16. $\begin{vmatrix} 3 & 3 & -2 \\ -5 & -2 & -3 \\ 7 & 1 & 6 \end{vmatrix} = (-36 - 63 + 10) - (28 - 9 - 90)$

$= -89 + 71 = -18$

$x = \dfrac{\begin{vmatrix} -18 & 3 & -2 \\ -1 & -2 & -3 \\ 14 & 1 & 6 \end{vmatrix}}{-18}$

$= \dfrac{(216 - 126 + 2) - (56 + 54 - 18)}{-18}$

$= \dfrac{92 - 92}{-18} = 0$

$y = \dfrac{\begin{vmatrix} 3 & -18 & -2 \\ -5 & -1 & -3 \\ 7 & 14 & 6 \end{vmatrix}}{-18}$

$= \dfrac{(-18 + 378 + 140) - (14 - 126 + 540)}{-18}$

$= \dfrac{500 - 428}{-18} = \dfrac{72}{-18} = -4$

$z = \dfrac{\begin{vmatrix} 3 & 3 & -18 \\ -5 & -2 & -1 \\ 7 & 1 & 14 \end{vmatrix}}{-18}$

$= \dfrac{(-84 - 21 + 90) - (252 - 3 - 210)}{-18}$

$= \dfrac{-15 - 39}{-18} = \dfrac{-54}{-18} = 3$

$(0, -4, 3)$

17. $A = \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 5 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix}$

$= \pm\frac{1}{2}[(0 + 0 + 30) - (6 + 0 + 0)]$

$= \pm\frac{1}{2}(30 - 6) = \pm\frac{1}{2}(24) = 12 \text{ ft}^2$

Lesson 4.4

Developing Concepts Activity 4.4 (p. 222)
Exploring the Concept

1. $AI = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

$BI = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}$

$CI = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix};$

When the matrix is mutliplied bv the identity matrix, the result is the original matrix.

2. $IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

$IB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}$

$IC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix};$

Yes

3. $DE = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Each … is the … matr…

$ED = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Drawing Conclusions (p. 222)

1. $AI = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Multip… the iden… matrix … commu…

$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Any matrix times the identity will result in the … matrix.

2. Multiplying a 2 × 2 matrix by $I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ res… original matrix, just as multiplying a real numbe… 1 results in the original number.

3. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$

Sample answer: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

4. The product of a matrix and its inverse is the ide… matrix; the product of a nonzero real number an… reciprocal notes

5. No; $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$; you can never multiply ze… number to get the number 1 needed for the iden… matrix.

6. $\begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2a + 7c & 2b + 7d \\ a + 4c & b + 4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2a + 7c = 1$

$2b + 7d = 0$

$a + 4c = 0$

$b + 4d = 1$

$b = 1 - 4d$

$2(1 - 4d) + 7d = 0$

$-8d + 7d = -2$

$-d = -2$

$d = 2$

$b = 1 - 4 \cdot 2$

$b = -7$

$a + 4c = 0$

$a = -4c$

$2(-4c) + 7c = 1$

$-8c + 7c = 1$

$-c = 1$

$c = -1$

$a = -4(-1)$

$a = 4$

$(4, -7, -1, 2)$ $\quad \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix}$

4.4 Guided Practice (p. 227)

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. $AB = BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. *Sample answer:* To find the inverse of a 2 × 2 matrix A, first calculate *det A* and check that it is not zero. Then switch the upper left and lower right entries of *A*. Negate the lower left and the upper right entries of A. Multiply this new matrix by the reciprocal of the *det A*.

4. *Sample answer:* In order for *AX* to be defined, the number of rows in *X* must also equal the number of columns in *A*, so *X* has 2 rows. The number of columns in the product *B* must be equal the number of columns in *X*, so *X* has 2 columns.

5. $\det B = (8 - 8) = 0$; No

6. $\frac{1}{-8 + 9}\begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}$

7. $\frac{1}{3}\begin{bmatrix} -1 & -2 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} \\ 0 & -1 \end{bmatrix}$

8. $-\frac{1}{4}\begin{bmatrix} 4 & 0 \\ -6 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ \frac{3}{2} & \frac{1}{4} \end{bmatrix}$

9. $\frac{1}{\frac{65}{8}}\begin{bmatrix} \frac{1}{4} & -4 \\ 2 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{65} & -\frac{32}{65} \\ \frac{16}{65} & \frac{4}{65} \end{bmatrix}$

10. $\frac{1}{-5.5}\begin{bmatrix} 4 & -3 \\ -2.5 & 0.5 \end{bmatrix} = \begin{bmatrix} -\frac{8}{11} & \frac{6}{11} \\ \frac{5}{11} & -\frac{1}{11} \end{bmatrix}$

11. $\frac{1}{-6.08}\begin{bmatrix} 0.2 & -2 \\ -3.2 & 1.6 \end{bmatrix} = \begin{bmatrix} -0.0329 & 0.3289 \\ 0.5263 & -0.2632 \end{bmatrix}$

12. $\det D = -5 + 6 = 1$

$D^{-1} = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$

$[-71 \quad 39]\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [7 \quad 18]$ GR

$[-35 \quad 20]\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [5 \quad 5]$ EE

$[-118 \quad 69]\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [20 \quad 9]$ TI

$[-84 \quad 49]\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [14 \quad 7]$ NG

$[-95 \quad 57]\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [19 \quad 0]$ S_

GREETINGS

4.4 Practice and Applications (pp. 227-229)

13. $\frac{1}{1}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

14. $\frac{1}{2}\begin{bmatrix} 3 & -2 \\ -8 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -1 \\ -4 & 3 \end{bmatrix}$

15. $\frac{1}{-1}\begin{bmatrix} 7 & -8 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 1 & -1 \end{bmatrix}$

16. $\frac{1}{1}\begin{bmatrix} -3 & -17 \\ -1 & -6 \end{bmatrix} = \begin{bmatrix} -3 & -17 \\ -1 & -6 \end{bmatrix}$

17. $\frac{1}{1}\begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$

18. $\frac{1}{-15}\begin{bmatrix} 1 & 2 \\ 4 & -7 \end{bmatrix} = \begin{bmatrix} -\frac{1}{15} & -\frac{2}{15} \\ -\frac{4}{15} & \frac{7}{15} \end{bmatrix}$

19. $\frac{1}{2}\begin{bmatrix} 2 & 7 \\ -2 & -6 \end{bmatrix} = \begin{bmatrix} 1 & \frac{7}{2} \\ -1 & -3 \end{bmatrix}$

20. $\frac{1}{4}\begin{bmatrix} 4 & 4 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & \frac{5}{4} \end{bmatrix}$

21. $\frac{1}{6}\begin{bmatrix} 3 & 3 \\ 9 & 11 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{3}{2} & \frac{11}{6} \end{bmatrix}$

22. $\frac{1}{\frac{5}{2}}\begin{bmatrix} 1 & -\frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$

23. $\frac{1}{2}\begin{bmatrix} 10 & -2.5 \\ -8 & 2.2 \end{bmatrix} = \begin{bmatrix} 5 & -1.25 \\ -4 & 1.1 \end{bmatrix}$

24. $\frac{1}{\frac{11}{4}}\begin{bmatrix} \frac{5}{2} & -\frac{3}{4} \\ 1 & \frac{4}{5} \end{bmatrix} = \begin{bmatrix} \frac{10}{11} & -\frac{3}{11} \\ \frac{4}{11} & \frac{16}{55} \end{bmatrix}$

25. $A^{-1} = -\frac{1}{25}\begin{bmatrix} 5 & 13 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & -\frac{13}{25} \\ 0 & \frac{1}{5} \end{bmatrix}$

$$\begin{bmatrix} -\frac{1}{5} & -\frac{13}{25} \\ 0 & \frac{1}{5} \end{bmatrix}\begin{bmatrix} -5 & -13 \\ 0 & 5 \end{bmatrix}x = \begin{bmatrix} -\frac{1}{5} & -\frac{13}{25} \\ 0 & \frac{1}{5} \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} \frac{37}{25} & -\frac{1}{5} \\ -\frac{4}{5} & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{37}{25} & -\frac{1}{5} \\ -\frac{4}{5} & 0 \end{bmatrix}$$

26. $A^{-1} = \frac{1}{18}\begin{bmatrix} 2 & 1 \\ -8 & 5 \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{18} \\ -\frac{4}{9} & \frac{5}{18} \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 8 & 2 \end{bmatrix}x = \begin{bmatrix} \frac{1}{9} & \frac{1}{18} \\ -\frac{4}{9} & \frac{5}{18} \end{bmatrix}\begin{bmatrix} 17 & 20 \\ 26 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} \frac{10}{3} & \frac{10}{3} \\ -\frac{1}{3} & -\frac{10}{3} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{10}{3} & \frac{10}{3} \\ -\frac{1}{3} & -\frac{10}{3} \end{bmatrix}$$

27. $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 & 6 \\ 3 & -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} -4 & 2 & -7 \\ 3 & -1 & 5 \end{bmatrix}$$

$$x = \begin{bmatrix} -4 & 2 & -7 \\ 3 & -1 & 5 \end{bmatrix}$$

28. $A^{-1} = \frac{1}{7}\begin{bmatrix} 1 & 3 \\ -4 & -5 \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ -\frac{4}{7} & -\frac{5}{7} \end{bmatrix}\begin{bmatrix} -5 & -3 \\ 4 & 1 \end{bmatrix}x = \begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ -\frac{4}{7} & -\frac{5}{7} \end{bmatrix}\begin{bmatrix} -12 & -5 & 18 \\ 4 & -3 & -13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} 0 & -2 & -3 \\ 4 & 5 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 & -2 & -3 \\ 4 & 5 & -1 \end{bmatrix}.$$

29. $A^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -7 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{7}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$

$$\begin{bmatrix} \frac{4}{5} & -\frac{7}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix}x = \begin{bmatrix} \frac{4}{5} & -\frac{7}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} -1 & -8 \\ -3 & -24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} \frac{17}{5} & \frac{136}{5} \\ -\frac{8}{5} & -\frac{64}{5} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{17}{5} & \frac{136}{5} \\ -\frac{8}{5} & -\frac{64}{5} \end{bmatrix}$$

30. $A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix}$

$$\begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix}\begin{bmatrix} -7 & -9 \\ 4 & 5 \end{bmatrix}x = \begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix}\begin{bmatrix} -2 & 5 \\ 2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} 8 & -2 \\ -6 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 8 & -2 \\ -6 & 1 \end{bmatrix}$$

31. $A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & -\frac{1}{2} \end{bmatrix}$

$$\begin{bmatrix} 3 & -1 \\ 2 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -1 & 2 \\ -4 & 6 \end{bmatrix}x = \begin{bmatrix} 3 & -1 \\ 2 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} 11 & -2 \\ 8 & -\frac{3}{2} \end{bmatrix}$$

$$x = \begin{bmatrix} 11 & -2 \\ 8 & -\frac{3}{2} \end{bmatrix}$$

32. $A^{-1} = \frac{1}{10}\begin{bmatrix} -2 & 3 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} -0.2 & 0.3 \\ -0.6 & 0.4 \end{bmatrix}$

$$\begin{bmatrix} -0.2 & 0.3 \\ -0.6 & 0.4 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 6 & -2 \end{bmatrix}x = \begin{bmatrix} -0.2 & 0.3 \\ -0.6 & 0.4 \end{bmatrix}\begin{bmatrix} 3 & [illegible] \\ 13 & [illegible] \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x = \begin{bmatrix} 3.3 & 1.3 \\ 3.4 & -0.6 \end{bmatrix}$$

$$x = \begin{bmatrix} 3.3 & 1.3 \\ 3.4 & -0.6 \end{bmatrix}$$

33. $A^{-1} = -1\begin{bmatrix} -1 & 3 \\ -2 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & -10 \end{bmatrix}$

$$B^{-1} = -\frac{1}{19}\begin{bmatrix} -10 & -3 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{10}{19} & \frac{3}{19} \\ \frac{3}{19} & -\frac{1}{19} \end{bmatrix}$$

not inverses of each other

34.

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 5 & 2 & 3 \\ 7 & 3 & 4 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -1 & -11 & 8 \\ 1 & 7 & -5 \\ 1 & 14 & -10 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & -10 & 8 \\ 11 & 7 & -5 \\ 1 & 12 & -10 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 0.333 & 0.133 & [illegible] \\ -3.5 & -0.4 & [illegible] \\ -4.167 & -0.467 & [illegible] \end{bmatrix}$$

not inverses of each other

35. $C = \begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} 3 & -4 & 2 \\ 0 & 2 & 1 \\ 4 & -5 & 3 \end{bmatrix}$

$$D = \begin{bmatrix} 3 & -4 & 2 \\ 0 & 2 & 1 \\ 4 & -5 & 3 \end{bmatrix} \qquad D^{-1} = \begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix}$$

inverses of each other

36. $E = \begin{bmatrix} 10 & 2 & -25 \\ 4 & 1 & -10 \\ -9 & -2 & 23 \end{bmatrix}$ $E^{-1} = \begin{bmatrix} 3 & 4 & 5 \\ -2 & 5 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

$H = \begin{bmatrix} 3 & 4 & 5 \\ -2 & 5 & 0 \\ 1 & 2 & 2 \end{bmatrix}$ $H^{-1} = \begin{bmatrix} 10 & 2 & -25 \\ 4 & 1 & -10 \\ -9 & -2 & 23 \end{bmatrix}$

inverses of each other

37.

$A = \begin{bmatrix} -3 & 4 & 5 \\ 1 & 5 & 0 \\ 5 & 2 & 2 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -0.065 & -0.013 & 0.163 \\ 0.013 & 0.203 & -0.033 \\ 0.15 & -0.17 & 0.124 \end{bmatrix}$

$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

38.

$B = \begin{bmatrix} -7 & 0 & -6 \\ -4 & 1 & 3 \\ 11 & -3 & -9 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} 0 & -3 & -1 \\ 0.5 & -21.5 & -7.5 \\ -0.167 & 3.5 & 1.167 \end{bmatrix}$

$B \cdot B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

39.

$A = \begin{bmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 12 & -7 & 3 \\ -20 & 12 & -5 \\ 1.5 & -1 & 0.5 \end{bmatrix}$

$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

40. JOB WELL DONE

[10 15][2 0][23 5][12 12][0 4][15 14][5 0]

$[10\ \ 15]\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-20\ \ 55]$

$[2\ \ 0]\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [2\ \ -4]$

$[23\ \ 5]\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [13\ \ -21]$

$[12\ \ 12]\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-12\ \ 36]$

$[0\ \ 4]\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-8\ \ 20]$

$[15\ \ 14]\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-13\ \ 40]$

$[5\ \ 0]\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [5\ \ -10]$

−20, 55, 2, −4, 13, −21, −12, 36, −8, 20, −13, 40, 5, −10

41. STAY THERE

[19 20][1 25][0 20][8 5][18 5]

$[19\ \ 20]\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [39\ \ 98]$

$[1\ \ 25]\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [26\ \ 77]$

$[0\ \ 20]\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [20\ \ 60]$

$[8\ \ 5]\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [13\ \ 31]$

$[18\ \ 5]\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [23\ \ 51]$

39, 98, 26, 77, 20, 60, 13, 31, 23, 51

42. COME TO DINNER

[3 15][13 5][0 20][15 0][4 9][14 14][5 18]

$[3\ \ 15]\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-33\ \ 12]$

$[13\ \ 5]\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [37\ \ -8]$

$[0\ \ 20]\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-60\ \ 20]$

$[15\ \ 0]\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [60\ \ -15]$

$[4\ \ 9]\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-11\ \ 5]$

$[14\ \ 14]\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [14\ \ 0]$

$[5\ \ 18]\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-34\ \ 13]$

−33, 12, 37, −8, −60, 20, 60, −15, −11, 5, 14, 0, −34, 13

43. HAPPY BIRTHDAY

[8 1][16 16][25 0][2 9][18 20][8 4][1 25]

$[8\ \ 1]\begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [36\ \ -14]$

$[16\ \ 16]\begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [16\ \ 0]$

$[25\ \ 0]\begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [125\ \ -50]$

$[2\ \ 9]\begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [-26\ \ -50]$

$[18\ \ 20]\begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [10\ \ 4]$

$[8\ \ 4]\begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [24\ \ -8]$

$[1\ \ 25]\begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [-95\ \ 48]$

36, −14, 16, 0, 125, −50, −26, 14, 10, 4, 24, −8, −95, 48

Chapter 4 *continued*

44. $D = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ $\quad D^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$(-1 \quad 4) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [2 \quad 5]$ BE

$(30 \quad -41) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [19 \quad 8]$ SH

$(39 \quad -58) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \quad 1]$ TA

$(22 \quad -33) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [11 \quad 0]$ K_

$(31 \quad -46) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [16 \quad 1]$ PA

$(23 \quad -34) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [12 \quad 1]$ LA

$(1 \quad 1) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [3 \quad 5]$ CE

BESHTAK PALACE

45. $(21 \quad -31) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [11 \quad 1]$ KA

$(22 \quad -26) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [18 \quad 14]$ RN

$(-9 \quad 19) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [1 \quad 11]$ AK

$(-20 \quad 40) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [1 \quad 11]$ _T

$(-3 \quad 11) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \quad 13]$ EM

$(20 \quad -24) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [16 \quad 12]$ PL

$(10 \quad -15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \quad 0]$ E_

KARNAK TEMPLE

46. $(39 \quad -58) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \quad 1]$ TA

$(-2 \quad 12) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [8 \quad 18]$ HR

$(0 \quad 9) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [9 \quad 18]$ IR

$(-19 \quad 38) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [0 \quad 19]$ _S

$(13 \quad -9) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [17 \quad 21]$ QU

$(-16 \quad 33) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [1 \quad 18]$ AR

$(\;\; -15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \quad 0]$ E_

TAHRIR SQUARE

47. $(32 \quad -44) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \quad 1]$ TH

$(10 \quad -15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \quad 0]$ E_

$(-4 \quad 15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [7 \quad 18]$ GR

$(9 \quad -13) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \quad 1]$ EA

$(40 \quad -60) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \quad 0]$ T_

$(22 \quad -25) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [19 \quad 16]$ SP

$(7 \quad -6) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [8 \quad 9]$ HI

$(4 \quad 6) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [14 \quad 24]$ NX

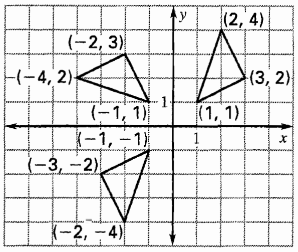
THE GREAT SPHINX

48. Egypt

49. a. $AT = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -4 & - \\ 1 & 2 & \end{bmatrix}$

$AAT = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & - \\ -1 & - \end{bmatrix}$

$T = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix}$



90° rotatio

b. *Sample answer:* Find A^{-1} and then multipl A^{-1} on the left. $A^{-1}AAT = IAT = AT$. No AT by A^{-1} on the left: $A^{-1}AT = IT = T$.

50. *Sample answer:* Write the equation in the form Find A^{-1} and multiply both sides of the equatio on the left.

51. $\frac{1}{2}\begin{vmatrix} 6 & 2 \\ -7 & -2 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ -\frac{7}{2} & -1 \end{vmatrix}$

D

52. $\begin{bmatrix}-1 & 2\\ 3 & -5\end{bmatrix}\begin{bmatrix}5 & 2\\ 3 & 1\end{bmatrix}x = \begin{bmatrix}-1 & 2\\ 3 & -5\end{bmatrix}\begin{bmatrix}4 & 43\\ 2 & 25\end{bmatrix}$

$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}x = \begin{bmatrix}0 & 7\\ 2 & 4\end{bmatrix}$

$x = \begin{bmatrix}0 & 7\\ 2 & 4\end{bmatrix}$

A

53. a. $45w - 35y = 10$

$38w - 30y = 8$

$\begin{vmatrix}45 & -35\\ 38 & -30\end{vmatrix} = -20$

$w = \dfrac{\begin{vmatrix}10 & -35\\ 8 & -30\end{vmatrix}}{-20} = \dfrac{-300 + 280}{-20} = 1$

$y = \dfrac{\begin{vmatrix}45 & 10\\ 38 & 8\end{vmatrix}}{-20} = \dfrac{360 - 380}{-20} = 1$

$45x - 35y = 15$

$38x - 30z = 14$

$\begin{vmatrix}45 & -35\\ 38 & -30\end{vmatrix} = -20$

$x = \dfrac{\begin{vmatrix}15 & -35\\ 14 & -30\end{vmatrix}}{-20} = \dfrac{-450 + 490}{-20} = -2$

$z = \dfrac{\begin{vmatrix}45 & 15\\ 38 & 14\end{vmatrix}}{-20} = \dfrac{630 - 570}{-20} = -3$

$A^{-1} = \begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix}$

b. $[45\ \ -35]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [10\ \ 15]$ JO

$[38\ \ -30]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [8\ \ 14]$ HN

$[18\ \ -18]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [0\ \ 18]$ _R

$[35\ \ -30]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [5\ \ 20]$ ET

$[81\ \ -60]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [21\ \ 18]$ UR

$[42\ \ -28]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [14\ \ 0]$ N_

$[75\ \ -55]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [20\ \ 15]$ TO

—CONTINUED—

53. —CONTINUED—

$[2\ \ -2]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [0\ \ 2]$ _B

$[22\ \ -21]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [1\ \ 19]$ AS

$[15\ \ -10]\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix} = [5\ \ 0]$ E_

JOHN RETURN TO BASE

4.4 Mixed Review (p. 229)

54. $3(11 - 4y) + 5y = 12$

$33 - 12y + 5y = 12$

$-7y = -21$

$y = 3$

$x = 11 - 4(3)$

$x = -1$

$(-1, 3)$

55. $4x - 12y = 2$

$-4x + 12y = -2$

$0 = 0$

infinitely many solutions

56. $-20x + 28y = 132$

$20x - 45y = -200$

$-17y = -68$

$y = 4$

$-5x + 7(4) = 33$

$x = -1$

$(-1, 4)$

57. $2x - 2(22 - 3z - 7x) + 9z = -10$

$2x - 44 + 6z + 14x + 9z = -10$

$16x + 15z = 34$ Eq 1

$-3x - 5(22 - 3z - 7x) - 10z = 8$

$-3x + 35x + 15z - 10z = 8 + 110$

$32x + 5z = 118$ Eq 2

$-32x - 30z = -68$ Eq 1

$32x + 5z = 118$ Eq 2

$-25z = 50$

$z = -2$

$32x + 5(2) = 118$

$32x = 128$

$x = 4$

$y = 22 - 3(-2) - 7(4)$

$y = 22 + 6 - 28$

$y = 0$

$(4, 0, -2)$

Chapter 4 *continued*

58. $-2(6 - 3z) + 3y + z = -11$

$-12 + 6z + 3y + z = -11$

$7z + 3y = 1$ Eq 1

$3(6 - 3z) - y + 2z = 13$

$18 - 9z - y + 2z = 13$

$-7z - y = -5$ Eq 2

$7z + 3y = 1$

$\underline{-7z - y = -5}$

$2y = -4$

$y = -2$

$-7z + 2 = -5$

$-7z = -7$

$z = 1$

$x = 6 - 3(1)$

$x = 3$

$(3, -2, 1)$

59. $4x - 3y + 8z = -8$

$\underline{-4x + 14y - 24z = 48}$

$11y - 16z = 40$ Eq 1

$2x + y - 4z = 4$

$\underline{-2x + 7y - 12z = 24}$

$8y - 16z = 28$ Eq 2

$11y - 16z = 40$

$\underline{-8y + 16z = -28}$

$3y = 12$

$y = 4$

$8(4) - 16z = 28$ $\quad 2x = 4 + 4(\frac{1}{4}) - 4$

$-16z = -4$ $\quad 2x = 1$

$z = \frac{1}{4}$ $\quad x = \frac{1}{2}$

$(\frac{1}{2}, 4, \frac{1}{4})$

60. $\begin{bmatrix} [illegible] & 0 & -1 \\ -[illegible] & & -1 \end{bmatrix}$

61. Not possible; the matrices have different dimensions.

62. $\begin{bmatrix} 8 & -24 & -32 \\ -8 & -64 & 0 \end{bmatrix}$ **63.** $\begin{bmatrix} 17 & -3 & -1 \\ 0 & 25 & 31 \end{bmatrix}$

64. $\begin{bmatrix} [illegible] & 2 & -2 \\ -5 & 13 & -5 \end{bmatrix}$ **65.** $\begin{bmatrix} 2 & 5 & 1 \\ 3 & 4 & 8 \end{bmatrix}$

66. $6x + 8y = 1000$

$x + y = 150$

$6(150 - y) + 8y = 1000$

$2y = 100$

$y = 50$

$x = 150 - 50$

$x = 100$

100 vegetarian, 50 chicken

Lesson 4.5

Activity (p. 230)

1. $\begin{bmatrix} 5x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$ **2.** $\begin{bmatrix} 2 & -1 \\ -4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

$5x - 4y = 8$

$x + 2y = 6$

a linear system

4.5 Guided Practice (p. 233)

1. *Sample answer:* A matrix of variables is a colu[illegible] containing only the variables of the equations i[illegible] system. A constant matrix is a column matrix c[illegible] only the constant terms of the equations in a li[illegible] tem. To solve a linear system that has been wr[illegible] matrix equation, solve for the matrix of variab[illegible] tiplying (on the left) the matrix of constants b[illegible] inverse of the coefficient matrix.

2. $X = A^{-1}B$

3. *Sample answer:* Because matrix multiplicatio[illegible] commutative, $A \times A^{-1}$ cannot be simplified t[illegible] $AA^{-1}X = X$; Therefore, the matrix equation ca[illegible] solved by multiplying both sides by A^{-1} on the [illegible]

4. $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$ **5.** $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} [illegible] \\ [illegible] \end{bmatrix}$

6. $\begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 8 \end{bmatrix}$

7. $A = \begin{bmatrix} 1 & 1 \\ 7 & 8 \end{bmatrix}$ $A^{-1} = \frac{1}{1}\begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix} = [illegible]$

$x = \begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 21 \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$

$(-5, 7)$