52. $y + 5 = -\frac{5}{7}(x - 0)$

$y = -\frac{5}{7}x - 5$

53. $y + 6 = \frac{-6}{-4}(x - 0)$

$y = \frac{3}{2}x - 6$

54. $y = 6 - 4x$

$-3x - 2(6 - 4x) = 8$

$-3x - 12 + 8x = 8$

$5x = 20$

$x = 4$

$y = 6 - 16 = -10$

$(4, -10)$

55. $y = -9 - 2x$

$3x + 5(-9 - 2x) = 4$

$3x - 10x = 49$

$-7x = 49$

$x = -7$

$y = -9 + 14$

$y = 5$

$(-7, 5)$

56. $-9x + 5y = 1$

$\underline{9x - 6y = 6}$

$-y = 7$

$y = -7$

$3x - 2y = 2$

$3x = 2 + 2y$

$3x = 2 + 2(-7)$

$3x = -12$

$x = -4$

$(-4, -7)$

57. $x - y = 1$

$x = 1 + y$

$2(1 + y) - 2y = 8$

$2 + 2y - 2y = 8$

$2 \neq 8$

no solution

58. $-6x + 8y = -2$

$\underline{6x + 2y = 7}$

$10y = 5$

$y = \frac{1}{2}$

$6x + 1 = 7$

$6x = 6$

$x = 1$

$\left(1, \frac{1}{2}\right)$

59. $20x + 8y = -40$

$\underline{-\ 3x - 8y = 40}$

$17x = 0$

$x = 0$

$-8y = 40$

$y = -5$

$(0, -5)$

60. $14x + 6y = 22$

$\underline{-\ 14x + 35y = 224}$

$41y = 246$

$y = 6$

$7x + 3y = 11$

$7x = 11 + (-3y)$

$7x = 11 + (-3)(6)$

$7x = 11 - 18$

$7x = -7$

$x = -1$

$(-1, 6)$

61. $10x - 8y = -2$

$\underline{-\ 10x + 45y = -50}$

$37y = -52$

$y = \frac{-52}{37}$

$2x - 9\left(-\frac{52}{37}\right) = 10$

$2x = -\frac{98}{37}$

$x = -\frac{49}{37}$

$\left(-\frac{49}{37}, -\frac{52}{37}\right)$

62. $x = 7y + 49$

$12(7y + 49) + y = -24$

$85y = -612$

$y = -\frac{36}{5}$

$x = 7\left(\frac{-36}{5}\right) + 49$

$x = \frac{-252}{5} + \frac{245}{5} = \frac{-7}{5}$

$\left(-\frac{7}{5}, -\frac{36}{5}\right)$

Section 4.3

4.3 Guided Practice (p. 218)

1. *Sample answer:* Cramer's rule is a method of so[illegible] system of linear equations by using the determin[illegible] the coefficient matrix associated with the system. [illegible] find the determinant of the coefficient matrix an[illegible] that it does not equal 0. Each coordinate of the s[illegible] of the system is found as a fraction. The denom[illegible] the fraction is the determinant of the coefficient [illegible] The numerator of the fraction is the determinant [illegible] matrix formed by using the coefficient matrix a[illegible] replacing the column of coefficients of the corre[illegible] variable with the column of constants from the [illegible]

2. Yes; *Sample answer:* $\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 6 & 1 \\ 2 & 2 \end{vmatrix} =$ [illegible]

3. a. The calculation should be $6 - (-5) = 11$

b. The calculation should be $12 - (-2) = 1$[illegible]

4. The determinant cannot be zero. **5.** $0 - 6 = -$[illegible]

6. $1 - 20 = -19$ **7.** $32 - 4 = 28$

8. $\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix} = -30 + 32 = 2$

$x = \dfrac{\begin{vmatrix} 4 & -8 \\ -4 & -5 \end{vmatrix}}{2} = \dfrac{-20 - 32}{2} = \dfrac{-52}{2} = -26$

$y = \dfrac{\begin{vmatrix} 6 & 4 \\ 4 & -4 \end{vmatrix}}{2} = \dfrac{-24 - 16}{2} = \dfrac{-40}{2} = -20$

$(-26, -20)$

9. $\begin{vmatrix} 2 & 7 \\ 3 & -8 \end{vmatrix} = -16 - 21 = -37$

$x = \dfrac{\begin{vmatrix} -3 & 7 \\ -23 & -8 \end{vmatrix}}{-37} = \dfrac{24 + 161}{-37} = -5$

$y = \dfrac{\begin{vmatrix} 2 & -3 \\ 3 & -23 \end{vmatrix}}{-37} = \dfrac{-46 + 9}{-37} = \dfrac{-37}{-37} = 1$

$(-5, 1)$

10. $\begin{vmatrix} 12 & -2 \\ -14 & 11 \end{vmatrix} = 12 \cdot 11 - (-2 \cdot -14)$

$= 132 - 28 = 104$

$x = \dfrac{\begin{vmatrix} 2 & -2 \\ 51 & 11 \end{vmatrix}}{104} = \dfrac{22 + 102}{104} = \dfrac{124}{104} = \dfrac{31}{26}$

$y = \dfrac{\begin{vmatrix} 12 & 2 \\ -14 & 51 \end{vmatrix}}{104} = \dfrac{612 + 28}{104} = \dfrac{640}{104} = \dfrac{80}{13}$

$\left(\dfrac{31}{26}, \dfrac{80}{13}\right)$

11. $A = \frac{1}{2}(50 \cdot 70) = 1750 \text{ in.}^2$

4.3 Practice and Applications (pp. 218–220)

12. $8 - 10 = -2$ **13.** $24 - 0 = 24$ **14.** $9 + 6 = 15$

15. $-14 + 77 = 63$ **16.** $16 - 0 = 16$

17. $9 - 40 = -31$ **18.** $-54 + 15 = -39$

19. $0 + 24 = 24$ **20.** $96 + 10 = 106$

21. $(36 + 40 + 16) - (-15 + 192 - 8) = 92 - 169 = -77$

22. $(10 + 0 + 16) - (0 + 5 - 36) = 26 + 31 = 57$

23. $(0 + 100 + 80) - (-130 + 0 - 50) = 180 + 180 = 360$

24. $(-16 + 224 - 40) - (-56 + 2 - 1280) = (168) - (-1334)$

$= 1502$

25. $(-64 + 0 + 0) - (0 - 180 + 0) = -64 + 180 = 116$

26. $(96 + 54 + 1404) - (81 + 936 + 96) = (1554) - (1113) = 441$

27. $(-54 + 0 - 60) - (45 + 0 - 240) = -114 + 195 = 81$

28. $(-324 + 396 - 3000) - (1980 - 810 - 240) = -2928 - 930$

$= -3858$

29. $(0 - 192 + 200) - (0 + 180 + 560) = 8 - 740 = -732$

30. $A = \pm\frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 2 & 7 & 1 \\ 5 & 5 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}[(0 + 5 + 10) - (35 + 0 + 2)]$

$= \pm\frac{1}{2}(15 - 37) = 11$

31. $A = \frac{1}{2}\begin{vmatrix} 3 & 6 & 1 \\ 3 & 0 & 1 \\ 1 & 3 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}[(0 + 6 + 9) - (0 + 9 + 18)]$

$= \pm\frac{1}{2}(15 - 27) = 6$

32. $A = \pm\frac{1}{2}\begin{vmatrix} 6 & -1 & 1 \\ 2 & 2 & 1 \\ 4 & 8 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}[(12 - 4 + 16) - (8 + 48 - 2)]$

$= \pm\frac{1}{2}(24 - 54) = 15$

33. $A = \pm\frac{1}{2}\begin{vmatrix} -4 & 2 & 1 \\ 3 & -1 & 1 \\ -2 & -2 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}([4 - 4 - 6) - (2 + 8 + 6)]$

$= \pm\frac{1}{2}(-6 - 16) = 11$

34. $A = \pm\frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ -1 & -4 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}[(-8 + 0 - 2) - (0 + 4 + 6)]$

$= \pm\frac{1}{2}(-10 - 10) = 10$

35. $A = \pm\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ -1 & 1 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}[(6 - 3 - 2) - (-6 + 1 - 6)$

$= \pm\frac{1}{2}[1 + 11] = 6$

36. $\begin{vmatrix} 2 & 1 \\ 5 & 6 \end{vmatrix} = 12 - 5 = 7$

$x = \dfrac{\begin{vmatrix} 3 & 1 \\ 4 & 6 \end{vmatrix}}{7} = \dfrac{18 - 4}{7} = 2$

$y = \dfrac{\begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix}}{7} = \dfrac{8 - 15}{7} = -1$

$(2, -1)$

37. $\begin{vmatrix} 7 & -5 \\ 3 & 10 \end{vmatrix} = 70 + 15 = 85$

$x = \dfrac{\begin{vmatrix} 11 & -5 \\ -56 & 10 \end{vmatrix}}{85} = \dfrac{110 - 280}{85} = -2$

$y = \dfrac{\begin{vmatrix} 7 & 11 \\ 3 & -56 \end{vmatrix}}{85} = \dfrac{-392 - 33}{85} = -5$

$(-2, -5)$

38. $\begin{vmatrix} 9 & 2 \\ 4 & -3 \end{vmatrix} = -27 - 8 = -35$

$x = \dfrac{\begin{vmatrix} 7 & 2 \\ 42 & -3 \end{vmatrix}}{-35} = \dfrac{-21 - 84}{-35} = 3$

$y = \dfrac{\begin{vmatrix} 9 & 7 \\ 4 & 42 \end{vmatrix}}{-35} = \dfrac{378 - 28}{-35} = -10$

$(3, -10)$

39. $\begin{vmatrix} 1 & 7 \\ 3 & -5 \end{vmatrix} = -5 - 21 = -26$

$x = \dfrac{\begin{vmatrix} -3 & 7 \\ 17 & -5 \end{vmatrix}}{-26} = \dfrac{15 - 119}{-26} = 4$

$y = \dfrac{\begin{vmatrix} 1 & -3 \\ 3 & 17 \end{vmatrix}}{-26} = \dfrac{17 + 9}{-26} = -1$

$(4, -1)$

40. $\begin{vmatrix} -1 & -12 \\ 12 & -15 \end{vmatrix} = 15 + 144 = 159$

$x = \dfrac{\begin{vmatrix} 44 & -12 \\ -51 & -15 \end{vmatrix}}{159} = \dfrac{-660 - 612}{159} = -8$

$y = \dfrac{\begin{vmatrix} -1 & 44 \\ 12 & -51 \end{vmatrix}}{159}$

$y = \dfrac{51 - 528}{159} = -3$

$(-8, -3)$

41. $\begin{vmatrix} 4 & -3 \\ 8 & -7 \end{vmatrix} = -28 + 24 = -4$

$x = \dfrac{\begin{vmatrix} 18 & -3 \\ 34 & -7 \end{vmatrix}}{-4} = \dfrac{-126 + 102}{-4} = 6$

$y = \dfrac{\begin{vmatrix} 4 & 18 \\ 8 & 34 \end{vmatrix}}{-4} = \dfrac{136 - 144}{-4} = 2$

$(6, 2)$

42. $\begin{vmatrix} 4 & -5 \\ 2 & -7 \end{vmatrix} = -28 + 10 = -18$

$x = \dfrac{\begin{vmatrix} 13 & -5 \\ 24 & -7 \end{vmatrix}}{-18} = \dfrac{-91 + 120}{-18} = \dfrac{29}{-18}$

$y = \dfrac{\begin{vmatrix} 4 & 13 \\ 2 & 24 \end{vmatrix}}{-18} = \dfrac{96 - 26}{-18} = \dfrac{35}{-9}$

$\left(-\dfrac{29}{18}, -\dfrac{35}{9}\right)$

43. $\begin{vmatrix} 8 & -9 \\ -5 & 7 \end{vmatrix} = 56 - 45 = 11$

$x = \dfrac{\begin{vmatrix} 32 & -9 \\ 40 & 7 \end{vmatrix}}{11} = \dfrac{224 + 360}{11} = \dfrac{584}{11}$

$y = \dfrac{\begin{vmatrix} 8 & 32 \\ -5 & 40 \end{vmatrix}}{11} = \dfrac{320 + 160}{11} = \dfrac{480}{11}$

$\left(\dfrac{584}{11}, \dfrac{480}{11}\right)$

44. $\begin{vmatrix} 3 & 10 \\ 12 & 15 \end{vmatrix} = 45 - 120 = -75$

$x = \dfrac{\begin{vmatrix} 50 & 10 \\ 64 & 15 \end{vmatrix}}{-75} = \dfrac{750 - 640}{-75} = \dfrac{110}{-75} = -\dfrac{22}{15}$

$y = \dfrac{\begin{vmatrix} 3 & 50 \\ 12 & 64 \end{vmatrix}}{-75} = \dfrac{192 - 600}{-75} = \dfrac{136}{25}$

$\left(-\dfrac{22}{15}, \dfrac{136}{25}\right)$

45. $\begin{vmatrix} 1 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 4 & -4 \end{vmatrix} = (4 + 6 - 12) - (9 + 4 - 8) = -$

$x = \dfrac{\begin{vmatrix} -2 & 2 & -3 \\ -1 & -1 & 1 \\ 4 & 4 & -4 \end{vmatrix}}{-7} = \dfrac{(-8 + 8 + 12) - (12 -}{-7}$

$= \dfrac{12 - 12}{-7} = 0$

$y = \dfrac{\begin{vmatrix} 1 & -2 & -3 \\ 1 & -1 & 1 \\ 3 & 4 & -4 \end{vmatrix}}{-7} = \dfrac{(4 - 6 - 12) - (9 + 4 -}{-7}$

$= \dfrac{-14 - 21}{-7} = 5$

$z = \dfrac{\begin{vmatrix} 1 & 2 & -2 \\ 1 & -1 & -1 \\ 3 & 4 & 4 \end{vmatrix}}{-7} = \dfrac{(4 - 6 - 8) - (6 - 4 -}{-7}$

$= \dfrac{-18 - 10}{-7} = 4$

$(0, 5, 4)$

46. $\begin{vmatrix} 1 & 3 & -1 \\ -2 & -6 & 1 \\ 3 & 5 & -2 \end{vmatrix} = (12 + 9 + 10) - (18 + 5 + 12) = -4$

$$x = \frac{\begin{vmatrix} 1 & 3 & -1 \\ -3 & -6 & 1 \\ 4 & 5 & -2 \end{vmatrix}}{-4}$$

$$= \frac{(12 + 12 + 15) - (24 + 5 + 18)}{-4} \frac{39 - 47}{-4} = 2$$

$$y = \frac{\begin{vmatrix} 1 & 1 & -1 \\ -2 & -3 & 1 \\ 3 & 4 & -2 \end{vmatrix}}{-4} = \frac{(6 + 3 + 8) - (9 + 4 + 4)}{-4}$$

$$= \frac{17 - 17}{-4} = 0$$

$$z = \frac{\begin{vmatrix} 1 & 3 & 1 \\ -2 & -6 & -3 \\ 3 & 5 & 4 \end{vmatrix}}{-4}$$

$$= \frac{(-24 - 27 - 10) - (-18 - 15 - 24)}{-4}$$

$$= \frac{-61 + 57}{-4}$$

$$= 1$$

(2, 0, 1)

47. $\begin{vmatrix} 3 & 2 & -5 \\ 6 & 0 & -1 \\ 0 & -1 & 3 \end{vmatrix} = (0 + 0 + 30) - (0 + 3 + 36) = -9$

$$x = \frac{\begin{vmatrix} -10 & 2 & -5 \\ 8 & 0 & -1 \\ -2 & -1 & 3 \end{vmatrix}}{-9}$$

$$= \frac{(0 + 4 + 40) - (0 - 10 + 48)}{-9} = \frac{44 - 38}{-9}$$

$$= -\frac{6}{9} = -\frac{2}{3}$$

$$y = \frac{\begin{vmatrix} 3 & -10 & -5 \\ 6 & 8 & -1 \\ 0 & -2 & 3 \end{vmatrix}}{-9}$$

$$= \frac{(72 + 0 + 60) - (0 + 6 - 180)}{-9} = \frac{132 + 174}{-9}$$

$$= \frac{306}{-9} = -34$$

$$z = \frac{\begin{vmatrix} 3 & 2 & -10 \\ 6 & 0 & 8 \\ 0 & -1 & -2 \end{vmatrix}}{-9}$$

—CONTINUED—

47. —CONTINUED—

$$= \frac{(0 + 0 + 60) - (0 - 24 - 24)}{-9} = \frac{60 + 48}{-9} = -12$$

$\left(-\frac{2}{3}, -34, -12\right)$

48. $\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 0 & -2 \end{vmatrix} = (-2 + 10 + 0) - (5 + 0 - 4) = 7$

$$x = \frac{\begin{vmatrix} 9 & 2 & 1 \\ 3 & 1 & 1 \\ -1 & 0 & -2 \end{vmatrix}}{7}$$

$$= \frac{(-18 - 2 + 0) - (-1 + 0 - 12)}{7} = \frac{-20 + 13}{7} = -1$$

$$y = \frac{\begin{vmatrix} 1 & 9 & 1 \\ 1 & 3 & 1 \\ 5 & -1 & -2 \end{vmatrix}}{7}$$

$$= \frac{(-6 + 45 - 1) - (15 - 1 - 18)}{7} = \frac{38 + 4}{7} = 6$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 9 \\ 1 & 1 & 3 \\ 5 & 0 & -1 \end{vmatrix}}{7}$$

$$= \frac{(-1 + 30 + 0) - (45 + 0 - 2)}{7} = \frac{29 - 43}{7} = -2$$

(−1, 6, −2)

49. $\begin{vmatrix} 4 & 1 & 6 \\ 3 & 3 & 2 \\ -1 & -1 & 1 \end{vmatrix} = (12 - 2 - 18) - (-18 - 8 + 3)$

$= 15$

$$x = \frac{\begin{vmatrix} 7 & 1 & 6 \\ 17 & 3 & 2 \\ -9 & -1 & 1 \end{vmatrix}}{15}$$

$$= \frac{(21 - 18 - 102) - (-162 - 14 + 17)}{15}$$

$$= \frac{-99 + 159}{15} = 4$$

$$y = \frac{\begin{vmatrix} 4 & 7 & 6 \\ 3 & 17 & 2 \\ -1 & -9 & 1 \end{vmatrix}}{15}$$

$$= \frac{(68 - 14 - 162) - (-102 - 72 + 21)}{15}$$

$$= \frac{-108 + 153}{15} = 3$$

—CONTINUED—

49. —CONTINUED—

$$z = \frac{\begin{vmatrix} 4 & 1 & 7 \\ 3 & 3 & 17 \\ -1 & -1 & -9 \end{vmatrix}}{15}$$

$$= \frac{(-108 - 17 - 21) - (-21 - 68 - 27)}{15}$$

$$= \frac{-146 + 116}{15} = -2$$

$(4, 3, -2)$

50. $\begin{vmatrix} 1 & 4 & -1 \\ 2 & -1 & 2 \\ -3 & 1 & -3 \end{vmatrix} = (3 - 24 - 2) - (-3 + 2 - 24) = 2$

$$x = \frac{\begin{vmatrix} -7 & 4 & -1 \\ 15 & -1 & 2 \\ -22 & 1 & -3 \end{vmatrix}}{2}$$

$$= \frac{(-21 - 176 - 15) - (-22 - 14 - 180)}{2}$$

$$= \frac{-212 + 216}{2} = 2$$

$$y = \frac{\begin{vmatrix} 1 & -7 & -1 \\ 2 & 15 & 2 \\ -3 & -22 & -3 \end{vmatrix}}{2}$$

$$= \frac{(-45 + 42 + 44) - (45 + -44 + 42)}{2}$$

$$= \frac{41 - 43}{2} = -1$$

$$z = \frac{\begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 15 \\ -3 & 1 & -22 \end{vmatrix}}{2}$$

$$= \frac{(22 - 180 - 14) - (-21 + 15 - 176)}{2}$$

$$= \frac{-172 + 182}{2} = 5$$

$(2, -1, 5)$

51. $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 4 & -2 \\ 6 & 5 & 0 \end{vmatrix} = (0 - 12 + 5) - (24 - 20 + 0) = -11$

$$x = \frac{\begin{vmatrix} 5 & 1 & 1 \\ 9 & 4 & -2 \\ 16 & 5 & 0 \end{vmatrix}}{-11}$$

$$= \frac{(0 + 45 - 32) - (64 - 50 + 0)}{-11} = \frac{13 - 14}{-11} = \frac{1}{11}$$

—CONTINUED—

51. —CONTINUED—

$$y = \frac{\begin{vmatrix} 2 & 5 & 1 \\ 1 & 9 & -2 \\ 6 & 16 & 0 \end{vmatrix}}{-11}$$

$$= \frac{(0 - 60 + 16) - (54 - 64 + 0)}{-11} = \frac{-44 + 1}{-11} =$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 5 \\ 1 & 4 & 9 \\ 6 & 5 & 16 \end{vmatrix}}{-11}$$

$$= \frac{(128 + 54 + 25) - (120 + 90 + 16)}{-11}$$

$$= \frac{207 - 226}{-11} = \frac{19}{11}$$

$\left(\frac{1}{11}, \frac{34}{11}, \frac{19}{11}\right)$

52. $\begin{vmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{vmatrix} = (2 - 12 + 70) - (-21 + 1$

$= 63$

$$x = \frac{\begin{vmatrix} 13 & 2 & 7 \\ -2 & -1 & -2 \\ -14 & 5 & 2 \end{vmatrix}}{63}$$

$$= \frac{(-26 + 56 - 70) + (98 - 130 - 8)}{63} = \frac{-40 -}{63}$$

$$y = \frac{\begin{vmatrix} -1 & 13 & 7 \\ 2 & -2 & -2 \\ 3 & -14 & 2 \end{vmatrix}}{63}$$

$$= \frac{(4 - 78 - 196) - (-42 - 28 + 52)}{63}$$

$$= \frac{-270 + 18}{63} = -4$$

$$z = \frac{\begin{vmatrix} -1 & 2 & 13 \\ 2 & -1 & -2 \\ 3 & 5 & -14 \end{vmatrix}}{63}$$

$$= \frac{(-14 - 12 + 130) - (-39 + 10 - 56)}{63}$$

$$= \frac{104 + 85}{63} = 3$$

$(0, -4, 3)$

53. $\begin{vmatrix} -3 & 1 & 2 \\ 9 & -1 & 2 \\ 8 & 5 & -4 \end{vmatrix} = (-12 + 16 + 90) - (-16) - 30 - 36$

$= 176$

$x = \dfrac{\begin{vmatrix} -14 & 1 & 2 \\ -8 & -1 & 2 \\ 6 & 5 & -4 \end{vmatrix}}{176}$

$= \dfrac{(-56 + 12 - 80) - (-12 - 140 + 32)}{176}$

$= \dfrac{-124 + 120}{176} = \dfrac{-4}{176} = -\dfrac{1}{44}$

$y = \dfrac{\begin{vmatrix} -3 & -14 & 2 \\ 9 & -8 & 2 \\ 8 & 6 & -4 \end{vmatrix}}{176}$

$= \dfrac{(-96 - 224 + 108) - (-128 - 36 + 504)}{176}$

$= \dfrac{-212 - 340}{176} = -\dfrac{552}{176} = -\dfrac{69}{22}$

$z = \dfrac{\begin{vmatrix} -3 & 1 & -14 \\ 9 & -1 & -8 \\ 8 & 5 & 6 \end{vmatrix}}{176}$

$= \dfrac{(18 - 64 - 630) - (112 + 120 + 54)}{176}$

$= \dfrac{-676 - 286}{176} = -\dfrac{962}{176} = -\dfrac{481}{88}$

$\left(-\dfrac{1}{44}, -\dfrac{69}{22}, -\dfrac{481}{88}\right)$

54. $A = \pm\dfrac{1}{2}\begin{vmatrix} 35 & 220 & 1 \\ 112 & 56 & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$= \pm\dfrac{1}{2}[(1960 + 0 + 0) - (0 + 0\ 24{,}640)]$

$= \pm\dfrac{1}{2}(-22{,}680) = 11{,}340 \text{ mi}^2$

55. $A = \pm\dfrac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 12 & 2 & 1 \\ 12 & 26 & 1 \end{vmatrix}$

$= \pm\dfrac{1}{2}[(0 + 24 + 312) - (24 + 0 + 24)]$

$= \pm\dfrac{1}{2}(336 - 48) = 144 \text{ ft}^2$

56. $A = \pm\dfrac{1}{2}\begin{vmatrix} 14 & 2 & 1 \\ 22 & 2 & 1 \\ 14 & 18 & 1 \end{vmatrix}$

$= \pm\dfrac{1}{2}[(28 + 28 + 396) - (28 + 252 + 44)$

$= \pm\dfrac{1}{2}(452 - 324) = 64 \text{ ft}^2$

57. $144 \text{ ft}^2 \times \left(\dfrac{1 \text{ in.}}{6 \text{ ft}}\right)^2 = 144 \text{ ft}^2 \times \dfrac{1 \text{ in.}^2}{36 \text{ ft}^2} = 4 \text{ in.}^2$

58. $A = \pm\dfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 100 & 120 & 1 \\ 140 & 20 & 1 \end{vmatrix}$

$= \pm\dfrac{1}{2}[(0 + 0 + 2000) - (16{,}800 + 0 + 0)]$

$= \pm\dfrac{1}{2}(2000 - 16{,}800) = 7400 \text{ mi}^2$

59. $10x + 2y = 13.56$ let x = price of premium gas/gal

$x = y + 0.12$ let y = price of regular gas/gal

$\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix} = -10 - 2 = -12$

$x = \dfrac{\begin{vmatrix} 13.56 & 2 \\ 0.12 & -1 \end{vmatrix}}{-12} = \dfrac{-13.56 - 0.24}{-12} = \dfrac{-13.80}{-12} = \$1.15$

$1.15 premium gas per gallon

$y = \dfrac{\begin{vmatrix} 10 & 13.56 \\ 1 & 0.12 \end{vmatrix}}{-12} = \dfrac{1.20 - 13.56}{-12} = \dfrac{-12.36}{-12} = \$1.03$

$1.03 regular gas per gallon

60. $4S + 4N = 184$

$S + 6F = 146$

$2N + 4F = 104$

$\begin{vmatrix} 4 & 4 & 0 \\ 1 & 0 & 6 \\ 0 & 2 & 4 \end{vmatrix} = (0 + 0 + 0) - (0 + 48 + 16) = -64$

$S = \dfrac{\begin{vmatrix} 184 & 4 & 0 \\ 146 & 0 & 6 \\ 104 & 2 & 4 \end{vmatrix}}{-64}$

$= \dfrac{(0 + 2496 + 0) - (0 + 2208 + 2336)}{-64}$

$= \dfrac{2496 - 4544}{-64} = 32$

—CONTINUED—

60. —CONTINUED—

$$N = \frac{\begin{vmatrix} 4 & 184 & 0 \\ 1 & 146 & 6 \\ 0 & 104 & 4 \end{vmatrix}}{-64}$$

$$= \frac{(2336 + 0 + 0) - (0 + 2496 + 736)}{-64}$$

$$= \frac{2336 - 3232}{-64} = 14$$

$$F = \frac{\begin{vmatrix} 4 & 4 & 184 \\ 1 & 0 & 146 \\ 0 & 2 & 104 \end{vmatrix}}{-64}$$

$$= \frac{(0 + 0 + 368) - (0 + 1168 + 416)}{-64}$$

$$= \frac{368 - 1584}{-64} = 19$$

61. The determinant is multiplied by -1. Proof for 2×2 matrices:

$$-1\begin{vmatrix} a & b \\ c & d \end{vmatrix} = -(ad - bc) = bc - ad = 1\begin{vmatrix} b & a \\ d & c \end{vmatrix}$$

62. A: $A = \pm\frac{1}{2}\begin{vmatrix} -3 & 4 & 1 \\ 4 & 2 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$$= \pm\frac{1}{2}[(-6 + 4 - 8) - (2 + 6 + 16)]$$

$$= \pm\frac{1}{2}(-10 - 24) = 17$$

$$A = \pm\frac{1}{2}\begin{vmatrix} 4 & 2 & 1 \\ 1 & -2 & 1 \\ 3 & -4 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}[(-8 + 6 - 4) - (-6 - 16 + 2)]$$

$$= \pm\frac{1}{2}(-6 + 20) = 7$$

63. C $\begin{vmatrix} -5 & 6 \\ -2 & 10 \end{vmatrix} = -50 + 12 = -38$

$$\begin{vmatrix} -7 & 1 \\ 3 & 5 \end{vmatrix} = -35 - 3 = -38$$

64. a. $\det AB = (\det A)(\det B)$

b. $\det kA = k^2 \det A$

4.3 Mixed Review (p. 221)

65. $\ldots = 7 - 10 = -3$

66. $\ldots = 3(-2) + 7 = -6 + 7 = 1$

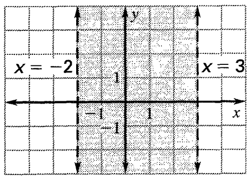
67. $\ldots = -(-1)^2 + 5 = -1 + 5 = 4$

68. $\ldots = (7)^2 - 2(7) - 4 = 49 - 18 = 31$

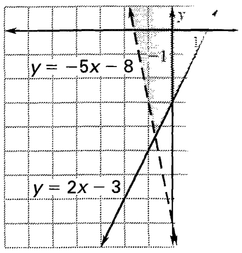
69. $\ldots = (\frac{1}{2})^2 + 4(\frac{1}{2}) - 1 = 1\frac{1}{4}$

70. $f(3) = 3^5 - 2 \cdot 3 - 10 = 243 - 16 = 227$

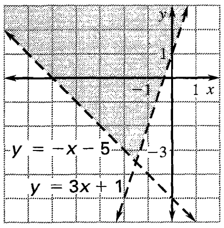
71.



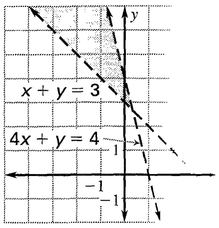
72.



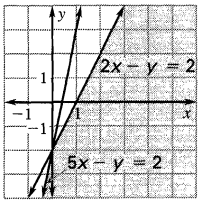
73.



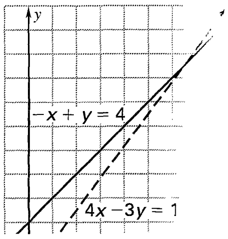
74.



75.



76.



77. $\begin{bmatrix} -2(6) - 4(3) & -2(-1) - 4(-3) \\ 5(6) + 1(3) & 5(-1) + 1(-3) \end{bmatrix} = \begin{bmatrix} -24 & \ldots \\ 33 & \ldots \end{bmatrix}$

78. $\begin{bmatrix} 7(0) - 1(4) & 7(-3) - 1(8) \\ 4(0) - 10(4) & 4(-3) - 10(8) \end{bmatrix} = \begin{bmatrix} -4 & \ldots \\ -40 & \ldots \end{bmatrix}$

79. $\begin{bmatrix} 11(-8) - 2(8) & 11(3) - 2(-1) \\ 0(-8) + 4(8) & 0(3) + 4(-1) \end{bmatrix} = \begin{bmatrix} -104 & \ldots \\ 32 & \ldots \end{bmatrix}$

80. $\begin{bmatrix} 3(10) - 5(12) & 3(9) - 5(16) \\ -7(10) + 2(12) & -7(9) + 2(16) \end{bmatrix} = \begin{bmatrix} -30 & \ldots \\ -46 & \ldots \end{bmatrix}$

81. $\begin{bmatrix} 0.5(0) + 3(4) & 0.5(0.6) + 3(0.8) \\ 0.2(0) + 1(4) & 0.2(0.6) + 1(0.8) \end{bmatrix} = \begin{bmatrix} 12 & \ldots \\ 4 & \ldots \end{bmatrix}$

82. $\begin{bmatrix} -2(1.6) + 1.3(-4) & -2(6) + 1.3(1.9) \\ 1.5(1.6) - 3(-4) & 1.5(6) - 3(1.9) \end{bmatrix}$

$$= \begin{bmatrix} -8.4 & -9.53 \\ 14.4 & 3.3 \end{bmatrix}$$

Quiz (p. 221)

1. $\begin{bmatrix} -5 & 4 & 15 \\ 2 & -14 & 1 \end{bmatrix}$ **2.** $\begin{bmatrix} -5 & -7 \\ 0 & -1 \end{bmatrix}$

3. $\begin{bmatrix} -14 & 4 \\ -8 & -18 \end{bmatrix} + \begin{bmatrix} 12 & -6 \\ -10 & +6 \end{bmatrix} = \begin{bmatrix} -2 & \ldots \\ -18 & \ldots \end{bmatrix}$

4. $\begin{bmatrix} 4 & -6 & 10 \\ 3 & 6 & 0 \\ 9 & -4 & 5 \end{bmatrix} + \begin{bmatrix} -8 & 4 & 12 \\ 0 & -24 & 20 \\ 8 & 0 & -4 \end{bmatrix} = \begin{bmatrix} \ldots \end{bmatrix}$

5. $\begin{bmatrix} 8(3) - (-2) & 8(7) - (0) \\ 6(3) - 2(-2) & 6(7) - 2(0) \end{bmatrix} = \begin{bmatrix} 26 & 56 \\ 22 & 42 \end{bmatrix}$

6. $\begin{bmatrix} 2(1) - 1(9) + 3(4) & 2(0) - 1(-3) + 3(-6) \\ 2(1) + 4(9) + 0(4) & 2(0) + 4(-3) + 0(-6) \end{bmatrix} =$

$\begin{bmatrix} 5 & -15 \\ 38 & -12 \end{bmatrix}$

7. $-8 + 18 = 10$ **8.** $-18 + 18 = 0$

9. $(0 - 24 + 120) - (0 + 16 + 10) = 96 - 26 = 70$

10. $(-72 + 15 + 0) - (-12 + 0 - 30) = -57 + 42$

$= -15$

11. $\begin{vmatrix} -8 & 1 \\ -5 & 4 \end{vmatrix} = -32 + 5 = -27$

$x = \dfrac{\begin{vmatrix} -6 & 1 \\ 3 & 4 \end{vmatrix}}{-27} = \dfrac{-24 - 3}{-27} = 1$

$y = \dfrac{\begin{vmatrix} -8 & -6 \\ -5 & 3 \end{vmatrix}}{-27} = \dfrac{-24 - 30}{-27} = 2$

$(1, 2)$

12. $\begin{vmatrix} 3 & -2 \\ -6 & 1 \end{vmatrix} = 3 - 12 = -9$

$x = \dfrac{\begin{vmatrix} 10 & -2 \\ -7 & 1 \end{vmatrix}}{-9} = \dfrac{10 - 14}{-9} = \dfrac{4}{9}$

$y = \dfrac{\begin{vmatrix} 3 & 10 \\ -6 & -7 \end{vmatrix}}{-9} = \dfrac{-21 + 60}{-9} = \dfrac{39}{-9} = -\dfrac{13}{3}$

$\left(\dfrac{4}{9}, -\dfrac{13}{3}\right)$

13. $\begin{vmatrix} 5 & 4 \\ 3 & -6 \end{vmatrix} = -30 - 12 = -42$

$x = \dfrac{\begin{vmatrix} 12 & 4 \\ 3 & -6 \end{vmatrix}}{-42} = \dfrac{-72 - 12}{-42} = 2$

$y = \dfrac{\begin{vmatrix} 5 & 12 \\ 3 & 3 \end{vmatrix}}{-42} = \dfrac{15 - 36}{42} = \dfrac{1}{2}$

$\left(2, \dfrac{1}{2}\right)$

14. $\begin{vmatrix} 4 & 1 & 6 \\ 2 & 2 & 4 \\ -1 & -1 & 1 \end{vmatrix} = (8 - 4 - 12) - (-12 - 16 + 2) = 18$

$x = \dfrac{\begin{vmatrix} 2 & 1 & 6 \\ 1 & 2 & 4 \\ -5 & -1 & 1 \end{vmatrix}}{18} = \dfrac{(4 - 20 - 6) - (-60 - 8 + 1)}{18}$

$= \dfrac{-22 + 67}{18} = \dfrac{5}{2}$

$y = \dfrac{\begin{vmatrix} 4 & 2 & 6 \\ 2 & 1 & 4 \\ -1 & -5 & 1 \end{vmatrix}}{18}$

$= \dfrac{(4 - 8 - 60) - (-6 - 80 + 4)}{18} = \dfrac{-64 + 82}{18} = 1$

$z = \dfrac{\begin{vmatrix} 4 & 1 & 2 \\ 2 & 2 & 1 \\ -1 & -1 & -5 \end{vmatrix}}{18} = \dfrac{(-40 - 1 - 4) - (-4 - 4 - 10)}{18}$

$= \dfrac{-45 + 18}{18} = -\dfrac{3}{2}$

$\left(\dfrac{5}{2}, 1, -\dfrac{3}{2}\right)$

15. $\begin{vmatrix} 1 & 1 & 4 \\ 2 & -3 & -1 \\ -4 & 2 & 2 \end{vmatrix} = (-6 + 4 + 16) - (48 - 2 + 4)$

$= -36$

$x = \dfrac{\begin{vmatrix} 7 & 1 & 4 \\ -24 & -3 & -1 \\ 8 & 2 & 2 \end{vmatrix}}{-36}$

$= \dfrac{(-42 - 8 - 192) - (-96 - 14 - 48)}{-36}$

$= -\dfrac{242 + 158}{-36} = \dfrac{7}{3}$

$y = \dfrac{\begin{vmatrix} 1 & 7 & 4 \\ 2 & -24 & -1 \\ -4 & 8 & 2 \end{vmatrix}}{-36}$

$= \dfrac{(-48 + 28 + 64) - (384 - 8 + 28)}{-36} = \dfrac{44 - 404}{-36} = 10$

$z = \dfrac{\begin{vmatrix} 1 & 1 & 7 \\ 2 & -3 & -24 \\ -4 & 2 & 8 \end{vmatrix}}{-36}$

$= \dfrac{(-24 + 96 + 28) - (84 - 48 + 16)}{-36}$

$= \dfrac{100 - 52}{-36} = -\dfrac{4}{3}$

$\left(\dfrac{7}{3}, 10, -\dfrac{4}{3}\right)$

16. $\begin{vmatrix} 3 & 3 & -2 \\ -5 & -2 & -3 \\ 7 & 1 & 6 \end{vmatrix} = (-36 - 63 + 10) - (28 - 9 - 90)$

$= -89 + 71 = -18$

$x = \dfrac{\begin{vmatrix} -18 & 3 & -2 \\ -1 & -2 & -3 \\ 14 & 1 & 6 \end{vmatrix}}{-18}$

$= \dfrac{(216 - 126 + 2) - (56 + 54 - 18)}{-18}$

$= \dfrac{92 - 92}{-18} = 0$

$y = \dfrac{\begin{vmatrix} 3 & -18 & -2 \\ -5 & -1 & -3 \\ 7 & 14 & 6 \end{vmatrix}}{-18}$

$= \dfrac{(-18 + 378 + 140) - (14 - 126 + 540)}{-18}$

$= \dfrac{500 - 428}{-18} = \dfrac{72}{-18} = -4$

$z = \dfrac{\begin{vmatrix} 3 & 3 & -18 \\ -5 & -2 & -1 \\ 7 & 1 & 14 \end{vmatrix}}{-18}$

$= \dfrac{(-84 - 21 + 90) - (252 - 3 - 210)}{-18}$

$= \dfrac{-15 - 39}{-18} = \dfrac{-54}{-18} = 3$

$(0, -4, 3)$

17. $A = \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 5 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}[(0 + 0 + 30) - (6 + 0 + 0)]$

$= \pm\frac{1}{2}(30 - 6) = \pm\frac{1}{2}(24) = 12 \text{ ft}^2$

Lesson 4.4

Developing Concepts Activity 4.4 (p. 222)
Exploring the Concept

1. $AI = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

$BI = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}$

$CI = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix};$

When the matrix is mutliplied bv the identity matrix, the result is the original matrix.

2. $IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

$IB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}$

$IC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix};$

Yes

3. $DE = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Each prc… is the ide… matrix.

$ED = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Drawing Conclusions (p. 222)

1. $AI = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Multiplica… the identit… matrix is commutat…

$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Any matrix times the identity will result in the orig… matrix.

2. Multiplying a 2×2 matrix by $I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ resul… original matrix, just as multiplying a real number … 1 results in the original number.

3. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$

Sample answer: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

4. The product of a matrix and its inverse is the ide… matrix; the product of a nonzero real number an… reciprocal notes

5. No; $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$; you can never multiply ze… number to get the number 1 needed for the iden… matrix.