16x^2 - y^2 + 16y - 128 = 0$

$\underline{y^2 - 48x - 16y - 32 = 0}$

$16x^2 - 48x - 32 - 128 = 0$

$16x^2 - 48x - 160 = 0$

$16(x^2 - 3x - 10) = 0$

$16(x - 5)(x + 2) = 0$

$x = 5$ or $x = -2$

$y^2 - 48(5) - 16y - 32 = 0$

$y^2 - 16y - 240 - 32 = 0$

$y^2 - 16y - 272 = 0$

$y = \dfrac{16 \pm \sqrt{256 - (4)(1)(-272)}}{2(1)}$

$y = \dfrac{16 \pm \sqrt{256 + 1088}}{2}$

$y = \dfrac{16 \pm \sqrt{1344}}{2}$

$y = \dfrac{16 \pm \sqrt{64 \cdot 21}}{2}$

$y = \dfrac{16 \pm 8\sqrt{21}}{2}$

$y = 8 \pm 4\sqrt{21}$

$y^2 - 48(-2) - 16y - 32 = 0$

$y^2 - 16y + 96 - 32 = 0$

$y^2 - 16y + 64 = 0$

$(y - 8)(y - 8) = 0$

$y = 8$

$(-2, 8), (5, 8 \pm 4\sqrt{21})$

43. $x^2 - y^2 - 8x + 8y - 24 = 0$

$\underline{x^2 + y^2 - 8x - 8y + 24 = 0}$

$2x^2 - 16x = 0$

$2x(x - 8) = 0$

$x = 0$ or $x = 8$

$(0) - y^2 - 8(0) + 8y - 24 = 0$

$-y^2 + 8y - 24 = 0$

$-(y^2 - 8y + 24) = 0$

$y = \dfrac{8 \pm \sqrt{64 - 4(1)(24)}}{2(1)}$

$y = \dfrac{8 \pm \sqrt{-32}}{2}$, no real roots

$8^2 - y^2 - 8(8) + 8y - 24 = 0$

$64 - y^2 - 64 + 8y - 24 = 0$

$-y^2 + 8y - 24 = 0$

$-(y^2 - 8y + 24) = 0$

$y = 8 \pm \sqrt{64 - 4(1)24}$

$y = \dfrac{8 \pm \sqrt{-32}}{2}$, no real roots

44. $x^2 + 4y^2 - 4x - 8y + 4 = 0$

$x^2 + 4y - 4 = 0$

$x^2 - 4x + 4y^2 - 8y = -4$

$(x^2 - 4x + 4) + 4(y^2 - 2y + 1) = -4 + 4 + 4(1)$

$(x - 2)^2 + 4(y - 1)^2 = 4$

$x^2 + 4y - 4 = 0$

$x^2 = -4y + 4$

$x^2 = -4(y - 1)$

$(y - 1) = -\dfrac{x^2}{4}$

$(x - 2)^2 + 4(y - 1)^2 - 4 = 0$

$(x^2 - 4x + 4) + 4\left(-\dfrac{x^2}{4}\right)^2 - 4 = 0$

$(x^2 - 4x + 4) + 4\left(\dfrac{x^4}{16}\right) - 4 = 0$

$x^2 - 4x + 4 + \dfrac{x^4}{4} - 4 = 0$

$x^2 - 4x + \dfrac{x^4}{4} = 0$

$\dfrac{4(x^2 - 4x)}{4} + \dfrac{x^4}{4} = 0$

$\dfrac{4x^2 - 16x + x^4}{4} = 0$

$\dfrac{x^4 + 4x^2 - 16x}{4} = 0$

$\dfrac{1}{4}x^4 + x^2 - 4x = 0$

x	1	2	3
y	-2.75	0	5.75

$x\left(\dfrac{1}{4}x^3 + x - 4\right) = 0$ $x = 2$

$x = 0$ $\dfrac{1}{4}x^3 + x - 4 = 0$

$0^2 + 4y - 4 = 0$ $2^2 + 4y - 4 = 0$

$4y = 4$ $4y = 0$

$y = 1$ $y = 0$

$(0, 1), (2, 0)$

45. $4x^2 - 56x + 9y^2 + 160 = 0$

$$4x^2 + y^2 - 64 = 0$$

$4x^2 + y^2 - 64 = 0 \rightarrow y^2 = -4x^2 + 64$

$4x^2 - 56x + 9y^2 + 160 = 0$

$9y^2 = -4x^2 + 56x - 160$

$y^2 = \dfrac{-4x^2 + 56x - 160}{9}$

$\dfrac{-4x^2 + 56x - 160}{9} = -4x^2 + 64$

$-4x^2 + 56x - 160 = 9(-4x^2 + 64)$

$-4x^2 + 56x - 160 = -36x^2 + 576$

$-4x^2 + 36x^2 + 56x - 160 - 576 = 0$

$32x^2 + 56x - 736 = 0$

$8(4x^2 + 7x - 92) = 0$

$8(4x + 23)(x - 4) = 0$

$x = 4 \text{ or } x = -\dfrac{23}{4}$

$y^2 = -4x^2 + 64$

$y^2 = -4(4)^2 + 64$

$y^2 = -64 + 64 = 0 \rightarrow y = 0$

$y^2 = -4\left(-\dfrac{23}{4}\right)^2 + 64$

$y^2 = -4\left(\dfrac{529}{16}\right) + 64$

$y^2 = -\dfrac{529}{4} + 64 = -132.25 + 64 = -68.25,$

no real roots

$(4, 0)$

46. $x^2 + y^2 - 16x + 39 = 0$

$$x^2 - y^2 - 9 = 0$$

$y^2 = -x^2 + 16x - 39$

$y^2 = x^2 - 9$

$-x^2 + 16x - 39 = x^2 - 9$

$-2x^2 + 16x - 39 + 9 = 0$

$-2x^2 + 16x - 30 = 0$

$2x^2 - 16x + 30 = 0$

$2(x^2 - 8x + 15) = 0$

$2(x - 5)(x - 3) = 0$

$x = 5 \text{ or } x = 3$

$y^2 = x^2 - 9$

$y^2 = (5)^2 - 9 = 25 - 9 = 16$

$y = \pm 4$

$y^2 = (3)^2 - 9 = 9 - 9 = 0$

$y = 0$

$(5, \pm 4), (3, 0)$

47. $x^2 - 4y^2 - 20x - 64y - 172 = 0$

$$4x^2 + y^2 - 80x + 16y + 400 = 0$$

$x^2 - 20x - 4y^2 - 64y = 172$

$(x^2 - 20x + 100) - 4(y^2 + 16y + 64)$

$\qquad\qquad = 172 + 100 - 4(64)$

$(x - 10)^2 - 4(y + 8)^2 = 272 - 256$

$(x - 10)^2 - 4(y + 8)^2 = 16$

$\dfrac{(x - 10)^2}{16} - \dfrac{(y + 8)^2}{4} = 1$

$4x^2 + y^2 - 80x + 16y + 400 = 0$

$(4x^2 - 80x) + (y^2 + 16y) = -400$

$4(x^2 - 20x + 100) + (y^2 + 16y + 64)$

$\qquad = -400 + 4(100) + 64$

$4(x - 10)^2 + (y + 8)^2 = 64$

$\dfrac{(x - 10)^2}{16} + \dfrac{(y + 8)^2}{64} = 1$

$\dfrac{(x - 10)^2}{16} - \dfrac{(y + 8)^2}{4}$

$= \dfrac{(x - 10)^2}{16} + \dfrac{(y + 8)^2}{64}$

$\dfrac{(x - 10)^2}{16} - \dfrac{(x - 10)^2}{16} - \dfrac{(y + 8)^2}{4} = \dfrac{(y + 8)^2}{64}$

$\dfrac{(y + 8)^2}{64} + \dfrac{(y + 8)^2}{4} = 0$

$(y + 8)^2 + 16(y + 8)^2 = 0$

$(y^2 + 16y + 64) + 16(y^2 + 16y + 64) = 0$

$y^2 + 16y + 64 + 16y^2 + 256y + 1024 = 0$

$17y^2 + 272y + 1088 = 0$

$(17y + 136)(y + 8) = 0$

$y = -8$

$x^2 - 4(-8)^2 - 20x - 64(-8) - 172 = 0$

$x^2 - 256 - 20x + 512 - 172 = 0$

$x^2 - 20x + 84 = 0$

$(x - 14)(x - 6) = 0$

$x = 14 \quad \text{or} \quad x = 6$

$(14, -8), (6, -8)$

48. $x^2 - 2x + 4 + y^2 - 10 = 0$

$$2y^2 - x + 3 = 0$$

$x^2 - 2x + y^2 - 6 = 0$

$-2x^2 + 4x - 2y^2 + 12 = 0$

$\underline{2y^2 - x + 3 = 0}$

Chapter 10 *continued*

48. **—CONTINUED—**

$$-2x^2 + 3x + 15 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(-2)(15)}}{2(-2)}$$

$$x = \frac{-3 \pm \sqrt{9 + 120}}{-4}$$

$$x = \frac{-3 \pm \sqrt{129}}{-4}$$

$$x = \frac{3 \pm \sqrt{129}}{4}$$

$$2y^2 - x + 3 = 0$$

$$2y^2 = x - 3$$

$$y^2 = \frac{x - 3}{2}$$

$$y^2 = \frac{\left(\frac{3 + \sqrt{129}}{4}\right) - 3}{2}$$

$$y^2 = \frac{1}{2}\left(\frac{3 + \sqrt{129}}{4} - \frac{12}{4}\right)$$

$$y^2 = \frac{1}{2}\left(\frac{3 - 12 + \sqrt{129}}{4}\right)$$

$$y^2 = \frac{1}{2}\left(\frac{-9 + \sqrt{129}}{4}\right)$$

$$y = \pm\sqrt{\frac{-9 + \sqrt{129}}{8}}$$

$$y = \pm\frac{1}{2}\sqrt{\frac{-9 + \sqrt{129}}{2}}$$

$$y^2 = \frac{\left(\frac{3 - \sqrt{129}}{4}\right) - 3}{2}$$

$$y^2 = \frac{1}{2}\left(\frac{3 - \sqrt{129}}{4} - \frac{12}{4}\right)$$

$$y^2 = \frac{1}{2}\left(\frac{-9 - \sqrt{129}}{4}\right)$$

$$y = \pm\sqrt{\frac{-9 - \sqrt{129}}{8}}, \text{ no real roots}$$

$$\left(\frac{3 + \sqrt{129}}{4}, \pm\frac{1}{2}\sqrt{\frac{-9 + \sqrt{129}}{2}}\right)$$

49.

$$4x^2 - y^2 - 8x + 6y - 9 = 0$$

$$2x^2 - 3y^2 + 4x + 18y - 43 = 0$$

$$-12x^2 + 3y^2 + 24x - 18y + 27 = 0$$

$$\underline{2x^2 - 3y^2 + 4x + 18y - 43 = 0}$$

$$-10x^2 + 28x - 16 = 0$$

$$-2(5x^2 - 14x + 8) = 0$$

$$-2(5x - 4)(x - 2) = 0$$

$$x = 2 \text{ or } x = \frac{4}{5}$$

$$2(2)^2 - 3y^2 + 4(2) + 18y - 43 = 0$$

$$8 - 3y^2 + 8 + 18y - 43 = 0$$

$$-3y^2 + 16 + 18y - 43 = 0$$

$$-3y^2 + 18y - 27 = 0$$

$$-3(y^2 - 6y + 9) = 0$$

$$3(y - 3)(y - 3) = 0$$

$$y = 3$$

$$2x^2 - 3y^2 + 4x + 18y - 43 = 0$$

$$2\left(\frac{4}{5}\right)^2 - 3y^2 + 4\left(\frac{4}{5}\right) + 18y - 43 = 0$$

$$2\left(\frac{16}{25}\right) - 3y^2 + \left(\frac{16}{5}\right) + 18y - 43 = 0$$

$$\frac{32}{25} - 3y^2 + \frac{80}{25} + 18y - \frac{1075}{25} = 0$$

$$-3y^2 + 18y - \frac{1075}{25} + \frac{112}{25} = 0$$

$$-3y^2 + 18y - \frac{963}{25} = 0$$

$$y = \frac{-18 \pm \sqrt{324 - 4(-3)\left(-\frac{963}{25}\right)}}{2(-3)}$$

$$y = \frac{-18 \pm 1\sqrt{324 - 462.24}}{-6}$$

$$y = \frac{-18 \pm \sqrt{-138.24}}{-6}, \text{ no real roots}$$

$$(2, 3)$$

50. $10x^2 - 25y^2 - 100x = -160$

$$y^2 - 2x + 16 = 0$$

$$10x^2 - 25y^2 - 100x + 160 = 0$$

$$\underline{25y^2 - 50x + 400 = 0}$$

$$10x^2 - 150x + 560 = 0$$

$$10(x^2 - 15x + 56) = 0$$

$$10(x - 7)(x - 8) = 0$$

$$x = 7 \text{ or } x = 8$$

$$y^2 - 2x + 16 = 0$$

$$y^2 = 2x - 16$$

$$y^2 = 2(7) - 16$$

$$y^2 = 14 - 16$$

$$y^2 = -2, \text{ no real roots}$$

$$y^2 = 2(8) - 16$$

$$y^2 = 16 - 16$$

$$y^2 = 0$$

$$y = 0$$

$$(8, 0)$$

51 $x^2 - y - 4 = 0$

$x^2 + 3y^2 - 4y - 10 = 0$

$x^2 + 3y^2 - 4y - 10 = 0$

$\underline{-x^2 + y + 4 = 0}$

$3y^2 - 3y - 6 = 0$

$3(y^2 - y - 2) = 0$

$3(y + 1)(y - 2) = 0$

$y = -1$ or $y = 2$

$x^2 - y - 4 = 0$

$x^2 = y + 4$	$x^2 = y + 4$
$x^2 = (-1) + 4$	$x^2 = (2) + 4$
$x^2 = 3$	$x^2 = 6$
$x = \pm\sqrt{3}$	$x = \pm\sqrt{6}$

$(\pm\sqrt{3}, -1), (\pm\sqrt{6}, 2)$

52. $x^2 + y^2 + 8x + 7 = 0$

$x^2 + y^2 + 4x + 4y - 5 = 0$

$x^2 + y^2 = 1$

$x^2 + y^2 + 4x + 4y - 5 = 0$

$\underline{-(x^2 + y^2 \qquad = 1)}$

$4x + 4y - 5 = -1$

$4x + 4y = 4$

$x^2 + y^2 + 8x + 7 = 0$

$\underline{-(x^2 + y^2 \qquad = 1)}$

$8x + 7 = -1$

$8x = -8$

$8x = -8$

$4x + 4y = 4$

$8x \qquad = -8$

$\underline{-8x - 8y = -8}$

$-8y = -16$

$y = -2$

$4x + 4(-2) = 4$

$4x + (-8) = 4$

$4x = 12$

$x = 3$

$x^2 + y^2 \stackrel{?}{=} 1$

$(3)^2 + (-2)^2 \stackrel{?}{=} 1$

$9 + 4 \stackrel{?}{=} 1$

$13 \neq 1$

no intersection

53. $x^2 + y^2 - 8 = 0$

$x^2 + y^2 - 3x + y = 0$

$2x^2 + 2y^2 - 5x - 10 = 0$

$2x^2 + 2y^2 - 5x - 10 = \qquad 0$

$\underline{-(2x^2 + 2y^2 \qquad\qquad = 16)}$

$-5x - 10 = -16$

$-5x \qquad = -6$

$x^2 + y^2 - 3x + y = \quad 0$

$\underline{-(x^2 + y^2 \qquad\qquad = 8)}$

$-3x + y = -8$

$-3x + y = -8$

$-5x \qquad = -6$

$15x - 5y = \quad 40$

$\underline{-15x \qquad = -18}$

$-5y = \quad 22$

$y = -\dfrac{22}{5}$

$-3x + \left(-\dfrac{22}{5}\right) = -8$

$-3x = -8 + \dfrac{22}{5}$

$-3x = -\dfrac{40}{5} + \dfrac{22}{5}$

$-3x = -\dfrac{18}{5}$

$x = -\dfrac{18}{5}\left(-\dfrac{1}{3}\right)$

$x = \dfrac{6}{5}$

$x^2 + y^2 - 8 = 0$

$\left(\dfrac{6}{5}\right)^2 + \left(-\dfrac{22}{5}\right)^2 - 8 = 0$

$\dfrac{36}{25} - \dfrac{484}{25} = 8$

$-\dfrac{348}{25} = 8$

$13.92 \neq 8$

no intersection

54.
$x^2 + 3y^2 = 16$	$3(2)^2 + y^2 = 16$
$3x^2 + y^2 = 16$	$12 + y^2 = 16$
$y = -x$	$y = \pm 2$
$x^2 + 3(-x)^2 = 16$	$3(-2)^2 + y^2 = 16$
$x^2 + 3x^2 = 16$	$12 + y^2 = 16$
$4x^2 = 16$	$y = \pm 2$

—CONTINUED—

Chapter 10 *continued*

54. —CONTINUED—

$$x^2 = 4$$
$$x = \pm 2$$

$y = -x$

$y = -(-2) = 2$

$y = -(2) = -2$

$(-2, 2), (2, -2)$

55. $x^2 + y^2 - 4x - 4y = 26$

$x^2 + y^2 - 4x = 54$

$y = 3x - 8$

$$
\begin{aligned}
x^2 + y^2 - 4x - 4y &= 26 \\
-(x^2 + y^2 - 4x &= 54) \\
\hline
-4y &= -28 \\
y &= 7
\end{aligned}
$$

$3x = y + 8$

$x = \dfrac{y + 8}{3}$

$x = \dfrac{7 + 8}{3} = \dfrac{15}{3} = 5$

$(5, 7)$

56. Use rotation by 180°. If one is at (a, b), the other is at $(-a, -b)$.

57.

0 points	2 points	4 points
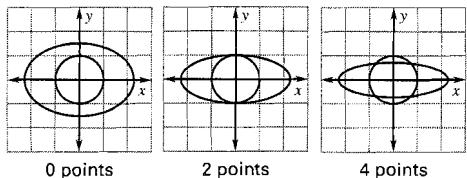
58.

$$d = 60t$$
$$d = 3600t^2$$
$$3600t^2 = 60t$$
$$3600t^2 - 60t = 0$$
$$60t(60t - 1) = 0$$
$$t = 0, \; t = \frac{1}{60}h = 1 \text{ min}$$

59. $x^2 + y^2 = 1620$

$y = -\frac{1}{3}x + 30$

$x^2 + \left(-\frac{1}{3}x + 30\right)^2 = 1620$

$x^2 + \frac{1}{9}x^2 - 20x + 900 - 1620 = 0$

$\frac{10}{9}x^2 - 20x - 720 = 0$

$10x^2 - 180x - 6480 = 0$

$10(x^2 - 18x - 648) = 0$

$10(x - 36)(x + 18) = 0$

$x = 36 \quad \text{or} \quad x = -18$

$y = -\dfrac{1}{3}x + 30$

$y = -\dfrac{1}{3}(36) + 30 = -12 + 30 = 18$

$y = -\dfrac{1}{3}(-18) + 30 = 6 + 30 = 36$

$(36, 18), (-18, 36)$

$d = \sqrt{(-18 - 36)^2 + (36 - 18)^2} = \sqrt{2916 + 324}$

$d = \sqrt{3240} \approx 59.6 \text{ miles}$

60. Slope of line containing Clark Street: $m = \dfrac{0 - (-1)}{5 - (-2)} = \dfrac{1}{7}$

Equation of the line: $y - 0 = \dfrac{1}{7}(x - 5)$

$$y = \frac{1}{7}x - \frac{5}{7}$$

Equation of circle: $x^2 + y^2 = 1$

$x^2 + y^2 = 1$

$y = \dfrac{1}{7}x - \dfrac{5}{7}$

$x^2 + \left(\dfrac{1}{7}x - \dfrac{5}{7}\right)^2 = 1$

$x^2 + \dfrac{1}{49}x^2 - \dfrac{10}{49}x + \dfrac{25}{49} = 1$

$\dfrac{50}{49}x^2 - \dfrac{10}{49}x - \dfrac{24}{49} = 0$

$50x^2 - 10x - 24 = 0$

$2(25x^2 - 5x - 12) = 0$

$2(5x + 3)(5x - 4) = 0$

$x = -\dfrac{3}{5} \quad \text{or} \quad x = \dfrac{4}{5}$

$y = \dfrac{1}{7}\left(-\dfrac{3}{5}\right) - \dfrac{5}{7}$ $\qquad$ $y = \dfrac{1}{7}\left(\dfrac{4}{5}\right) - \dfrac{5}{7}$

$y = -\dfrac{3}{35} - \dfrac{5}{7}$ $\qquad$ $y = \dfrac{4}{35} - \dfrac{5}{7}$

$y = -\dfrac{4}{5}$ $\qquad\qquad$ $y = -\dfrac{3}{5}$

$\left(-\dfrac{3}{5}, -\dfrac{4}{5}\right), \left(\dfrac{4}{5}, -\dfrac{3}{5}\right)$

—CONTINUED—

Chapter 10 *continued*

60. —CONTINUED—

$$d = \sqrt{\left(-\frac{3}{5} - \frac{4}{5}\right)^2 + \left(-\frac{4}{5} - \left(-\frac{3}{5}\right)\right)^2}$$

$$= \sqrt{\frac{49}{25} + \frac{1}{25}}$$

$$= \sqrt{\frac{50}{25}}$$

$$= \sqrt{2}$$

$$d \approx 1.41 \text{ mi}$$

61. $xy - 24 = 0$

$$x^2 - 25y^2 + 100 = 0$$

$$xy = 24$$

$$x = \frac{24}{y}$$

$$\left(\frac{24}{y}\right)^2 - 25y^2 + 100 = 0$$

$$\frac{576}{y^2} - 25y^2 + 100 = 0$$

$$\frac{576 - 25y^4 + 100y^2}{y^2} = 0$$

$$25y^4 - 100y^2 - 576 = 0$$

$$(5y^2 - 36)(5y^2 + 16) = 0$$

$$5y^2 = 36 \qquad 5y^2 = -16$$

$$y^2 = \frac{36}{5} \qquad y^2 = -\frac{16}{5}, \text{ no real roots}$$

$$y = \pm\frac{6}{\sqrt{5}}$$

$$x\left(\pm\frac{6}{\sqrt{5}}\right) - 24 = 0$$

$$\pm\frac{6}{\sqrt{5}}x = 24$$

$$x = 24\left(\pm\frac{\sqrt{5}}{6}\right)$$

$$x = \pm 4\sqrt{5}$$

$$\left(4\sqrt{5}, \frac{6\sqrt{5}}{5}\right) \approx (8.9, 2.7)$$

62. $m = \dfrac{8 - 0}{0 - 10} = -\dfrac{8}{10} = -\dfrac{4}{5}$

Equation of line: $y - 8 = -\dfrac{4}{5}(x - 0)$

$$y = -\frac{4}{5}x + 8$$

Equation of hyperbola: $\dfrac{x^2}{36} - \dfrac{y^2}{64} = 1$

$$\frac{x^2}{36} - \frac{y^2}{64} = 1$$

$$y = -\frac{4}{5}x + 8$$

$$\frac{x^2}{36} - \frac{\left(-\frac{4}{5}x + 8\right)^2}{64} = 1$$

$$16x^2 - 9\left(-\frac{4}{5}x + 8\right)^2 = 576$$

$$16x^2 - 9\left(\frac{16}{25}x^2 - \frac{64}{5}x + 64\right) = 576$$

$$16x^2 - \frac{144}{25}x^2 + \frac{576}{5}x - 576 - 576 = 0$$

$$400x^2 - 144x^2 + 2880x - 14{,}400 - 14{,}400 = 0$$

$$256x^2 + 2880x - 28{,}880 = 0$$

$$64(4x^2 + 45x - 450) = 0$$

$$x = \frac{-45 \pm \sqrt{2025 - 4(4)(-450)}}{2(4)}$$

$$x = \frac{-45 \pm \sqrt{9225}}{8}$$

$$x = \left(\frac{-45 \pm 15\sqrt{41}}{8}\right)$$

$$y = -\frac{4}{5}\left(\frac{-45 + 15\sqrt{41}}{8}\right) + 8$$

$$y = \frac{9 - 3\sqrt{41}}{2} + 8$$

$$y = \frac{9 - 3\sqrt{41} + 16}{2}$$

$$y = \frac{25 - 3\sqrt{41}}{2}$$

$$\left(-\frac{45 + 15\sqrt{41}}{8}, \frac{25 - 3\sqrt{41}}{2}\right) \approx (6.38, 2.90)$$

63. Location 1: $x^2 + y^2 = (13)^2$

$$x^2 + y^2 = 169$$

Location 2: $(x - 12)^2 + (y + 19)^2 = 169$

$$x^2 - 24x + 144 + y^2 + 38y + 361 = 169$$

Location 3: $(x - 11)^2 + (y - 10)^2 = 625$

$$x^2 - 22x + 121 + y^2 - 20y + 100 = 625$$

$$
\begin{aligned}
x^2 - 24x + 144 + y^2 + 38y + 361 &= 169 \\
-(x^2 \qquad\qquad + y^2) &= (-169) \\
\hline
-24x + 144 + \qquad 38y + 361 &= 0 \\
-24x \qquad + \qquad 38y + 505 &= 0 \\
-24x \qquad + \qquad 38y &= -505
\end{aligned}
$$

Chapter 10 *continued*

63. —CONTINUED—

$$x^2 - 22x + 121 + y^2 - 20y + 100 = 625$$
$$\underline{-(x^2 \qquad\qquad + y^2) \qquad\qquad = (-169)}$$
$$-22x + 121 \quad -20y + 100 = 456$$
$$-22x \qquad\qquad -20y + 221 = 456$$
$$\underline{-22x \qquad\qquad -20y \qquad = 235}$$

$$-24x + \quad 38y = -505$$
$$\underline{-22x - \quad 20y = \quad 235}$$
$$528x - \quad 836y = 11{,}110$$
$$\underline{-528x - \quad 480y = \quad 5640}$$
$$-1316y = 16{,}750$$
$$y \approx -12.73$$

$$-22x - 20y = 235$$
$$-22x - 20(-12.73) = 235$$
$$-22x = 235 - 254.6$$
$$-22x = 19.6$$
$$x \approx 0.89$$

$(0.89, -12.73)$

Epicenter is approximately 100 kilometers east, 1300 kilometers south of location 1.

64. $\quad x^2 + y^2 = 6$
$$2x^2 + 4y^2 = 7$$
$$-2x^2 - 2y^2 = -12$$
$$\underline{2x^2 + 4y^2 = \qquad 7}$$
$$2y^2 = \quad -5$$
$$y^2 = -\tfrac{5}{2}, \text{ no real roots}$$

A

65. $25x^2 + 36y^2 - 900 = 0$
$$-2x^2 + y + 5 = 0$$
$$y = 2x^2 - 5$$
$$25x^2 + 36(2x^2 - 5)^2 - 900 = 0$$
$$25x^2 + 36(4x^4 - 20x^2 + 25) - 900 = 0$$
$$25x^2 + 144x^4 - 720x^2 + 900 - 900 = 0$$
$$144x^4 - 695x^2 = 0$$
$$x^2(144x^2 - 695) = 0$$
$$x^2 = 0 \qquad 144x^2 = 695$$
$$x = 0 \qquad x^2 = \frac{695}{144}$$
$$x = \pm\frac{\sqrt{695}}{12}$$
$$y = 2x^2 - 5 = 2(0)^2 - 5 = -5$$
$(0, 5)$

E

66. *Sample answer:* $\dfrac{x^2}{16} + \dfrac{(y-6)^2}{4} = 1$

$$y = \frac{3}{8}x^2$$

$$(x + 4)^2 + (y - 3)^2 = 9$$

10.7 Mixed Review (p. 638)

67. $2(4) + 5 = 8 + 5 = 13$

68. $\dfrac{1}{(2)^3} - 1 = \dfrac{1}{8} - 1 = \dfrac{1}{8} - \dfrac{8}{8} = -\dfrac{7}{8}$

69. $(-2)^{5-1} = (-2)^4 = 16$

70. $\dfrac{3}{(-3)^{4-2}} = \dfrac{3}{(-3)^2} = \dfrac{3}{9} = \dfrac{1}{3}$

71. $f(x) = (x - 3)(x + 3)(x - 1)$
$$= (x^2 - 9)(x - 1)$$
$$= x^3 - x^2 - 9x + 9$$

72. $f(x) = x(x - 2)^2(x - 4)$
$$= x(x^2 - 4x + 4)(x - 4)$$
$$= x(x^3 - 4x^2 + 4x - 4x^2 + 16x - 16)$$
$$= x(x^3 - 8x^2 + 20x - 16)$$
$$= x^4 - 8x^3 + 20x^2 - 16x$$

73. $f(x) = (x - 2i)(x + 2i)$
$$= x^2 - 4i^2$$
$$= x^2 + 4$$

74. $f(x) = (x - 3 - i)(x - 3 + i)$
$$= x^2 - 3x + xi - 3x + 9 - 3i - xi + 3i - i^2$$
$$= x^2 - 6x + 9 + 1$$
$$= x^2 - 6x + 10$$

75. $f(x) = (x - 2)(x + 1)(x + 1 + i)(x + 1 - i)$
$$= (x^2 - x - 2)(x^2 + x - xi + x + 1 - i + xi + i - i^2)$$
$$= (x^2 - x - 2)(x^2 + 2x + 1 + 1)$$
$$= (x^2 - x - 2)(x^2 + 2x + 2)$$
$$= x^4 + 2x^3 + 2x^2 - x^3 - 2x^2 - 2x - 2x^2 - 4x - 4$$
$$= x^4 + x^3 - 2x^2 - 6x - 4$$

76. $f(x) = (x + 2)(x + 3)(x - i)(x + i)(x - i)(x + i)$
$$= (x^2 + 5x + 6)(x^2 - i^2)(x^2 - i^2)$$
$$= (x^2 + 5x + 6)(x^2 + 1)(x^2 + 1)$$
$$= (x^2 + 5x + 6)(x^4 + 2x^2 + 1)$$
$$= x^6 + 2x^4 + x^2 + 5x^5 + 10x^3 + 5x + 6x^4 + 12x^2 + 6$$
$$= x^6 + 5x^5 + 8x^4 + 10x^3 + 13x^2 + 5x + 6$$

77. $f(x) = \sqrt{2x + 3}$

x	$-\frac{3}{2}$	0	$\frac{1}{2}$	3
y	0	$\sqrt{3}$	2	3

domain: $x \geq -\frac{3}{2}$

range: $y \geq 0$
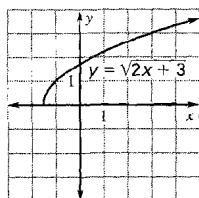
78. $f(x) = 5\sqrt{x - 8}$

x	8	9	12
y	0	5	10

domain: $x \geq 8$

range: $y \geq 0$
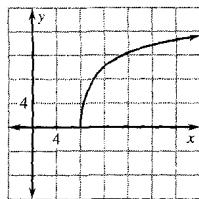
79. $f(x) = -(x + 4)^{1/2} + 2$

x	0	-4	5
y	0	2	-1

domain: $x \geq -4$

range: $y \leq 2$
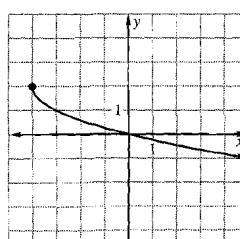
80. $f(x) = -3\sqrt[3]{x + 1}$

x	-1	7	-9
y	0	-6	6

domain: all reals

range: all reals
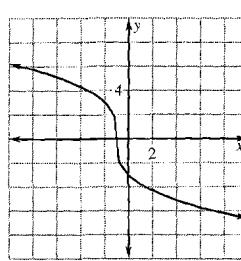
81. $f(x) = \sqrt[3]{4x + 1} + 2$

x	$\frac{7}{4}$	1
y	4	3.71

domain: all reals

range: all reals
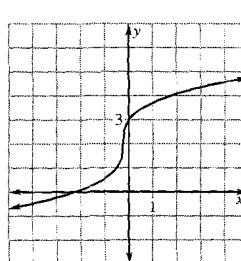
82. $f(x) = 5(x - 1)^{1/3}$

x	1	9	-7
y	0	10	-10

domain: all reals

range: all reals
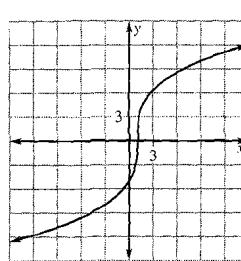
83. $A = 3, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(3)(1) = -12$

$B^2 - 4AC < 0, B = 0, A \neq C$

ellipse

84. $A = 4, B = 0, C = -1$

$B^2 - 4AC = 0^2 - 4(4)(-1) = 16$

$B^2 - 4AC > 0$

hyperbola

85. $A = 1, B = 0, C = 0$

$B^2 - 4AC = 0^2 - 4(1)(0) = 0$

$B^2 - 4AC = 0$

parabola

86. $A = 1, B = 0, C = 1$

$B^2 - 4AC = 0^2 - 4(1)(1) = -4$

$B^2 - 4AC < 0, B = 0, A = C$

circle

Quiz 3 (p. 638)

1. $(x - h)^2 + (y - k)^2 = r^2$

$(x + 3)^2 + (y + 5)^2 = 64$

2. Horizontal: $\dfrac{(x - h)^2}{a^2} + \dfrac{(y - k)^2}{b^2} = 1$

$(h, k) = \left(\dfrac{-7 + 6}{2}, \dfrac{2 + 2}{2}\right) = \left(-\dfrac{1}{2}, 2\right)$

$a = \sqrt{\left(-7 + \dfrac{1}{2}\right)^2 + (2 - 2)^2}$

$a = \sqrt{\left(-\dfrac{14}{2} + \dfrac{1}{2}\right)^2} = \sqrt{\left(-\dfrac{13}{2}\right)^2} = \sqrt{\dfrac{169}{4}} = \dfrac{13}{2}$

$c = \sqrt{\left(4 + \dfrac{1}{2}\right)^2 + (2 - 2)^2}$

$c = \sqrt{\left(\dfrac{8}{2} + \dfrac{1}{2}\right)^2} = \sqrt{\left(\dfrac{9}{2}\right)^2} = \dfrac{9}{2}$

$b^2 = a^2 - c^2$

$\quad = \left(\dfrac{13}{2}\right)^2 - \left(\dfrac{9}{2}\right)^2 = \dfrac{169}{4} - \dfrac{81}{4} = \dfrac{88}{4} = 22$

$\dfrac{(x + 0.5)^2}{42.25} + \dfrac{(y - 2)^2}{22} = 1$

3. parabola opens right $p > 0$

$(y - k)^2 = 4p(x - h)$

vertex $(4, -1)$, so $h = 4, k = -1$

$|p| = \sqrt{(7 - 4)^2 + (-1 + 1)^2} = \sqrt{(3)^2} = 3 \rightarrow p = 3$

$(y - k)^2 = 4p(x - h)$

$(y + 1)^2 = 4(3)(x - 4)$

$(y + 1)^2 = 12(x - 4)$

Chapter 10 *continued*

4. Vertical: $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

$(h,k) = \left(\dfrac{2+2}{2}, \dfrac{3+4}{2}\right)$

$(h,k) = \left(2, \dfrac{7}{2}\right) = (2, 3.5)$

$a = \sqrt{(2-2)^2 + (3-3.5)^2}$

$\quad = \sqrt{(-0.5)^2} = \sqrt{0.25} = 0.5$

$c = \sqrt{(2-2)^2 + (-1-3.5)^2} = \sqrt{(-4.5)^2}$

$\quad = \sqrt{20.25} = 4.5$

$b^2 = c^2 - a^2 = 4.5^2 - 0.5^2 = 20.25 - 0.25 = 20$

$\dfrac{(y-3.5)^2}{0.25} - \dfrac{(x-2)^2}{20} = 1$

5. $A = 1, B = 0, C = 4$

$B^2 - 4AC = 0^2 - 4(1)(4) = -16$

$B^2 - 4AC < 0, \ B = 0, \ A \neq C$

ellipse

6. $A = -1, B = 0, C = -1$

$B^2 - 4AC = 0^2 - 4(-1)(-1) = -4$

$B^2 - 4AC < 0, \ B = 0, \ A \neq C$

circle

7. $A = 0, B = 0, C = -2$

$B^2 - 4AC = 0^2 - 4(0)(-2) = 0$

$B^2 - 4AC = 0$

parabola

8. $A = -6, B = 0, C = 4$

$B^2 - 4AC = 0^2 - 4(-6)(4) = 96$

$B^2 - 4AC > 0$

hyperbola

9. $3x^2 - 4x - y + 2 = 0$

$y = -5x + 4$

$3x^2 - 4x - (-5x + 4) + 2 = 0$

$3x^2 - 4x + 5x - 4 + 2 = 0$

$3x^2 + x - 2 = 0$

$(3x - 2)(x + 1) = 0$

$x = \dfrac{2}{3}$ or $x = -1$

$y = -5\left(\dfrac{2}{3}\right) + 4 = -\dfrac{10}{3} + \dfrac{12}{3} = \dfrac{2}{3}$

$y = -5(-1) + 4 = 5 + 4 = 9$

$\left(\dfrac{2}{3}, \dfrac{2}{3}\right), (-1, 9)$

10. $\begin{aligned} -x^2 + y^2 + 4x - 6y + 4 &= 0 \\ \underline{x^2 + y^2 - 4x - 6y + 12} &= 0 \\ 2y^2 - 12y + 16 &= 0 \end{aligned}$

$2(y^2 - 6y + 8) = 0$

$2(y - 4)(y - 2) = 0$

$y = 4$ or $y = 2$

$x^2 + (4)^2 - 4x - 6(4) + 12 = 0$

$x^2 + 16 - 4x - 24 + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x = 2$

$x^2 + (2)^2 - 4x - 6(2) + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x = 2$

$(2, 4), (2, 2)$

11. $\begin{aligned} x^2 + y^2 + 4y - 12 &= 0 \\ x^2 - 16y^2 - 64y - 80 &= 0 \end{aligned}$

$\begin{aligned} x^2 - 16y^2 - 64y - 80 &= 0 \\ \underline{-x^2 - y^2 - 4y + 12} &= 0 \\ -17y^2 - 68y - 68 &= 0 \end{aligned}$

$-17(y^2 + 4y + 4) = 0$

$(y + 2)(y + 2) = 0$

$y = -2$

$x^2 + (-2)^2 + 4(-2) - 12 = 0$

$x^2 + 4 - 8 - 12 = 0$

$x^2 - 16 = 0$

$(x - 4)(x + 4) = 0$

$x = \pm 4$

$(-4, -2), (4, -2)$

12. $\begin{aligned} y^2 - 6x - 2y - 3 &= 0 \\ 2y^2 - 4y + x + 6 &= 0 \end{aligned}$

$\begin{aligned} y^2 - 6x - 2y - 3 &= 0 \\ \underline{12y^2 - 24y + 6x + 36} &= 0 \\ 13y^2 - 26y + 33 &= 0 \end{aligned}$

$y = \dfrac{26 \pm \sqrt{676 - 4(13)(33)}}{2(13)}$

$y = \dfrac{26 \pm \sqrt{676 - 1716}}{26}$, no real roots

13. The epicenter of the earthquake is 50 miles due west of the seismograph.