CHAPTER 4

Think & Discuss *(p. 197)*

1.
$$\begin{array}{r} 216.46 \\ -\ 204.96 \\ \hline \end{array}$$
$11.50 billion increase

$$\begin{array}{r} 168.36 \\ -\ 167.14 \\ \hline \end{array}$$
$1.22 billion decrease

2.

Change in Sales from 1997 to 1998	
Age	Sales (billions)
10-14	16.09
15-19	11.50
20-24	−1.22
25-29	13.44
30-34	21.98
35-39	31.00
40-44	6.35
45+	46.67

Study Guide *(p. 198)*

1. $4 + (-5) = -1$ **2.** $-10 - 3 = -13$

3. $(-1)6 - 4(2) = -6 - 8 = -14$

4. $12(5) + 2(-10) = 60 - 20 = 40$

5. commutative property of multiplication

6. commutative property of addition

7. distributive property of subtraction

8. $2x = 30$

$x = 15$

$x + 4y = 27$

$15 + 4y = 27$

$4y = 12$

$y = 3$

$(15, 3)$

9. $x = 7 + y$

$3(7 + y) - 7y = 61$

$21 + 3y - 7y = 61$

$-4y = 40$

$y = -10$

$x = 7 - 10 = -3$

$(-3, -10)$

10.
$$\begin{array}{r} -x + 2y = -24 \\ x + 3y = 20 \\ \hline 0 + 5y = -4 \end{array}$$

$y = -\frac{4}{5}$

$x - 2\left(-\frac{4}{5}\right) = 24$

$x = \frac{112}{5}$

$\left(\frac{112}{5}, -\frac{4}{5}\right)$

11.
$$\begin{array}{r} 9x + 3y = -24 \\ 8x - 3y = -10 \\ \hline 17x = -34 \end{array}$$

$x = -2$

$3(-2) + y = -8$

$-6 + y = -8$

$y = -2$

$(-2, -2)$

Lesson 4.1

4.1 Guided Practice (p. 203)

1. A matrix is a rectangular arrangement of numbers in rows and columns.

A row matrix is a matrix with only 1 row: $[3\quad -2\quad 0\quad 4]$

A column matrix is a matrix with only 1 column: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

A square matrix is a matrix with the same number of rows as columns: $\begin{bmatrix} 1 & 2 & 3 \\ 8 & 0 & 1 \\ 2 & -1 & 2 \end{bmatrix}$

2. Yes; the dimensions of both matrices are 3×2 and all the corresponding entries are equal.

3. The matrices must have the same dimensions.

4. $-2\left(\begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}\right) = -2\begin{bmatrix} -3 & 3 \\ 6 & -5 \\ -6 & 11 \end{bmatrix}$

$= \begin{bmatrix} 6 & -6 \\ -12 & 10 \\ 12 & -22 \end{bmatrix}$;

no; part (b) in Example 3 is $-2A + B$, which is not the same as $-2(A + B)$

5.

This Year

	Comprehensive	HMO Standard	HMO Plus
Individual	$694.32	$451.80	$489.48
Family	$1725.36	$1187.76	$1248.12

Next Year

	Comprehensive	HMO Standard	HMO Plus
Individual	$683.91	$463.10	$499.27
Family	$1699.48	$1217.45	$1273.08

6. $\begin{bmatrix} 20 \\ -22 \\ 9 \end{bmatrix} - \begin{bmatrix} -11 \\ -10 \\ -6 \end{bmatrix} = \begin{bmatrix} (20 + 11) \\ (-22 + 10) \\ (9 + 6) \end{bmatrix} = \begin{bmatrix} 31 \\ -12 \\ 15 \end{bmatrix}$

7. $\begin{bmatrix} -6 & -7 & 4 \\ -4 & 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -5 & 8 \\ 9 & 12 & -9 \end{bmatrix}$

$= \begin{bmatrix} (-6 - 1) & (-7 - 5) & (4 + 8) \\ (-4 + 9) & (0 + 12) & (-1 - 9) \end{bmatrix}$

$= \begin{bmatrix} -7 & -12 & 12 \\ 5 & 12 & -10 \end{bmatrix}$

8. $-4\begin{bmatrix} 2 & 0 \\ -4 & -5 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 16 & 20 \end{bmatrix}$

9. $6\begin{bmatrix}-5 & -1\\ 2 & 0\end{bmatrix} - 5\begin{bmatrix}-1 & 0\\ 4 & -3\end{bmatrix}$

$= \begin{bmatrix}-30 & -6\\ 12 & 0\end{bmatrix} - \begin{bmatrix}-5 & 0\\ 20 & -15\end{bmatrix} = \begin{bmatrix}(-30+5) & (-6-0)\\ (12-20) & (0+15)\end{bmatrix}$

$= \begin{bmatrix}-25 & -6\\ -8 & 15\end{bmatrix}$

10. $1.04\begin{bmatrix}\$683.91 & \$1699.48\\ \$463.10 & \$1217.45\\ \$499.27 & \$1273.08\end{bmatrix} = \begin{bmatrix}\$711.27 & \$1767.46\\ \$481.62 & \$1266.15\\ \$519.24 & \$1324.00\end{bmatrix}$

Monthly payments:

$\frac{1}{12}\begin{bmatrix}\$711.27 & \$1767.46\\ \$481.62 & \$1266.15\\ \$519.24 & \$1324.00\end{bmatrix} = \begin{bmatrix}\$59.27 & \$147.29\\ \$40.14 & \$105.51\\ \$43.27 & \$110.33\end{bmatrix}$

4.1 Practice and Applications (pp. 203–206)

11. not equal **12.** equal **13.** not equal **14.** equal

15. $\begin{bmatrix}(1+3) & (-4+5)\\ (-7-5) & (2+2)\end{bmatrix} = \begin{bmatrix}4 & 1\\ -12 & 4\end{bmatrix}$

16. Not possible; the two matrices do not have the same dimensions.

17. $\begin{bmatrix}(-8+4) & (-2-5)\\ (6-1) & (-6+1)\end{bmatrix} = \begin{bmatrix}-4 & -7\\ 5 & -5\end{bmatrix}$

18. $\begin{bmatrix}(-3+2) & (5-7)\\ (0-4) & (-1+9)\end{bmatrix} = \begin{bmatrix}-1 & -2\\ -4 & 8\end{bmatrix}$

19. $\begin{bmatrix}(1.2+4.1) & (3.5+8.7)\\ (0.2+2.6) & (5.1+5.3)\end{bmatrix} = \begin{bmatrix}5.3 & 12.2\\ 2.8 & 10.4\end{bmatrix}$

20. $\begin{bmatrix}(7-3) & (-1+6) & (4+3)\\ (11+10) & (-9+1) & (2-5)\end{bmatrix} = \begin{bmatrix}4 & 5 & 7\\ 21 & -8 & -3\end{bmatrix}$

21. Not possible; the two matrices do not have the same dimensions.

22. $\begin{bmatrix}(\frac{1}{2}-2) & (\frac{1}{4}-\frac{3}{4})\\ (3-\frac{1}{2}) & (8-5)\end{bmatrix} = \begin{bmatrix}-\frac{3}{2} & -\frac{1}{2}\\ \frac{5}{2} & 3\end{bmatrix}$

23. $\begin{bmatrix}4 & 12 & -28\\ 16 & 0 & -24\end{bmatrix}$ **24.** $\begin{bmatrix}-10 & -30\\ 15 & 5\end{bmatrix}$

25. $\begin{bmatrix}4 & 12 & 36\\ -20 & 20 & 60\\ -12 & -20 & -44\end{bmatrix}$ **26.** $\begin{bmatrix}0 & 0\\ 18 & 18\\ -3 & -4\end{bmatrix}$

27. $\begin{bmatrix}-1 & -1 & 2\\ \frac{1}{8} & \frac{3}{11} & -5\end{bmatrix}$ **28.** $\begin{bmatrix}-21.5 & 8.5\\ 3 & -12.75\\ -12 & 11\\ 25 & -20\end{bmatrix}$

29. $\begin{bmatrix}12 & -8\\ 0 & 5\\ 0 & 3\end{bmatrix} + \begin{bmatrix}-4 & 0\\ 12 & -8\\ -16 & 20\end{bmatrix} = \begin{bmatrix}8 & -8\\ 12 & -3\\ -16 & 23\end{bmatrix}$

30.

$\begin{bmatrix}-12 & -20 & 4\\ 8 & -14 & -8\end{bmatrix} - \begin{bmatrix}-1 & 5 & 13\\ -3 & -6 & 19\end{bmatrix} = \begin{bmatrix}-11 & \dots\\ 11 & \dots\end{bmatrix}$

31. $\begin{bmatrix}14 & -14\\ -2 & 6\end{bmatrix} + \begin{bmatrix}8 & -16\\ -20 & -24\end{bmatrix} = \begin{bmatrix}22 & -30\\ -22 & -18\end{bmatrix}$

32. $\begin{bmatrix}-21 & 3 & 0\\ 24 & -18 & -6\end{bmatrix} - \begin{bmatrix}8 & -2 & -14\\ -6 & -10 & 10\end{bmatrix}$

$= \begin{bmatrix}-29 & 5 & 14\\ 30 & -8 & -16\end{bmatrix}$

33. $-2x = 6$

$x = -3$

$-8 = y$

34. $3x - 4 = -16$

$3x = -12$

$x = -4$

35. $\begin{bmatrix}-6x & 8x\\ -22x & 10x\end{bmatrix} = \begin{bmatrix}12 & -16\\ y & -20\end{bmatrix}$

$-6x = \dots$

$x = -\dots$

$-22\dots =$

$-22(-\dots =$

$\dots =$

36. $-3 + x = -8$ $8 - 7 = y$

$x = -5$ $y = 1$

37.–41. Matrices can also be written with the rows . . . columns switched.

37.

	Before All-Stars		After All-Stars	
	Wins	Losses	Wins	Losses
Braves	59	29	47	2…
Mariners	37	51	39	34
Cubs	48	39	42	3…

38.

	Totals	
	Wins	Losses
Braves	106	56
Mariners	76	85
Cubs	90	73

39.

	1996		
	Number shipped (millions)	Value (millions)	Number shipped (millions)
CDs	20,779	$268,441	26,27…
Cassettes	15,299	$122,329	17,79…
Videos	45	$916	70

40.

Total	
Number shipped (millions)	Value (millions)
47,056	$613,138
33,098	$266,974
115	$2176

41. Changes from . . .

Number shipped (millions)	Value (m…
5498	$76,…
2500	$22,…
25	$…

42. Total Cost

$$\begin{array}{l} \\ \text{Public 2 yr college} \\ \text{Public 4 yr college} \\ \text{Private 2 yr college} \\ \text{Private 4 yr college} \end{array} \begin{array}{c} \begin{array}{ccc} 1995 & 1996 & 1997 \end{array} \\ \begin{bmatrix} 4{,}136 & 4{,}217 & 4{,}411 \\ 6{,}671 & 7{,}014 & 7{,}331 \\ 11{,}170 & 11{,}563 & 11{,}889 \\ 16{,}602 & 17{,}611 & 18{,}475 \end{bmatrix} \end{array}$$

43. Total Scores

$$\begin{array}{l} \\ 1993 \\ 1994 \\ 1995 \\ 1996 \end{array} \begin{array}{c} \begin{array}{cc} \text{Sophomores} & \text{Juniors} \end{array} \\ \begin{bmatrix} 146.8 & 148.4 \\ 146.1 & 147.8 \\ 146.8 & 148.4 \\ 146.2 & 148.1 \end{bmatrix} \end{array}; 2V + M$$

44. $\dfrac{148.4 + 147.8 + 148.4 + 148.1}{4} = 148.175$

45. 1991

$$\begin{array}{r} \\ \text{Northeast} \\ \text{Midwest} \\ \text{South} \\ \text{Mountain} \\ \text{Pacific} \end{array} \begin{array}{c} \begin{array}{ccc} 0\text{–}17 & 18\text{–}65 & \text{over } 65 \end{array} \\ \begin{bmatrix} 4.8 & 12.6 & 2.8 \\ 6.3 & 14.5 & 3.1 \\ 8.9 & 21.2 & 4.3 \\ 1.6 & 3.4 & 0.6 \\ 4.2 & 9.9 & 1.7 \end{bmatrix} \end{array}$$

2010

$$\begin{array}{r} \\ \text{Northeast} \\ \text{Midwest} \\ \text{South} \\ \text{Mountain} \\ \text{Pacific} \end{array} \begin{array}{c} \begin{array}{ccc} 0\text{–}17 & 18\text{–}65 & \text{over } 65 \end{array} \\ \begin{bmatrix} 4.2 & 11.4 & 2.5 \\ 5.3 & 13.8 & 3.0 \\ 8.5 & 22.6 & 5.0 \\ 1.7 & 4.2 & 0.9 \\ 4.6 & 10.5 & 1.9 \end{bmatrix} \end{array}$$

46. Change (percentage)

$$\begin{bmatrix} -0.6 & -1.2 & -0.3 \\ -1.0 & -0.7 & -0.1 \\ -0.4 & 1.4 & 0.7 \\ 0.1 & 0.8 & 0.3 \\ 0.4 & 0.6 & 0.2 \end{bmatrix}$$

47. South: 18–65, over 65

Mountain: 0–17, 18–65, over 65

Pacific: 0–17, 18–65, over 65

48. a. $B - A = \begin{bmatrix} -46{,}000 & 12.17 \\ 111{,}000 & 15.42 \\ 2{,}000 & -4.06 \\ -20{,}000 & -4.38 \end{bmatrix}$

111,000 more; $4.06 less

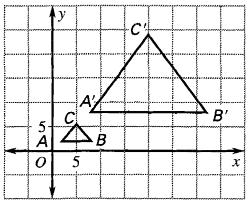
b.

$$\begin{array}{l} \\ \text{Art} \\ \text{Law} \\ \text{Music} \\ \text{Travel} \end{array} \begin{array}{c} \begin{array}{cc} \text{Volumes} & \text{Average Price} \end{array} \\ \begin{bmatrix} 2{,}186{,}000 & 94.63 \\ 1{,}543{,}000 & 161.60 \\ 504{,}000 & 82.48 \\ 378{,}000 & 72.22 \end{bmatrix} \end{array};$$

yes; no; For each subject area, adding the volumes sold in 1995 and 1996 gives the total of volumes sold over the two year period, but adding the price does not give the total average price over the two year period.

c. The number of art volumes sold has decreased and the price has increased over the two year period. The number of law volumes has increased and so has their average price. The number of music volumes sold has increased slightly, but the price has decreased. The number of travel volumes sold has decreased and so has the average price per volume. There is no direct relationship between the number of volumes sold and the average price per volume.

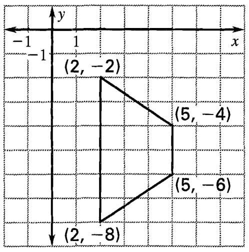
49. $4 \cdot \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{bmatrix} 2 & 2 \\ 8 & 2 \\ 5 & 6 \end{bmatrix} \end{array} = \begin{bmatrix} 8 & 8 \\ 32 & 8 \\ 20 & 24 \end{bmatrix};$



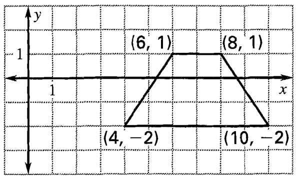
The "new" triangle has the vertices $A' = (8.8)$, $B' = (32, 8)$, and $C' = (20, 24)$; the two triangles are similar with each side of $\Delta A'B'C'$ being parallel to and 4 times as long as the corresponding side of ΔABC.

4.1 Mixed Review (p. 206)

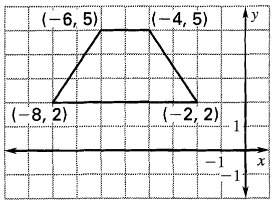
50.



51.



52.



53. 20 **54.** −16 **55.** 7

56. $-\frac{7}{2}$ **57.** $\frac{5}{14}$ **58.** $3.2(2.4 + 8.1) = 3.2(10.5) = 33.6$

Chapter 4 *continued*

59. $0 + 6 \le -3$; no
$-5 + 2 \le -3$; yes

60. $-25 - 0 > 2$; no
$25 - 23 > 2$; no

61. $8 - 3 < 5$; no
$-24 + 27 < 5$; yes

62. $42 - 30 > 4$; yes
$-21 - 0 > 4$; no

63. *Sample answer:* (1, 2) **64.** *Sample answer:* (0, −3)

65. *Sample answer:* (5, 5)

Technology Activity (p. 207)

1. $\begin{bmatrix} 6.6 & -6.1 \\ 15.33 & 1.72 \end{bmatrix}$ **2.** $\begin{bmatrix} -94 & 59 \\ 24 & 268 \\ -589 & -153 \end{bmatrix}$

3. $\begin{bmatrix} 6.4666 & 1.6688 \\ 23.0503 & 7.301 \end{bmatrix}$ **4.** $\begin{bmatrix} 0.23417 & 6.34636 \\ 16.0816 & -2.3397 \end{bmatrix}$

5. Change

	R	C	J	E
CDs	−8	−1	0	−1
Tapes	−3	−2	−1	0

; none; Rock CDs, Country CDs, Easy Listening CDs, Rock tapes, Country tapes, Jazz tapes

Section 4.2

4.2 Guided Practice (p. 211)

1. columns; rows

2. AB is defined because the number of columns in $A(1)$ is equal to the number of rows in $B(1)$. BA is not defined because the number of columns in $B(2)$ is not equal to the number of rows in $A(6)$.

3. False; matrix multiplication is not commutative. If you multiply out both sides of the equation you get

$$\begin{bmatrix} 10 & 3 \\ -6 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ -3 & 5 \end{bmatrix}$$

4. defined; 3 × 3 **5.** defined; 3 × 3 **6.** not defined

7. $\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1(2) + 0(1) & 1(0) + 0(3) \\ (-2)(2) + (-1)(1) & (-2)(0) + (-1)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ -5 & -3 \end{bmatrix}$$

8. $[-20]$

9. $\begin{bmatrix} -3 & 3 \\ 3 & -2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} (-3)(1) + 3(-2) & (-3)(0) + 3(-1) \\ 3(1) + (-2)(-2) & 3(0) + (-2)(-1) \\ 0(1) + (-1)(-2) & 0(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -3 \\ 7 & 2 \\ 2 & 1 \end{bmatrix}$$

10.

	Bats	Balls	Uniforms
Women	16	42	16
Men	14	43	15

Bats	21
Balls	4
Uniforms	30

$$\begin{bmatrix} 16 & 42 & 16 \\ 14 & 43 & 15 \end{bmatrix}\begin{bmatrix} 21 \\ 4 \\ 30 \end{bmatrix} = \begin{bmatrix} 16(21) + 42(4) + 16(30) \\ 14(21) + 43(4) + 15(30) \end{bmatrix}$$

$$= \begin{bmatrix} 336 + 168 + 480 \\ 294 + 172 + 450 \end{bmatrix}$$

$$= \begin{bmatrix} \$984 \\ \$916 \end{bmatrix} \begin{matrix} \text{Women's team} \\ \text{Men's team} \end{matrix}$$

4.2 Practice and Applications (pp. 211–213)

11. defined; 1 × 2 **12.** defined; 2 × 3 **13.** not defined

14. defined; 5 × 4 **15.** defined; 3 × 1 **16.** not defined

17. $\left[-\frac{1}{6}(12) + \frac{1}{2}(0) - \frac{1}{3}(-12)\right] = [-2 + 4] = [2]$

18. Not defined; the number of columns in the first matrix ... and this is not equal to the number of rows in the second matrix.

19. $\begin{bmatrix} 1(4) - 4(0) & 1(-1) - 4(-3) \\ 3(4) - 2(0) & 3(-1) - 2(-3) \end{bmatrix} = \begin{bmatrix} 4 - 0 & \cdots \\ 12 - 0 & \cdots \end{bmatrix}$

$$= \begin{bmatrix} 4 & 11 \\ 12 & 3 \end{bmatrix}$$

20. $\begin{bmatrix} -6(-1) - 2(-5) & -6(4) - 2(3) \\ 0(-1) + 3(-5) & 0(4) + 3(3) \end{bmatrix}$

$$= \begin{bmatrix} 6 + 10 & -24 - 6 \\ -15 & 9 \end{bmatrix} = \begin{bmatrix} 16 & -30 \\ -15 & 9 \end{bmatrix}$$

21. Not defined; the number of columns in the first matrix is not equal to the number of rows in the second matrix.

22. $\begin{bmatrix} 6(1) + 0(1.5) & 6(0) + 0(-0.5) \\ -0.2(1) + 0.2(1.5) & -0.2(0) + 0.2(-0.5) \\ 2.9(1) + 0.3(1.5) & 2.9(0) + 0.3(-0.5) \end{bmatrix}$

$$= \begin{bmatrix} 6 & 0 \\ -0.2 + 0.3 & -0.1 \\ 2.9 + 0.45 & -0.15 \end{bmatrix} = \begin{bmatrix} 6 & \cdots \\ 0.1 & -\cdots \\ 3.35 & -0.\cdots \end{bmatrix}$$

23 $\begin{bmatrix} \text{[illegible]} - 1.25(0) \\ \text{[illegible]} - 0.25(0) \end{bmatrix} = \begin{bmatrix} -1.2 - 0.1 \\ 1.2 - 0.3 \end{bmatrix}$

$= \begin{bmatrix} -1.3 \\ 0.9 \end{bmatrix}$

24. $\begin{bmatrix} -6(0) + (-7) + (-1) & -6(-1) + (-2) + (3) & -6(3) + (4) + (4) \\ -2(0) + 3(-7) + 8(-1) & -2(-1) + 3(-2) + 8(3) & -2(3) + 3(4) + 8(4) \\ 0.1(0) + 7(-7) + (-1) & 0.1(-1) + 7(-2) + (3) & 0.1(3) + 7(4) + (4) \end{bmatrix}$

$= \begin{bmatrix} -7 - 1 & 6 - 2 + 3 & -18 + 8 \\ -21 - 8 & 2 - 6 + 24 & -6 + 12 + 32 \\ -49 - 1 & -0.1 - 14 + 3 & 0.3 + 28 + 4 \end{bmatrix}$

$= \begin{bmatrix} -8 & 7 & -10 \\ -29 & 20 & 38 \\ -50 & -11.1 & 32.3 \end{bmatrix}$

25. $\begin{bmatrix} 6(-4) - 2(4) & 6(-2) - 2(-6) & 6(5) - 2(-1) \\ 1(-4) + 4(4) & 1(-2) + 4(-6) & 1(5) + 4(-1) \\ 0(-4) + 5(4) & 0(-2) + 5(-6) & 0(5) + 5(-1) \end{bmatrix}$

$= \begin{bmatrix} -24 - 8 & -12 + 12 & 30 + 2 \\ -4 + 16 & -2 - 24 & 5 - 4 \\ 20 & -30 & -5 \end{bmatrix}$

$= \begin{bmatrix} -32 & 0 & 32 \\ 12 & -26 & 1 \\ 20 & -30 & -5 \end{bmatrix}$

26. $\begin{bmatrix} 0(5) + 1(3) + 0(-4) & 0(-7) + 1(12) + 0(-5) & 0(4) + 1(6) + 0(-12) \\ 6(5) - 3(3) - 1(-4) & 6(-7) - 3(12) - 1(-5) & 6(4) - 3(6) - 1(-12) \\ -2(5) + 5(3) + 3(-4) & -2(-7) + 5(12) + 3(-5) & -2(4) + 5(6) + 3(-12) \end{bmatrix}$

$= \begin{bmatrix} 3 & 12 & 6 \\ 30 - 9 + 4 & -42 - 36 + 5 & 24 - 18 + 12 \\ -10 + 15 - 12 & 14 + 60 - 15 & -8 + 30 - 36 \end{bmatrix}$

$= \begin{bmatrix} 3 & 12 & 6 \\ 25 & -73 & 18 \\ -7 & 59 & -14 \end{bmatrix}$

27. $2AB = 2\begin{bmatrix} 4 & -2 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$

$= 2\begin{bmatrix} 4(1) - 2(-2) & 4(0) - 2(4) \\ 6(1) - 1(-2) & 6(0) - 1(4) \end{bmatrix}$

$= 2\begin{bmatrix} 8 & -8 \\ 8 & -4 \end{bmatrix}$

$= \begin{bmatrix} 16 & -16 \\ 16 & -8 \end{bmatrix}$

28. $AC = \begin{bmatrix} 4 & -2 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4(-1) - 2(-2) & 4(3) - 2(1) \\ 6(-1) - 1(-2) & 6(3) - 1(1) \end{bmatrix}$

$= \begin{bmatrix} -4 + 4 & 12 - 2 \\ -6 + 2 & 18 - 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix}$

$AB + AC = \begin{bmatrix} 8 & -8 \\ 8 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 4 & 13 \end{bmatrix}$

29. $D + E = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 2 & 4 \\ -2 & -3 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 5 & 6 \\ -1 & 4 & 2 \\ 3 & 1 & -4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 7 \\ -2 & 6 & 6 \\ 1 & -2 & -1 \end{bmatrix}$

$D(D + E) = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 2 & 4 \\ -2 & -3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 7 \\ -2 & 6 & 6 \\ 1 & -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3(1) - 2(-2) + 1(1) & 3(3) - 2(6) + 1(-2) & 3(7) - 2(6) + 1(-1) \\ -1(1) + 2(-2) + 4(1) & -1(3) + 2(6) + 4(-2) & -1(7) + 2(6) + 4(-1) \\ -2(1) - 3(-2) + 3(1) & -2(3) - 3(6) + 3(-2) & -2(7) - 3(6) + 3(-1) \end{bmatrix}$

$= \begin{bmatrix} 3 + 4 + 1 & 9 - 12 - 2 & 21 - 12 - 1 \\ -1 - 4 + 4 & -3 + 12 - 8 & -7 + 12 - 4 \\ -2 + 6 + 3 & -6 - 18 - 6 & -14 - 18 - 3 \end{bmatrix}$

$= \begin{bmatrix} 8 & -5 & 8 \\ -1 & 1 & 1 \\ 7 & -30 & -35 \end{bmatrix}$

30. $\begin{bmatrix} 1 & 3 & 7 \\ -2 & 6 & 6 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} -2 & 5 & 6 \\ -1 & 4 & 2 \\ 3 & 1 & -4 \end{bmatrix}$

$= \begin{bmatrix} 1(-2) + 3(-1) + 7(3) & 1(5) + 3(4) + 7(1) & 1(6) + 3(2) + 7(-4) \\ -2(-2) + 6(-1) + 6(3) & -2(5) + 6(4) + 6(1) & -2(6) + 6(2) + 6(-4) \\ 1(-2) - 2(-1) - 1(3) & 1(5) - 2(4) - 1(1) & 1(6) - 2(2) - 1(-4) \end{bmatrix}$

$= \begin{bmatrix} -2 - 3 + 21 & 5 + 12 + 7 & 6 + 6 - 28 \\ 4 - 6 + 18 & -10 + 24 + 6 & -12 + 12 - 24 \\ -2 + 2 - 3 & 5 - 8 - 1 & 6 - 4 + 4 \end{bmatrix}$

$= \begin{bmatrix} 16 & 24 & -16 \\ 16 & 20 & -24 \\ -3 & -4 & 6 \end{bmatrix}$

31. $-3(AC) = -3\begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix} = \begin{bmatrix} 0 & -30 \\ 12 & -51 \end{bmatrix}$

32. $0.5AB + 2AC = 0.5\begin{bmatrix} 8 & -8 \\ 8 & -4 \end{bmatrix} + 2\begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix}$

$= \begin{bmatrix} 4 & -4 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 20 \\ -8 & 34 \end{bmatrix}$

$= \begin{bmatrix} 4 & 16 \\ -4 & 32 \end{bmatrix}$

33. $\begin{bmatrix} -2(1) + 1(x) + 2(3) \\ 3(1) + 2(x) + 4(3) \\ 0(1) - 2(x) + 4(3) \end{bmatrix} = \begin{bmatrix} -2 + x + 6 \\ 3 + 2x + 12 \\ -2x + 12 \end{bmatrix}$

$= \begin{bmatrix} x + 4 \\ 2x + 15 \\ -2x + 12 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ y \end{bmatrix}$

$x + 4 = 6$

$x = 2$

$-2x + 12 = y$

$-2(2) + 12 = y$

$8 = y$

34. $\begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix}\begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 4(9) + 1(2) + 3(-1) & 4(-2) + 1(1) + 3(4) \\ -2(9) + x(2) + 1(-1) & -2(-2) + x(1) + 1(4) \end{bmatrix}$

$= \begin{bmatrix} 35 & 5 \\ 2x - 19 & x + 8 \end{bmatrix}$

$= \begin{bmatrix} y & 5 \\ -13 & 11 \end{bmatrix}$

$y = 35 \quad 2x - 19 = -13$

$2x = 6$

$x = 3$

35. Grain Production

	Wheat	Rice	Maize
China	0.201	0.348	0.180
India	0.22	0.215	0.017
CLS	0.073	0.001	0.005
US	0.113	0.014	0.405

36. $\begin{bmatrix} 0.201 & 0.348 & 0.180 \\ 0.220 & 0.215 & 0.017 \\ 0.073 & 0.001 & 0.005 \\ 0.113 & 0.014 & 0.405 \end{bmatrix}\begin{bmatrix} 608{,}846 \\ 570{,}906 \\ 586{,}923 \end{bmatrix}$

$= \begin{bmatrix} 122{,}378 & 198{,}675 & 105{,}646 \\ 133{,}946 & 122{,}745 & 9{,}978 \\ 44{,}446 & 571 & 2{,}935 \\ 68{,}800 & 7{,}993 & 237{,}704 \end{bmatrix}$

Total Weight

China	426,699.474
India	266,668.601
C.I.S.	47,951.279
U.S.	314,496.097

37. Matrix B $\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$

38. $AB = \begin{bmatrix} 3(6) + 5(5) + 4(4) \\ 5(6) + 2(5) + 5(4) \\ 4(6) + 6(5) + 2(4) \end{bmatrix} = \begin{bmatrix} 59 \\ 60 \\ 62 \end{bmatrix}$

39. Team 3; 62 points

40. $[2 \;\; 0.5 \;\; 3]\begin{bmatrix} 109 & 136 \\ 127 & 159 \\ 64 & 79 \end{bmatrix}$

$= [218 + 63.5 + 192 \quad 272 + 79.5 + 237]$

$= [473.5 \quad 588.5]$

The 120 lb person burned about 474 calories and the 150 lb person about 589 calories.

41. *Sample answer:* Check that the product is defined. To find the entry in the n^{th} row, m^{th} column, use the n^{th} row of the left matrix and the m^{th} column on the right matrix. Multiply each pair of corresponding entries and find the sum of these products.

42. $\begin{bmatrix} 0 & -1 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} 7 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -28 + 2 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -26 & 8 \end{bmatrix}$

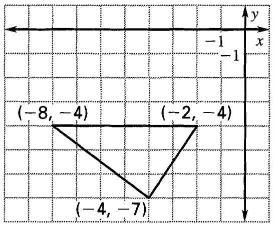
C

43. B

44. a. $AB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -7 & -4 & -4 \\ 4 & 8 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0(-7) + (-1)(4) & 0(-4) + (-1)(8) & 0(-4) + (-1)(2) \\ 1(-7) + 0(4) & 1(-4) + 0(8) & 1(-4) + 0(2) \end{bmatrix}$

$= \begin{bmatrix} -4 & -8 & -2 \\ -7 & -4 & -4 \end{bmatrix}$



AB represents a 90° rotation of the original triangle.

b. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -4 & -8 & -2 \\ -7 & -4 & -4 \end{bmatrix}$

$= \begin{bmatrix} 0(-4) + (-1)(-7) & 0(-8) + (-1)(-4) & 0(-2) + (-1)(-4) \\ 1(-4) + 0(-7) & 1(-8) + 0(-4) & 1(-2) + 0(-4) \end{bmatrix}$

$= \begin{bmatrix} 7 & 4 & 4 \\ -4 & -8 & -2 \end{bmatrix};$

vertices: $(7, -4)(4, -8)(4, -2)$

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 7 & 4 & 4 \\ -4 & -8 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0(7) + (-1)(-4) & 0(4) + (-1)(-8) & 0(4) + (-1)(-2) \\ 1(7) + 0(-4) & 1(4) + 0(-8) & 1(4) + (0)(-2) \end{bmatrix}$

$= \begin{bmatrix} 4 & 8 & 2 \\ 7 & 4 & 4 \end{bmatrix};$

vertices: $(4, 7)(8, 4)(2, 4)$

4.2 Mixed Review (p. 213)

45. $A = l \times w = 15 \times 12 = 180 \text{ m}^2$

46. $A = \frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 4 = 12 \text{ m}^2$

47. $A = \pi r^2 = \pi 9 \approx 28.3 \text{ ft}^2$

48. $y - 8 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x + \frac{15}{2}$

49. $y - 4 = -\frac{1}{4}(x - 0)$

$y = -\frac{1}{4}x + 4$

50. $y + 6 = \frac{10 + 6}{2 - 3}(x - 3)$

$y + 6 = -16(x - 3)$

$y = -16x + 42$

51. $y - 5 = \frac{14 - 5}{4 - 1}(x - 1)$

$y - 5 = \frac{9}{3}(x - 1)$

$y = 3x + 2$

52. $y + 5 = -\frac{5}{7}(x - 0)$

$y = -\frac{5}{7}x - 5$

53. $y + 6 = \frac{-6}{-4}(x - 0)$

$y = \frac{3}{2}x - 6$

54. $y = 6 - 4x$

$-3x - 2(6 - 4x) = 8$

$-3x - 12 + 8x = 8$

$5x = 20$

$x = 4$

$y = 6 - 16 = -10$

$(4, -10)$

55. $y = -9 - 2x$

$3x + 5(-9 - 2x) = 4$

$3x - 10x = 49$

$-7x = 49$

$x = -7$

$y = -9 + 14$

$y = 5$

$(-7, 5)$

56. $-9x + 5y = 1$

$9x - 6y = 6$

$-y = 7$

$y = -7$

$3x - 2y = 2$

$3x = 2 + 2y$

$3x = 2 + 2(-7)$

$3x = -12$

$x = -4$

$(-4, -7)$

57. $x - y = 1$

$x = 1 + y$

$2(1 + y) - 2y = 8$

$2 + 2y - 2y = 8$

$2 \neq 8$

no solution

58. $-6x + 8y = -2$

$6x + 2y = 7$

$10y = 5$

$y = \frac{1}{2}$

$6x + 1 = 7$

$6x = 6$

$x = 1$

$(1, \frac{1}{2})$

59. $20x + 8y = -40$

$-\ 3x - 8y = 40$

$17x = 0$

$x = 0$

$-8y = 40$

$y = -5$

$(0, -5)$

60. $14x + 6y = 22$

$-\ 14x + 35y = 224$

$41y = 246$

$y = 6$

$7x + 3y = 11$

$7x = 11 + (-3y)$

$7x = 11 + (-3)(6)$

$7x = 11 - 18$

$7x = -7$

$x = -1$

$(-1, 6)$

61. $10x - 8y = -2$

$-\ 10x + 45y = -50$

$37y = -52$

$y = \frac{-52}{37}$

$2x - 9(-\frac{52}{37}) = 10$

$2x = -\frac{98}{37}$

$x = -\frac{49}{37}$

$(-\frac{49}{37}, -\frac{52}{37})$

62. $x = 7y + 49$

$12(7y + 49) + y = -24$

$85y = -612$

$y = -\frac{36}{5}$

$x = 7(\frac{-36}{5}) + 49$

$x = \frac{-252}{5} + \frac{245}{5} = \frac{-7}{5}$

$(-\frac{7}{5}, -\frac{36}{5})$

Section 4.3

4.3 Guided Practice (p. 218)

1. *Sample answer:* Cramer's rule is a method of so[illegible] system of linear equations by using the determin[illegible] the coefficient matrix associated with the system[illegible] find the determinant of the coefficient matrix an[illegible] that it does not equal 0. Each coordinate of the s[illegible] of the system is found as a fraction. The denomi[illegible] the fraction is the determinant of the coefficient [illegible] The numerator of the fraction is the determinan[illegible] matrix formed by using the coefficient matrix a[illegible] replacing the column of coefficients of the corre[illegible] variable with the column of constants from the [illegible]

2. Yes; *Sample answer:* $\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 6 & 1 \\ 2 & 2 \end{vmatrix} =$ [illegible]

3. a. The calculation should be $6 - (-5) = 11$

b. The calculation should be $12 - (-2) = 14$

4. The determinant cannot be zero. **5.** $0 - 6 =$ [illegible]

6. $1 - 20 = -19$ **7.** $32 - 4 = 28$

8. $\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix} = -30 + 32 = 2$

$$x = \frac{\begin{vmatrix} 4 & -8 \\ -4 & -5 \end{vmatrix}}{2} = \frac{-20 - 32}{2} = \frac{-52}{2} = -26$$

$$y = \frac{\begin{vmatrix} 6 & 4 \\ 4 & -4 \end{vmatrix}}{2} = \frac{-24 - 16}{2} = \frac{-40}{2} = -20$$

$(-26, -20)$

9. $\begin{vmatrix} 2 & 7 \\ 3 & -8 \end{vmatrix} = -16 - 21 = -37$

$$x = \frac{\begin{vmatrix} -3 & 7 \\ -23 & -8 \end{vmatrix}}{-37} = \frac{24 + 161}{-37} = -5$$

$$y = \frac{\begin{vmatrix} 2 & -3 \\ 3 & -23 \end{vmatrix}}{-37} = \frac{-46 + 9}{-37} = \frac{-37}{-37} = 1$$

$(-5, 1)$