CHAPTER 12

Think & Discuss *(p. 699)*

1. Beethoven & Mozart
   Beethoven & Dvorak
   Beethoven & Brahms
   Beethoven & Srauss
   Beethoven & Tchaikovsky

   Mozart & Dvorak
   Mozart & Brahms
   Mozart & Strauss
   Mozart & Tchaikovsky

   Dvorak & Brahms
   Dvorak & Strauss
   Dvorak & Tchaikovsky

   Brahms & Strauss
   Brahms & Tchaikovsky

   Strauss & Tchaikovsky

   15 choices
2. $15 + 6 = 21$

Skill Review *(p. 700)*

1. 0.5, 50%
2. 0.2, 20%
3. 0.15, 15%
4. 0.48, 48%
5. 0.194, 19.4%
6. 0.469, 46.9%
7. $A = 4^2\pi$
   $= 16\pi$
   $= 50.27$
8. $A = 5^2$
   $= 25$
9. $A = \frac{1}{2}(12)(8)$
   $= 48$
10. $10^x = 0.5$
    $\log 10^x = \log 0.5$
    $x \log 10 = \log 0.5$
    $x = \dfrac{\log 0.5}{\log 10}$
    $x = -0.301$
11. $(0.5)^x + 3 = 3.75$
    $(0.5)^x = 0.75$
    $x \log 0.5 = \log 0.75$
    $x = 0.415$
12. $1 - 9^x = 0.25$
    $0.75 = 9^x$
    $\log 0.75 = x \log 9$
    $x = -0.131$

Lesson 12.1

12.1 Guided Practice (p. 705)

1. A permutation of n objects is an ordering of those objects.
2. The Fundamental Counting Principle states that if one event can occur in n ways, another in m ways, and a third in p ways, the number of ways all can occur is $n \cdot m \cdot p$. The number of permutations of n objects is also a product, $n(n-1)(n-2)\cdot \ldots \cdot 3\cdot 2\cdot 1$.
3. Yes; $1! = 1$ so $\dfrac{4!}{2!} = \dfrac{4!}{(2!)(1!)(1!)}$.
4. Because we are interested in all numbers with three even digits, those digits can be repeated. Therefore the correct answer is $5 \times 5 \times 5 = 125$.
5. $2! = 2$
6. $6! = 720$
7. $1! = 1$
8. $4! = 24$
9. $\dfrac{6!}{(6-3)!} = 6\cdot 5\cdot 4 = 120$
10. $\dfrac{5!}{(5-1)!} = 5$
11. $\dfrac{3!}{(3-3)!} = 6$
12. $\dfrac{10!}{(10-2)!} = 90$
13. $\dfrac{7!}{4!} = 210$
14. $\dfrac{5!}{3!\cdot 2!} = 10$

12.1 Practice and Applications (pp. 705–707)

15. $3\cdot 1 = 3$ ways
16. $3\cdot 5 = 15$ ways
17. $2\cdot 4\cdot 5 = 40$ ways
18. $4\cdot 6\cdot 9\cdot 7 = 1512$ ways
19. (a) $26\cdot 26\cdot 26\cdot 10\cdot 10\cdot 10 = 17{,}576{,}000$
    (b) $26\cdot 25\cdot 24\cdot 10\cdot 9\cdot 8 = 11{,}232{,}000$
20. (a) $10\cdot 10\cdot 26\cdot 26\cdot 26\cdot 26 = 45{,}697{,}600$
    (b) $10\cdot 9\cdot 26\cdot 25\cdot 24\cdot 23 = 32{,}292{,}000$
21. (a) $10\cdot 10\cdot 10\cdot 10\cdot 26\cdot 26 = 6{,}760{,}000$
    (b) $10\cdot 9\cdot 8\cdot 7\cdot 26\cdot 25 = 3{,}276{,}000$
22. (a) $26\cdot 26\cdot 26\cdot 26\cdot 26\cdot 10 = 118{,}813{,}760$
    (b) $26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 10 = 78{,}936{,}000$
23. $8! = 40{,}320$
24. $5! = 120$
25. $10! = 3{,}628{,}800$
26. $9! = 362{,}880$
27. $0! = 1$
28. $7! = 5040$
29. $3! = 6$
30. $12! = 479{,}001{,}600$
31. $\dfrac{3!}{(3-3)!} = 6$
32. $\dfrac{5!}{(5-2)!} = 20$
33. $\dfrac{2!}{(2-1)!} = 2$
34. $\dfrac{7!}{(7-6)!} = 5040$
35. $\dfrac{8!}{(8-5)!} = 6720$
36. $\dfrac{9!}{(9-4)!} = 3024$
37. $\dfrac{12!}{(12-3)!} = 1320$
38. $\dfrac{16!}{(16-0)!} = 1$
39. $2! = 2$
40. $3! = 6$
41. $4! = 24$
42. $5! = 120$
43. $6! = 720$
44. $7! = 5040$
45. $8! = 40{,}320$
46. $9! = 362{,}880$
47. $\dfrac{3!}{2!} = 3$
48. $\dfrac{5!}{3!} = 20$

Chapter 12 *continued*

49. $\dfrac{6!}{2!} = 360$

50. $\dfrac{6!}{2!2!} = 180$

51. $\dfrac{7!}{2!} = 2520$

52. $\dfrac{7!}{4!} = 210$

53. $\dfrac{8!}{2!2!} = 10{,}080$

54. $\dfrac{11!}{3!2!2!} = 1{,}663{,}200$

55. $5 \cdot 8 \cdot 12 = 480$

56. $7 \cdot 11 \cdot 6 \cdot 3 \cdot 2 = 2772$

57. **(a)** $36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 = 2{,}176{,}782{,}336$

(b) $36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 = 1{,}402{,}410{,}240$

58. ${}_nP_0 = \dfrac{n!}{(n-0)!} = \dfrac{n!}{n!} = 1$

This makes sense because there is only one way to choose zero objects from any set: take none.

59. $24! = 6.20 \times 10^{23}$

60. $10! = 3{,}628{,}800$

61. **(a)** $6! = 720$

(b) $\dfrac{9!}{(9-6)!} = 60{,}480$

62. $\dfrac{7!}{3!2!2!} = 210$

63. $\dfrac{15!}{3!5!4!3!} = 12{,}612{,}600$

64. ${}_nP_{n-1} = \dfrac{n!}{(n-(n-1))!} = \dfrac{n!}{1} = n!$ is the same as the number of permutations of all n objects. This makes sense because it can be thought of as choosing one object to leave out (n ways) then doing permutations of the $n - 1$ objects remaining $[(n-1)!$ ways$]$. Now we have $n \times (n-1)! = n!$.

65. B

66. C

67. $\dfrac{n!}{n} = (n-1)!$; You can arrange n objects in a circle n different ways in the same order so divide $n!$ by n to find the number of unique circular permutations.

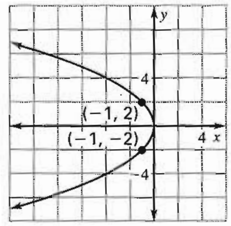
12.1 Mixed Review (p. 707)

68. $(x + 9)(x - 9) = x^2 - 81$

69. $(x^2 + 2)^2 = x^4 + 4x^2 + 4$

70. $(2x - 1)^3 = 8x^3 - 12x^2 + 6x - 1$

76.



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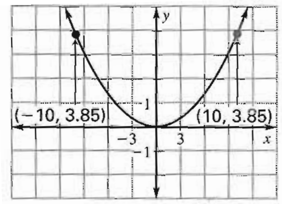

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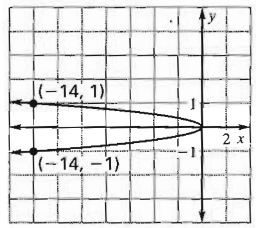

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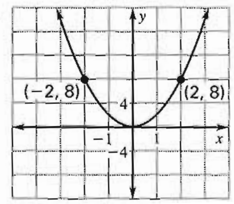


80. $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n = 6$

81. $\sum_{n=0}^{\infty} -4\left(\frac{1}{4}\right)^n = -\frac{16}{3}$

82. $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = 6$

83. $\sum_{n=1}^{\infty} 2\left(\frac{7}{5}\right)^{n-1}$ no sum

84. $\sum_{n=0}^{\infty} -5\left(\frac{1}{8}\right)^n = -\frac{40}{7}$

85. $\sum_{n=1}^{\infty} \frac{1}{2}(0.3)^{n-1} = 0.714$

86. $R = \dfrac{V}{I}$

$= \dfrac{120}{0.80}$

$= 150$ ohms

Lesson 12.2

Activity (p. 710)

1. **(a)** $a^2 + 2ab + b^2$

(b) $(a + b)(a^2 + 2ab + b^2) = a^3 + 3a^2b + 3ab^2 + b^3$

(c) $(a + b)(a^3 + 3a^2b + 3ab^2 + b^3)$

$= a^4 + 3a^3b + 3a^2b^2 + ab^3 + a^3b + 3a^2b^2 + 3ab^3 + b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

2. The coefficients for $(a + b)^n$ [redacted]

3. $(x+y)^4 = {}_4C_0x^4y^0 + {}_4C_1x^3y^1 + {}_4C_2x^2y^2 + {}_4C_3xy^3 + {}_4C_4x^0y^4$
$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x-y)^4 = {}_4C_0x^4(-y)^0 + {}_4C_1x^3(-y)^1 + {}_4C_2x^2(-y)^2 + {}_4C_3x(-y)^3 + {}_4C_4x^0(-y)^4$
$= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

The two expansions have the same variables raised to the same powers and the same coefficients. The signs of the two terms are different.

4. The problem should be $\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$. The six and five will cancel also.

5. ${}_8C_2 = \frac{8!}{6! \cdot 2!} = \frac{8 \cdot 7}{2 \cdot 1} = 28$

6. ${}_6C_5 = \frac{6!}{1! \cdot 5!} = \frac{6}{1} = 6$

7. ${}_5C_1 = \frac{5!}{4! \cdot 1!} = \frac{5}{1} = 5$

8. ${}_9C_9 = \frac{9!}{0! \cdot 9!} = 1$

9. $(x+y)^3 = {}_3C_0x^3y^0 + {}_3C_1x^2y^1 + {}_3C_2xy^2 + {}_3C_3x^0y^3$
$= x^3 + 3x^2y + 3xy^2 + y^3$

10. $(x+1)^4 = {}_4C_0x^4(1)^0 + {}_4C_1x^3(1)^1 + {}_4C_2x^2(1)^2 + {}_4C_3x(1)^3 + {}_4C_4x^0(1)^4$
$= x^4 + 4x^3 + 6x^2 + 4x + 1$

11. $(2x+4)^3 = {}_3C_0(2x)^3(4)^0 + {}_3C_1(2x)^2(4)^1 + {}_3C_2(2x)^1(4)^2 + {}_3C_3(2x)^0(4)^3$
$= 8x^3 + (3)(4x^2)(4) + (3)(2x)(16) + (1)(1)(64)$
$= 8x^3 + 48x^2 + 96x + 64$

12. $(2x+3y)^5$
$= {}_5C_0(2x)^5(3y)^0 + {}_5C_1(2x)^4(3y)^1 + {}_5C_2(2x)^3(3y)^2 + {}_5C_3(2x)^2(3y)^3 + {}_5C_4(2x)^1(3y)^4 + {}_5C_5(2x)^0(3y)^5$
$= (1)(32x^5)(1) + (5)(16x^4)(3y) + 10(8x^3)(9y^2) + (10)(4x^2)(27y^3) + 5(2x)(81y^4) + (1)(1)(243y^5)$
$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

13. $(x-y)^5 = {}_5C_0x^5(-y)^0 + {}_5C_1x^4(-y)^1 + {}_5C_2x^3(-y)^2 + {}_5C_3x^2(-y)^3 + {}_5C_4x^1(-y)^4 + {}_5C_5x^0(-y)^5$
$= x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

14. $(x-2)^3 = {}_3C_0x^3(-2)^0 + {}_3C_1x^2(-2)^1 + {}_3C_2x^1(-2)^2 + {}_3C_3x^0(-2)^3$
$= x^3 - 6x^2 + 12x - 8$

15. $(3x-1)^4 = {}_4C_0(3x)^4(-1)^0 + {}_4C_1(3x)^3(-1)^1 + {}_4C_2(3x)^2(-1)^2 + {}_4C_3(3x)^1(-1)^3 + {}_4C_4(3x)^0(-1)^4$
$= 81x^4 + 4(27x^3)(-1) + 6(9x^2)(1) + 4(3x)(-1) + 1$
$= 81x^4 - 108x^3 + 54x^2 - 12x + 1$

16. $(4x-4y)^3 = {}_3C_0(4x)^3(-4y)^0 + {}_3C_1(4x)^2(-4y)^1 + {}_3C_2(4x)^1(-4y)^2 + {}_3C_3(4x)^0(-4y)^3$
$= 64x^3 + (3)(16x^2)(-4y) + 3(4x)(16y^2) + (-64y^3)$
$= 64x^3 - 192x^2y + 192xy^2 - 64y^3$

17. ${}_5C_3(27) = 10(27) = 270$
${}_5C_5(3)^5 = 243$

12.2 Practice and Applications (pp. 712–714)

18. ${}_{10}C_2 = \frac{10!}{8! \cdot 2!} = \frac{10 \cdot 9}{2 \cdot 1} = 45$

19. ${}_8C_5 = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

20. ${}_5C_2 = \frac{5!}{3! \cdot 2!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$

21. ${}_8C_6 = \frac{8!}{2! \cdot 6!} = \frac{8 \cdot 7}{2 \cdot 1} = 28$

22. ${}_{12}C_4 = \frac{12!}{8! \cdot 4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 495$

23. ${}_{12}C_{12} = \frac{12!}{0! \cdot 12!} = 1$

24. ${}_{14}C_6 = \frac{14!}{8! \cdot 6!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 3003$

25. ${}_{11}C_3 = \frac{11!}{8! \cdot 3!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165$

26. ${}_{12}C_5 = \frac{12!}{7! \cdot 5!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 792$

27. ${}_4C_4 \cdot {}_{48}C_1 = 1(48)$
$= 48$

28. ${}_4C_1 \cdot {}_{48}C_4 = \dfrac{4!}{3! \cdot 1!} \cdot \dfrac{48!}{44! \cdot 4!}$
$= 4 \cdot 194{,}580$
$= 778{,}320$

29. ${}_4C_2 \cdot {}_4C_3 = 6 \cdot 4$
$= 24$

30. ${}_{13}C_1 \cdot {}_{48}C_1 = 13 \cdot 48$
$= 624$

31.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

32. $(x + 4)^6 = x^6 + 6(4)^1x^5 + 15(4)^2x^4 + 20(4)^3x^3 + 15(4)^4x^2 + 6(4)^5x^1 + 4^6$
$= x^6 + 24x^5 + 240x^4 + 1280x^3 + 3840x^2 + 6144x + 4096$

33. $(x - 3y)^6 = x^6 + 6(-3y)^1x^5 + 15(-3y)^2x^4 + 20(-3y)^3x^3 + 15(-3y)^4x^2 + 6(-3y)^5x^2 + (-3y)^6$
$= x^6 - 18x^5y + 135x^4y^2 - 540x^3y^3 + 1215x^2y^4 - 1458xy^5 + 729y^6$

34. $(x^2 + y)^7 = (x^2)^7 + 7(x^2)^6y^1 + 21(x^2)^5y^2 + 35(x^2)^4y^3 + 35(x^2)^3y^4 + 21(x^2)^2y^5 + 7(x^2)^1y^6 + y^7$
$= x^{14} + 7x^{12}y + 21x^{10}y^2 + 35x^8y^3 + 35x^6y^4 + 21x^4y^5 + 7x^2y^6 + y^7$

35. $(2x - y^3)^7 = (2x)^7 + 7(2x)^6(-y^3)^1 + 21(2x)^5(-y^3)^2 + 35(2x)^4(-y^3)^3 + 35(2x)^3(-y^3)^4 + 21(2x)^2(-y^3)^5 + 7(2x)^1(-y^3)^6 + (-y^3)^7$
$= 128x^7 - 448x^6y^3 + 672x^5y^6 - 560x^4y^9 + 280x^3y^{12} - 84x^2y^{15} + 14xy^{18} - y^{21}$

36. $(x - 2)^3 = {}_3C_0x^3(-2)^0 + {}_3C_1x^2(-2)^1 + {}_3C_2x^1(-2)^2 + {}_3C_3x^0(-2)^3$
$= x^3 - 6x^2 + 12x - 8$

37. $(x + 4)^5 = {}_5C_0x^5(4)^0 + {}_5C_1x^4(4)^1 + {}_5C_2x^3(4)^2 + {}_5C_3x^2(4)^3 + {}_5C_4x^1(4)^4 + {}_5C_5x^0(4)^5$
$= x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024$

38. $(x + 3y)^4 = {}_4C_0x^4(3y)^0 + {}_4C_1x^3(3y)^1 + {}_4C_2x^2(3y)^2 + {}_4C_3x^1(3y)^3 + {}_4C_4x^0(3y)^4$
$= x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$

39. $(2x - y)^6 = {}_6C_0(2x)^6(-y)^0 + {}_6C_1(2x)^5(-y)^1 + {}_6C_2(2x)^4(-y)^2 + {}_6C_3(2x)^3(-y)^3 + {}_6C_4(2x)^2(-y)^4 + {}_6C_5(2x)^1(-y)^5 + {}_6C_6(2x)^0(-y)^6$
$= 64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$

40. $(x^3 + 3)^5 = {}_5C_0(x^3)^5(3)^0 + {}_5C_1(x^3)^4(3)^1 + {}_5C_2(x^3)^3(3)^2 + {}_5C_3(x^3)^2(3)^3 + {}_5C_4(x^3)^1(3)^4 + {}_5C_5(x^3)^0(3)^5$
$= x^{15} + 15x^{12} + 90x^9 + 270x^6 + 405x^3 + 243$

41. $(3x^2 - 3)^4 = {}_4C_0(3x^2)^4(-3)^0 + {}_4C_1(3x^2)^3(-3)^1 + {}_4C_2(3x^2)^2(-3)^2 + {}_4C_3(3x^2)^1(-3)^3 + {}_4C_4(3x^2)^0(-3)^4$
$= 81x^8 - 324x^6 + 486x^4 - 324x^2 + 81$

42. $(2x - y^2)^7 = {}_7C_0(2x)^7 + {}_7C_1(2x)^6(-y^2)^1 + {}_7C_2(2x)^5(-y^2)^2 + {}_7C_3(2x)^4(-y^2)^3 + {}_7C_4(2x)^3(-y^2)^4 + {}_7C_5(2x)^2(-y^2)^5 + {}_7C_6(2x)^1(-y^2)^6 + {}_7C_7(-y^2)^7$
$= 128x^7 - 448x^6y^2 + 672x^5y^4 - 560x^4y^6 + 280x^3y^8 - 84x^2y^{10} + 14xy^{12} - y^{14}$

43. $(x^3 + y^2)^3 = {}_3C_0(x^3)^3(y^2)^0 + {}_3C_1(x^3)^2(y^2)^1 + {}_3C_2(x^3)^1(y^2)^2 + {}_3C_3(x^3)^0(y^2)^3$
$= x^9 + 3x^6y^2 + 3x^3y^4 + y^6$

44. $(x - 3)^7$; ${}_7C_2x^5(-3)^2$
$189x^5$
coefficient $= 189$

45. $(x + 2)^8$; ${}_8C_4x^4(2)^4$
$1120x^4$
coefficient $= 1120$

46. $(x^2 + 4)^{10}$; ${}_{10}C_7(x^2)^3(4)^7$
$(120)(16{,}384)x^6$
$1{,}966{,}080x^6$
coefficient $= 1{,}966{,}080$

47. ${}_{10}C_3 = \dfrac{10!}{7! \cdot 3!}$
$= \dfrac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$
$= 120$

48. $_{15}C_4 = \dfrac{15!}{11! \cdot 4!}$

$= \dfrac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1}$

$= 1365$

49. $_7C_1 \cdot {}_{10}C_2 = \dfrac{7!}{6! \cdot 1!} \cdot \dfrac{10!}{8! \cdot 2!}$

$= 7 \cdot \dfrac{10 \cdot 9}{2 \cdot 1}$

$= 315$

50. $_6C_1 \cdot {}_5C_4 = \dfrac{6!}{5! \cdot 1!} \cdot \dfrac{5!}{1! \cdot 4!}$

$= 6 \cdot 5$

$= 30$

51. $_{10}C_0 + {}_{10}C_1 + {}_{10}C_2 + {}_{10}C_3 + {}_{10}C_4 + {}_{10}C_5 + {}_{10}C_6 + {}_{10}C_7$

$= 1 + 10 + 45 + 120 + 210 + 252 + 210 + 120$

$= 968$

52. $_{20}C_{15} + {}_{20}C_{16} + {}_{20}C_{17} + {}_{20}C_{18} + {}_{20}C_{19} + {}_{20}C_{20}$

$= 15{,}504 + 4845 + 1140 + 190 + 20 + 1$

$= 21{,}700$

53. $_{10}C_3 + {}_{10}C_4 + {}_{10}C_5 + {}_{10}C_6 + {}_{10}C_7 + {}_{10}C_8 + {}_{10}C_9 +$

$_{10}C_{10} = 120 + 210 + 252 + 210 + 120 + 45 + 10 +$

$1 = 968$

54. $2^{12} - ({}_{12}C_0 + {}_{12}C_1 + {}_{12}C_2 + {}_{12}C_3)$

$= 4096 - (1 + 12 + 66 + 220)$

$= 3797$

55. combinations

$_8C_3 = \dfrac{8!}{5! \cdot 3!}$

$= 56$

56. permutations

$_{15}P_2 = \dfrac{15!}{13!}$

$= 210$

57. permutations

$_{16}P_4 = \dfrac{16!}{12!}$

$= 43{,}680$

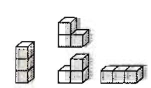
58. combinations

$_{12}C_6 = \dfrac{12!}{6! \cdot 6!}$

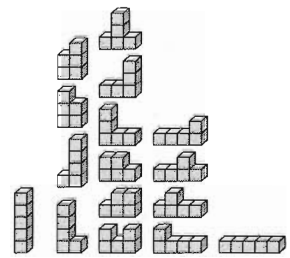
$= 924$

59. $_nP_r = r! \times {}_nC_r$

60.



61.



62. The numbers in row $(n - 1)$ of Pascal's triangle correspond to the number of ways to stack n blocks in r columns. The total number of ways is the sum of the numbers in row $(n - 1)$ of Pascal's triangle.

63. $_9C_0 + {}_9C_1 + {}_9C_2 + {}_9C_3 + {}_9C_4 + {}_9C_5 + {}_9C_6 + {}_9C_7 + {}_9C_8 + {}_9C_9$

$= 1 + 9 + 36 + 84 + 126 + 126 + 84 + 36 + 9 + 1$

$= 512$

64. The sum of the numbers in row n of Pascal's triangle is 2^n. Each number is the sum of the two numbers above it, so each internal number is added twice to the next row. For the ones on the ends, the additional ones on each end of the new row represents the second use of those values (this can be thought of as adding the one to an imaginary zero on the outside of the previous row).

65. $\displaystyle\sum_{n=0}^{20} 2^n = 2{,}097{,}151$

66. $S_n = S_{n-1} + S_{n-2} = 1, 1, 2, 3, 5, 8, \ldots$, which are the Fibonacci numbers.

67. a. $\dfrac{20!}{8! \cdot 7! \cdot 5!} = 99{,}768{,}240$

b. $\dfrac{20!}{8! \times 12!} \times \dfrac{12!}{7! \times 5!} \times \dfrac{5!}{5! \times 0!} = 125970 \times 792 \times 1$

$= 99{,}768{,}240$

c. They are the same. They are two different ways to count the same thing. The additional factorials in the numerators and denominators of part (b) simplify to become the expression in part (a).

68. $_nC_0 = \dfrac{n!}{n! \times 0!} = 1$

69. $_nC_n = \dfrac{n!}{0! \times n!} = 1$

70. $_nC_1 = \dfrac{n!}{(n-1)! \times 1!} = \dfrac{n!}{(n-1)!} = {}_nP_1$

71. $_nC_r = \dfrac{n!}{(n-r)! \times r!}$

$_nC_{n-r} = \dfrac{n!}{(n-(n-r))! \times (n-r)!}$

$= \dfrac{n!}{r! \times (n-r)!}$

Therefore $_nC_r = \dfrac{n!}{(n-r)! \times r!} = {}_nC_{n-r}$

72. $_nC_r \cdot {_rC_m} = \dfrac{n!}{(n-r)! \times r!} \cdot \dfrac{r!}{(r-m)! \times m!}$

$= \dfrac{n!}{(n-r)! \times (r-m)! \times m!}$

$_nC_m \cdot {_{n-m}C_{r-m}}$

$= \dfrac{n!}{(n-m)! \times m!} \times \dfrac{(n-m)!}{((n-m)-(r-m))! \times (r-m)!}$

$= \dfrac{n!}{m! \times (n-r)! \times (r-m)!}$

Therefore $_nC_r \cdot {_rC_m} = {_nC_m} \cdot {_{n-m}C_{r-m}}$

73. $_{n+1}C_r = {_nC_r} + {_nC_{r-1}}$

$_{n+1}C_r = \dfrac{(n+1)!}{((n+1)-r)! \cdot r!} = \dfrac{(n+1)!}{(n+1-r)!r!}$

$_nC_r = \dfrac{n!}{(n-r)!r!}$

$_nC_{r-1} = \dfrac{n!}{(n-(r-1))!(r-1)!} = \dfrac{n!}{(n+1-r)!(r-1)!}$

$_nC_r + {_nC_{r-1}}$

$= \dfrac{n!}{(n-r)!r!} + \dfrac{n!}{(n+1-r)!(r-1)!}$

$= \dfrac{n! \cdot (n+1-r)}{(n-r)! \cdot r! \cdot (n+1-r)}$

$+ \dfrac{n! \cdot r}{(n+1-r)! \cdot (r-1)! \cdot r}$

$= \dfrac{n! \cdot (n+1-r) + n! \cdot r}{(n+1-r)! \cdot r!}$

$= \dfrac{n! \cdot (n+1-r+r)}{(n+1-r)!\, r!}$

$= \dfrac{n! \cdot (n+1)}{(n+1-r)!\, r!}$

$= \dfrac{(n+1)!}{r!(n+1-r)!} = {_{n+1}C_r}$

12.2 Mixed Review (p. 715)

74. $A = \pi(18)^2$

$= 1017.88 \text{ cm}^2$

75. $A = (9.5) \times (11.3)$

$= 107.35 \text{ in.}^2$

76. $A = \frac{1}{2}(13)(9)$

$= 58.5 \text{ ft}^2$

77. $A = \frac{1}{2}(10 + 13)(27)$

$= 310.5 \text{ m}^2$

78. $\dfrac{x^2}{25} - \dfrac{y^2}{144} = 1$

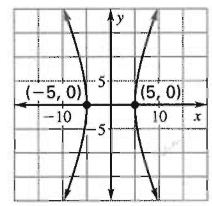
center: (0, 0)

asymptotes: $y = 0 + \dfrac{12}{5}(x - 0)$

$y = \dfrac{12}{5}x$

$y = 0 - \dfrac{12}{5}(x - 0)$

$y = -\dfrac{12}{5}x$



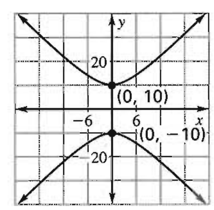
79. $\dfrac{y^2}{100} - \dfrac{x^2}{36} = 1$

center: (0, 0)

asymptotes: $y = 0 + \dfrac{10}{6}(x - 0)$

$y = \dfrac{5}{3}x$

$y = -\dfrac{5}{3}x$



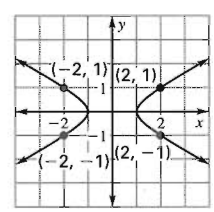
80. $x^2 - \dfrac{49y^2}{16} = 1$

center: (0, 0)

asymptotes: $y = 0 + \dfrac{7}{4}(x - 0)$

$y = \dfrac{7}{4}x$

$y = -\dfrac{7}{4}x$



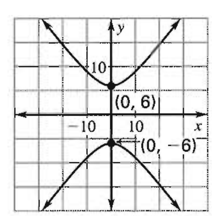
81. $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 9$

$\dfrac{y^2}{36} - \dfrac{x^2}{81} = 1$

center: (0, 0)

asymptotes: $y = \dfrac{2}{3}x$

$y = -\dfrac{2}{3}x$



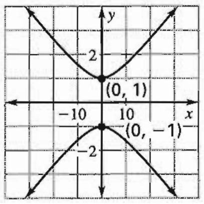
82. $64y^2 - x^2 = 64$

$\frac{y^2}{1} - \frac{x^2}{64} = 1$

center: (0, 0)

asymptotes: $y = \frac{1}{8}x$

$y = -\frac{1}{8}x$



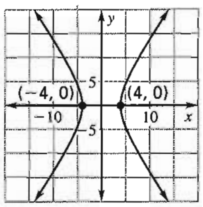
83. $9x^2 - 4y^2 = 144$

$\frac{x^2}{16} - \frac{y^2}{36} = 1$

center: (0, 0)

asymptotes: $y = \frac{3}{2}x$

$y = -\frac{3}{2}x$



84. geometric

3^n

85. arithmetic

$-4 + 7n$

86. geometric

$2(5)^{n-1}$

87. geometric

$(-2)^{n-1}$

88. arithmetic

$10 - 2n$

89. arithmetic

$-15 + 5n$

90. $3x + 4y = 70$

$3x + y = 40$

$y = -3x + 40$

$3x + 4(-3x + 40) = 70$

$3x - 12x + 160 = 70$

$90 = 9x$

$10 = x$

$3(10) + y = 40$

$y = 10$

10 plates, 10 bowls

Quiz 1 (p. 715)

1. $\frac{3!}{2!} = 3$

2. $4! = 24$

3. $5! = 120$

4. $\frac{6!}{2! \times 2!} = 180$

5. $7! = 5040$

6. $8! = 40{,}320$

7. $\frac{9!}{3!} = 60{,}480$

8. $\frac{10!}{2! \times 2!} = 907{,}200$

9. $(x + y)^6 = {}_6C_0x^6y^0 + {}_6C_1x^5y^1 + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5x^1y^5 + {}_6C_6y^6$

$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

10. $(x + 2)^4 = {}_4C_0x^4 + {}_4C_1x^3(2)^1 + {}_4C_2x^2(2)^2 + {}_4C_3x^1(2)^3 + {}_4C_4x^0(2)^4$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

11. $(x - 2y)^5 = {}_5C_0x^5 + {}_5C_1x^4(-2y)^1 + {}_5C_2x^3(-2y)^2 + {}_5C_3x^2(-2y)^3 + {}_5C_4x^1(-2y)^4 + {}_5C_5(-2y)^5$

$= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

12. $(3x - 4y)^3 = {}_3C_0(3x)^3 + {}_3C_1(3x)^2(-4y)^1 + {}_3C_2(3x)^1(-4y)^2 + {}_3C_3(-4y)^3$

$= 27x^3 - 108x^2y + 144xy^2 - 64y^3$

13. $(x^2 + 3y)^4 = {}_4C_0(x^2)^4 + {}_4C_1(x^2)^3(3y)^1 + {}_4C_2(x^2)^2(3y)^2 + {}_4C_3(x^2)^1(3y)^3 + {}_4C_4(3y)^4$

$= x^8 + 12x^6y + 54x^4y^2 + 108x^2y^3 + 81y^4$

14. $(4x^2 - 2)^6 = {}_6C_0(4x^2)^6 + {}_6C_1(4x^2)^5(-2)^1 + {}_6C_2(4x^2)^4(-2)^2 + {}_6C_3(4x^2)^3(-2)^3 + {}_6C_4(4x^2)^2(-2)^4 + {}_6C_5(4x^2)^1(-2)^5 + {}_6C_6(-2)^6$

$= 4096x^{12} - 12{,}288x^{10} + 15{,}360x^8 - 10{,}240x^6 + 3840x^4 - 768x^2 + 64$

15. $(x^3 - y^3)^3 = {}_3C_0(x^3)^3 + {}_3C_1(x^3)^2(-y^3)^1 + {}_3C_2(x^3)^1(-y^3)^2 + {}_3C_3(-y^3)^3$

$= x^9 - 3x^6y^3 + 3x^3y^6 - y^9$

16. $(2x^4 + 5y^2)^5 = {}_5C_0(2x^4)^5 + {}_5C_1(2x^4)^4(5y^2)^1 + {}_5C_2(2x^4)^3(5y^2)^2 + {}_5C_3(2x^4)^2(5y^2)^3 + {}_5C_4(2x^4)^1(5y^2)^4 + {}_5C_5(5y^2)^5$

$= 32x^{20} + 400x^{16}y^2 + 2000x^{12}y^4 + 5000x^8y^6 + 6250x^4y^8 + 3125y^{10}$

17. $(x + 3)^5$; ${}_5C_2x^3(3)^2$

$10(9)x^3$

coefficient = 90

18. $(5 - y^2)^3 = {}_3C_2(5)(-y^2)^2$

$(3)(5)y^4$

coefficient = 15

19. $6 \times 12 \times 6 \times 8 = 3456$

20. ${}_{12}C_3 \cdot {}_6C_1 = \frac{12!}{9! \times 3!} \times \frac{6!}{5! \times 1!}$

$= 220 \times 6$

$= 1320$

Lesson 12.3

12.3 Guided Practice (p. 719)

1. geometric
2. *B*; The event with the higher probability is more likely to occur.
3. A theoretical probability is based on the number of outcomes of the event and the total number of possible outcomes. An experimental probability is determined through an experiment, survey, or historical data about an event. The theoretical probability of rolling a 5 using a 6-sided die is $\frac{1}{6}$. If you actually rolled a 6-sided die 100 times, the experimental probability of rolling a 5 would be the number of fives you rolled divided by 100.
4. $\frac{2}{6} = \frac{1}{3}$
5. $\frac{1}{6}$
6. $\frac{4}{6} = \frac{2}{3}$
7. $\frac{5}{6}$
8. $P(\text{hitting shaded region}) = \frac{9\pi}{36} = \frac{\pi}{4} \approx 0.785$
9. $P(\text{hitting shaded region}) = \frac{(3\sqrt{2})^2}{3^2(\pi)} = \frac{18}{9\pi} \approx 0.637$
10. $P(\text{hitting shaded region}) = \frac{\frac{1}{2}(6)(6)}{6^2} = \frac{\frac{1}{2}(6)(6)}{6^2} = \frac{1}{2}$
11. **a.** $P(\text{under } 25) = \frac{94{,}507}{267{,}637} \approx 0.353$

    b. $P(\text{at least } 45) = \frac{89{,}522}{267{,}637} \approx 0.334$

12.3 Practice and Applications (pp. 719–722)

12. $\frac{10}{20} = 0.5$
13. $\frac{6}{20} = 0.3$
14. $\frac{4}{20} = 0.2$
15. $\frac{8}{20} = 0.4$
16. $\frac{6}{20} = 0.3$
17. $\frac{12}{20} = 0.6$
18. $\frac{1}{52} \approx 0.0192$
19. $\frac{4}{52} \approx 0.0769$
20. $\frac{13}{52} = 0.25$
21. $\frac{26}{52} = 0.5$
22. $\frac{48}{52} \approx 0.9231$
23. $\frac{12}{52} \approx 0.23$
24. experimental probability: $\frac{26}{120} \approx 0.217$

    theoretical probability: $\frac{1}{6} \approx 0.17$
25. experimental probability: $\frac{37}{120} \approx 0.308$

    theoretical probability: $\frac{2}{6} \approx 0.3$
26. experimental probability: $\frac{59}{120} \approx 0.4917$

    theoretical probability: $\frac{3}{6} = 0.5$
27. experimental probability: $\frac{61}{120} \approx 0.5083$

    theoretical probability: $\frac{3}{6} = 0.5$
28. experimental probability: $\frac{87}{120} = 0.725$

    theoretical probability: $\frac{4}{6} \approx 0.67$
29. experimental probability: $\frac{105}{120} = 0.875$

    theoretical probability: $\frac{5}{6} \approx 0.83$
30. $P(\text{red center}) = \frac{2^2\pi}{24^2} \approx 0.0218$
31. $P(\text{white border}) = \frac{24^2 - (10^2 \cdot \pi)}{24^2} \approx 0.455$
32. $P(\text{red center or white border}) = \frac{2^2 \cdot \pi + (24^2 - 10^2 \cdot \pi)}{24^2} \approx 0.477$
33. $P(\text{four rings or red center}) = \frac{10^2 \cdot \pi}{24^2} = 0.545$
34. $P(\text{the yellow or green ring}) = \frac{8^2 \cdot \pi}{24^2} - \frac{4^2 \cdot \pi}{24^2} = 0.262$
35. $\frac{1}{26} \approx 0.0385$
36. $\frac{4}{26} \approx 0.154$
37. $\frac{{}_{44}C_3 \cdot {}_{54}C_4}{{}_{100}C_7} = \frac{13244 \cdot 316251}{1.60075 \times 10^{10}} = 0.2617$
38. $\frac{9}{100} \cdot \frac{2}{99} \cdot \frac{2}{98} \cdot \frac{4}{97} \cdot \frac{12}{96} \cdot \frac{2}{95} \cdot \frac{3}{94} = 1.285 \times 10^{-10}$;
39. $\frac{1}{{}_{51}C_6} = 5.6 \times 10^{-8}$
40. $\frac{1}{10 \cdot 9 \cdot 8} = 0.001$

41. a. $\frac{672}{1211} \approx 0.555$

b. $\frac{46}{1211} \approx 0.038$

42. a. $\frac{4}{17} \approx 0.235$

b. $\frac{11}{17} \approx 0.647$

43. a. $\frac{6,069,589}{115,070,274} \approx 0.0527$

b. $\frac{99,830,336}{115,070,274} \approx 0.868$

44. $\frac{3-1}{5.5-0} = \frac{2}{5.5} \approx 0.364$

45. $\frac{0.25}{4} = 0.0625$

46. $\frac{3.6^2\pi}{18^2\pi} = 0.04$

47. $\frac{\pi}{26(50)} \approx 0.00242$

48. C

49. $\frac{25\pi - \frac{1}{2}(10)(5)}{25\pi} = 0.682$

E

50. 4 of the 6 graphs intersect the x-axis. $\frac{4}{6} = \frac{2}{3} = 0.667$

12.3 Mixed Review (p. 722)

51. $18 - 35 = -17$

52. $-18 - 0 = -18$

53. $3 - (-16) = 19$

54. $5 - 2(1) + 3(-7), -18$

55. $0 - 1(-2) + 5(-11), -53$

56. $-1(34) - (-2)(32) + 2(10), 50$

57. $\frac{6xy^2}{5x^3y} \cdot \frac{10y^4}{9xy} = \frac{4y^4}{3x^3}$

58. $\frac{(x+1)(x+2)}{(x-3)(x+2)} \cdot \frac{x(x-3)}{(x-2)(x+1)} = \frac{x}{x-2}$

59. $\frac{(5x-4)(5x+4)}{(5x-4)} \cdot \frac{(x-7)(x+3)}{x(5x+4)(x-7)} = \frac{x+3}{x}$

60. $\frac{4x(x-3)}{-(x-3)(x^2+3x+9)} \cdot (x^2+3x+9) = -4x$

61. 3, 10, 17, 24, 31

62. $-1, -3, -9, -27, -81$

63. 2; 8; 512; 134,217,728; 2.418×10^{24}

64. 1, 1, 2, 3, 5

65. $-2, 0, 2, 2, 0$

66. $1, -2, -2, 4, -8$

67. $2^{20} - ({}_{20}C_0 + {}_{20}C_1 + {}_{20}C_2 + {}_{20}C_3 + {}_{20}C_4)$

$= 1048576 - (1 + 20 + 190 + 1140 + 4845)$

$= 1,042,380$

12.3 Activity (p. 723)

1. Experimental results may vary.

Theoretical probabilities: $\frac{1}{6}$ for each number

Answers may vary but experimental probabilities should be close to theoretical probabilities.

2. Experimental results may vary.

Theoretical probabilities: $\frac{1}{13}$ for each card.

Answers may vary but experimental probabilites should be close to theoretical probabilities.

3. Experimental results may vary.

As the number of trials increases, the experimental results should more closely resemble the theoretical results.

Lesson 12.4

12.4 Guided Practice (p. 727)

1. Two events are mutually exclusive if there are no outcomes shared by both of them.

2. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. When A and B are mutually exclusive, $P(A \text{ and } B) = 0$ so the formula becomes $P(A \text{ or } B) = P(A) + P(B)$.

3. Yes, A' is defined as all outcomes not in A, so there is no intersection between events in A and events not in A.

4. $P(A \text{ or } B) = 0.2 + 0.3$

$= 0.5$

5. $P(A \text{ or } B) = 0.5 + 0.5$

$= 1$

6. $P(A \text{ or } B) = \frac{3}{8} + \frac{1}{8}$

$= \frac{1}{2}$

7. $P(A \text{ or } B) = \frac{1}{3} + \frac{1}{4}$

$= \frac{4}{12} + \frac{3}{12}$

$= \frac{7}{12}$

8. $P(A \text{ or } B) = 0.5 + 0.4 - 0.3$

$= 0.6$

9. $P(A \text{ or } B) = \frac{2}{5} + \frac{3}{5} - \frac{1}{5}$

$= \frac{4}{5}$

10. $0.8 = 0.7 + 0.2 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.1$

11. $\frac{9}{16} = \frac{5}{16} + \frac{7}{16} - P(A \text{ and } B)$

$P(A \text{ and } B) = \frac{3}{16}$

12. $P(A') = 1 - 0.5$

$= 0.5$

13. $P(A') = 1 - 0.75$

$= 0.25$

14. $P(A') = 1 - \frac{1}{3}$

$= \frac{2}{3}$

15. $P(A') = 1 - \frac{4}{7}$

$= \frac{3}{7}$

12.4 Practice and Applications (pp. 727–729)

16. $0.5 = 0.4 + 0.35 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.25$

no

17. $P(A \text{ or } B) = 0.6 + 0.2 - 0.1$

$= 0.7$

no

18. $0.70 = 0.25 + P(B) - 0$

$P(B) = 0.45$

yes

19. $\frac{14}{17} = \frac{13}{17} + P(B) - \frac{6}{17}$

$\frac{7}{17} = P(B)$

no

20. $\frac{7}{12} = \frac{1}{3} + \frac{1}{4} - P(A \text{ and } B)$

$P(A \textit{ and } B) = 0$

yes

21. $P(A \text{ or } B) = \frac{3}{4} + \frac{1}{3} - \frac{1}{4}$

$= \frac{9}{12} + \frac{4}{12} - \frac{3}{12}$

$= \frac{10}{12}$

$= \frac{5}{6}$

no

22. $P(A \text{ or } B) = 5 + 29 - 0$

$= 34\%$

yes

23. $10 = 30 + P(B) - 50$

$30\% = P(B)$

no

24. $32 = 16 + 24 - P(A \text{ and } B)$

$P(A \text{ and } B) = 8\%$

no

25. $P(A') = 1 - 0.34$

$= 0.66$

26. $P(A') = 1 - 0$

$= 1$

27. $P(A') = 1 - \frac{3}{4}$

$= \frac{1}{4}$

28. $P(A') = 1 - 1$

$= 0$

29. $P = \frac{1}{52}$

30. $P = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

31. $P = \frac{13}{52} + \frac{13}{52}$

$= \frac{26}{52}$

$= \frac{1}{2}$

32. $P = \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52}$

$= \frac{2}{13}$

33. $P = 0$

34. $P = \frac{4}{52} + \frac{12}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

35. $P(\text{sum is not } 3) = 1 - \frac{2}{36}$

$= \frac{34}{36}$

$= \frac{17}{18} \approx 0.944$

36. $P = 1 - \frac{6}{36}$

$= \frac{30}{36}$

$= \frac{5}{6} \approx 0.833$

37. $P = 1 - \frac{8}{36}$

$= \frac{28}{36}$

$= \frac{7}{9} \approx 0.778$

38. $P = 1 - \frac{3}{36}$

$= \frac{33}{36}$

$= \frac{11}{12} \approx 0.917$

39. $P = 1 - \frac{1}{36}$

$= \frac{35}{36} \approx 0.972$

40. $P = 1 - \frac{14}{36}$

$= \frac{22}{36}$

$= \frac{11}{18} \approx 0.611$

41. *Sample answer:* Not 3: 0.933; ≥ 5: 0.825; not 3 or 7: 0.783; ≤ 10: 0.942; > 2: 0.983; < 8 or > 11: 0.600; the experimental results are very similar to the theoretical results.

42. $P = 0.33 + 0.50$

$= 0.83$

43. $P = 0.40 + 0.35$

$= 0.75$

44. $P = \frac{42}{79} + \frac{44}{79} - \frac{28}{79}$

$= \frac{58}{79} \approx 0.734$

45. $P = \frac{48}{162} + \frac{78}{162} - \frac{14}{162}$

$= \frac{112}{162} \approx 0.691$

46. $0.85 = 0.45 + 0.50 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.10$

47. $0.50 = 0.20 + 0.40 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.10$

48. $P(\text{at least 2 share a dorm}) = 1 - \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{8^5}$

$= 0.795$

49. $P(\text{at least 2 share a dorm}) = 1 - \frac{14 \cdot 13 \cdot 12 \cdot 11}{14^4}$

$= 0.375$

50. A; A' is all outcomes not in A, so A has all outcomes not in A'.

51. a. $P = 1 - \frac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361}{365^5}$

$= 0.0271$

b. $P = 1 - \frac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360 \cdot 359 \cdot 358 \cdot 357 \cdot 356}{365^{10}}$

$= 0.117$

c. $1 - {}_{365}P_n\left(\frac{1}{365^n}\right)$

d. 23 (Probability ≈ 0.507)

52. $\frac{4}{1} = \frac{x}{5}$

$x = 20$

total marbles $= 5 + 20 = 25$

53. $\frac{4}{7}; \frac{7}{4}$

54. $P(E) = \frac{\text{odds in favor}}{1 + \text{odds in favor}}$

55. Odds in favor of Event $E = \frac{P(E)}{P(E')} = \frac{P(E)}{1 - P(E)}$

12.4 Mixed Review (p. 729)

56. $7^{2x} = 49^{16}$

$2x \log 7 = 16 \log 49$

$x = 8\frac{\log 49}{\log 7}$

$x = 16$

57. $9^x = 3^{x+1}$

$x \log 9 = (x + 1) \log 3$

$x = (x + 1)\frac{\log 3}{\log 9}$

$x = \frac{\log 3}{\log 9}$

$\frac{x}{x + 1} = 0.5$

$x = 0.5x + 0.5$

$0.5x = 0.5$

$x = 1$

58. $2^{4x+8} = 32^{19}$

$(4x + 8) \log 2 = 19 \log 32$

$4x + 8 = 19\frac{\log 32}{\log 2}$

$4x = 19(5) - 8$

$x = 21.75$

59. $5^x = 21$

$x \log 5 = \log 21$

$x = \frac{\log 21}{\log 5}$

$x = 1.892$

60. $10^{3x-1} - 13 = 8$

$10^{3x-1} = 21$

$(3x - 1) \log 10 = \log 21$

$3x - 1 = \frac{\log 21}{\log 10}$

$x = 0.774$

61. $72 = 91e^{-0.023x} + 50$

$\frac{22}{91} = e^{-0.023x}$

$\ln\left(\frac{22}{91}\right) = -0.023x$

$x = 61.73$

62. $r = \sqrt{(0 - 0)^2 + (7 - 0)^2}$

$= 7$

$x^2 + y^2 = 49$

63. $r = \sqrt{(3 - 0)^2 + (4 - 0)^2}$

$= 5$

$x^2 + y^2 = 25$

64. $r = \sqrt{(-1 - 0)^2 + (6 - 0)^2}$

$= \sqrt{37}$

$x^2 + y^2 = 37$

65. $r = \sqrt{(8 - 0)^2 + (-2 - 0)^2}$

$= \sqrt{68}$

$x^2 + y^2 = 68$

66. $r = \sqrt{(4 - 0)^2 + (4 - 0)^2}$

$= \sqrt{32}$

$x^2 + y^2 = 32$

67. $r = \sqrt{(3 - 0)^2 + (10 - 0)^2}$

$= \sqrt{109}$

$x^2 + y^2 = 109$

68. $r = \sqrt{(-4 - 0)^2 + (-2 - 0)^2}$

$= \sqrt{20}$

$x^2 + y^2 = 20$

69. $r = \sqrt{(16 - 0)^2 + (0)}$

$= 16$

$x^2 + y^2 = 256$

70. a. $26^3 \cdot 10^4 = 175{,}760{,}000$

b. $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 78{,}624{,}000$

71. a. $26^4 \cdot 10^3 = 456{,}976{,}000$

b. $26 \cdot 25 \cdot 24 \cdot 23 \cdot 10 \cdot 9 \cdot 8 = 258{,}336{,}000$