90. $25 = x(5x - 10)(2x - 1)$

$25 = x(10x^2 - 25x + 10)$

$25 = 10x^3 - 25x^2 + 10x$

$0 = 5x^2(2x - 5) + 5(2x - 5)$

$0 = (2x - 5)(5x^2 + 5)$

$x = 2\frac{1}{2}$

$2\frac{1}{2}$ ft $\times$ $2\frac{1}{2}$ ft $\times$ 4 ft

91. $250 = \frac{1}{3}(x - 5)^2\, 3x$

$250 = x(x^2 - 10x + 25)$

$0 = x^3 - 10x^2 + 25x - 250$

$0 = x^2(x - 10) + 25(x - 10)$

$0 = (x - 10)(x^2 + 25)$

$x = 10$

5 ft $\times$ 5 ft $\times$ 30 ft

92. $250 = (x - 5)^2 x$

$250 = (x^2 - 10x + 25)x$

$250 = x^3 - 10x^2 + 25x$

$x^3 - 10x^2 + 25x - 250 = 0$

$x^2(x - 10) + 25(x - 10) = 0$

$(x^2 + 25)(x - 10) = 0$

$x = 10$

5 in. by 5 in. by 10 in.

93. $(3x - 4)(9x^2 + 12x + 16)$ **94.** $x^3 - 8$

$27x^2 - 64$ $(x - 2)(x^2 + 2x + 4)$

C   D

95. $x^5 = 81x$

$x(x^4 - 81) = 0$

$x(x^2 - 9)(x^2 + 9) = 0$

$x(x - 3)(x + 3)(x^2 + 9) = 0$

$0, \pm 3$

D

96. *Sample answer:* If we think of the total volume equal to the 3 prisms: $a^2(a - b)$, $ab(a - b)$, and $b^2(a + b)$ Then $a(a - b)(a + b) + b^2(a + b)(a + b)(a^2 - ab + b^2) = a^3 + b^3$

97. $30x^2y + 36x^2 - 20xy - 24x$

$= 2x(15xy + 18x - 10y - 12)$

$= 2x(3x - 2)(5y + 6)$

98. $2x^7 - 127x = x(2x^6 - 127)$

6.4 Mixed Review (p. 351)

99. $\dfrac{6x^3y^9}{36x^3y^{-2}} = \dfrac{1}{6}y^{11}$ **100.** $\dfrac{5^{-2}x^2y^{-1}}{5^2xy^3} = \dfrac{x}{625y^4}$

101. $\dfrac{7^2x^{-3}y^2}{49x^{-3}y^{-2}} = y^4$

102. $f(3) = 3(3)^4 + 2(3)^3 - (3)^2 - 12(3) + 1$

$= 243 + 54 - 9 - 36 + 1$

$= 253$

103. $f(3) = 2(3)^5 - (3)^3 + 7(3) + 1$

$= 486 - 27 + 21 + 1$

$= 481$

104. $T = 4s + 6p + 8$

Math and History (p. 351)

1.

y	1	2	3	4	5
$f(y)$	2	12	36	80	150

y	6	7	8	9	10
$f(y)$	252	392	576	810	1100

2. $x^3 + x^2 = 252$

$x = 6$

3. $x^3 + 2x^2 = 288$

$\left(\dfrac{x}{2}\right)^3 + \left(\dfrac{x}{2}\right)^2 = 36$

$y = 3$

$x = \dfrac{(2)(3)}{1} = 6$

4. $3x^3 + x^2 = 90$

$27x^3 + x^2 = 810$

$(3x)^3 + (x)^2 = 810$

$y = 9$

$x = \dfrac{(1)(9)}{3} = 3$

5. $2x^3 + 5x^2 = 2500$

$\left(\dfrac{2x}{5}\right)^3 + \left(\dfrac{2x}{5}\right)^2 = 80$

$y = 4$

$x = \dfrac{(5)(4)}{2} = 10$

6. $7x^3 + 6x^2 = 1728$

$\left(\dfrac{7x}{6}\right)^3 + \left(\dfrac{7x}{6}\right)^2 = 392$

$y = 7$

$x = \dfrac{(6)(7)}{(7)} = 6$

7. $10x^3 + 3x^2 = 297$

$\left(\dfrac{10x}{3}\right)^3 + \left(\dfrac{10x}{3}\right)^2 = 1100$

$y = 10$

$x = \dfrac{(3)(10)}{10} = 3$

Lesson 6.5

6.5 Activity (p. 353)

1. $3x^2 + 4x + 10$; $3x^2 + 4x + 10 + \dfrac{15}{x - 2}$;

$x - 2\overline{\smash{\big)}\,3x^3 - 2x^2 + 2x - 5}$ $3x^2 + 4x + 10$; 15

$\underline{-3x^2 + 6x^2}$

$4x^2 + 2x$

$\underline{-4x^2 + 8x}$

$10x - 5$

$\underline{-10x + 20}$

15

Chapter 6 *continued*

2. 2 | 3   −2   2   −5
         .6   8   20
     3   4   10   15

They are equal; they match the coefficients of the quotient.

6.5 Guided Practice (p. 356)

1. For any number k, the remainder obtained when a polynomial $f(x)$ is divided by $x - k$ is the value of $f(x)$ when $x = k$.

2. Sample answer: $\dfrac{3x^3 + x^2 + 5x + 8}{3x - 4}$, $\dfrac{3x^3 + x^2 + 5x - 8}{x - 2}$

3. $x + 3; x^3 - 2x^2 - 9x + 18; x^2 - 5x + 6$

4.
$$
\begin{array}{r}
x^2 - 5x - 1 \\
2x + 3 \overline{)\, 2x^3 - 7x^2 - 17x - 3} \\
\underline{-2x^3 - 3x^2} \\
-10x^2 - 17x \\
\underline{10x^2 + 15x} \\
-2x - 3 \\
\underline{2x + 3} \\
0
\end{array}
$$

5.
$$
\begin{array}{r}
x^2 + x - 4 + \dfrac{14}{x+4} \\
x + 4 \overline{)\, x^3 + 5x^2 + 0x - 2} \\
\underline{-x^3 - 4x^2} \\
x^2 + 0x \\
\underline{-x^2 - 4x} \\
-4x - 2 \\
\underline{4x + 16} \\
14
\end{array}
$$

6.
$$
\begin{array}{r}
-3x^2 - 3x + 1 \\
x - 1 \overline{)\, -3x^3 + 0x^2 + 4x - 1} \\
\underline{3x^3 - 3x^2} \\
-3x^2 + 4x \\
\underline{3x^2 - 3x} \\
x - 1 \\
\underline{x - 1} \\
0
\end{array}
$$

7.
$$
\begin{array}{r}
-x + 2 + \dfrac{-3x + 5}{x^2 - 1} \\
x^2 - 1 \overline{)\, -x^3 + 2x^2 - 2x + 3} \\
\underline{x^3 \quad\quad - x} \\
2x^2 - 3x \\
\underline{-2x^2 \quad + 2} \\
-3x + 5
\end{array}
$$

8. −3 | 1   0   −8   3
          −3   9   −3
      1   −3   1   0
$x^2 - 3x + 1$

9. −4 | 1   0   −16   1   4
          −4   16   0   −4
      1   −4   0   1   0
$x^3 - 4x^2 + 1$

10. 3 | 1   2   15
         3   15
     1   5   30
$x + 5 + \dfrac{30}{x - 3}$

11. 2 | 1   7   −2
         2   18
     1   9   16
$x + 9 + \dfrac{16}{x - 2}$

12. 4 | 1   −8   4   48
          4   −16   −48
     1   −4   −12   0
$f(x) = (x - 4)(x^2 - 4x - 12)$
$f(x) = (x - 4)(x + 2)(x - 6)$
$4, -2, 6$

13. 10 | 2   −14   −56   −40
          20   60   40
     2   6   4   0
$f(x) = (x - 10)(2x^2 + 6x + 4)$
$\quad\quad = 2(x - 10)(x^2 + 3x + 2)$
$\quad\quad = 2(x - 10)(x + 1)(x + 2)$
$10, -1, -2$

14. $21 = -4x^3 + 25x$
$4x^3 - 25x + 21 = 0$

$$
\begin{array}{r}
4x^2 + 4x - 21 \\
x - 1 \overline{)\, 4x^3 + 0x^2 - 25x + 21} \\
\underline{-4x^3 + 4x^2} \\
4x^2 - 25x \\
\underline{-4x^2 + 4x} \\
-21x + 21 \\
\underline{21x - 21} \\
0
\end{array}
$$

$(x - 1)(4x^2 + 4x - 21) = 0$
$x = 1$ and $x = 1.85$
about 1.85 million radios

Chapter 6 *continued*

15.
$$x + 9 + \frac{13}{x - 2}$$
$$x - 2 \overline{) x^2 + 7x - 5}$$
$$\underline{-x^2 + 2x}$$
$$9x - 5$$
$$\underline{-9x + 18}$$
$$13$$

16.
$$3x + 20 + \frac{61}{x - 3}$$
$$x - 3 \overline{) 3x^2 + 11x + 1}$$
$$\underline{-3x^2 + 9x}$$
$$20x + 1$$
$$\underline{-20x + 60}$$
$$61$$

17.
$$2x - 5 + \frac{19}{x + 4}$$
$$x + 4 \overline{) 2x^2 + 3x - 1}$$
$$\underline{-2x^2 - 8x}$$
$$-5x - 1$$
$$\underline{5x + 20}$$
$$19$$

18.
$$x - 7 + \frac{11}{x + 1}$$
$$x + 1 \overline{) x^2 - 6x + 4}$$
$$\underline{-x^2 - x}$$
$$-7x + 4$$
$$\underline{7x + 7}$$
$$11$$

19.
$$x + 15 + \frac{147}{x - 10}$$
$$x - 10 \overline{) x^2 + 5x - 3}$$
$$\underline{-x^2 + 10x}$$
$$15x - 3$$
$$\underline{-15x + 150}$$
$$147$$

20.
$$x^2 - 2x - 1 - \frac{9}{x - 1}$$
$$x - 1 \overline{) x^3 - 3x^2 + x - 8}$$
$$\underline{-x^3 + x^2}$$
$$-2x^2 + x$$
$$\underline{2x^2 - 2x}$$
$$- x - 8$$
$$\underline{- x - 1}$$
$$-9$$

21.
$$2x^2 + 2 + \frac{9}{x^2 - 1}$$
$$x^2 - 1 \overline{) 2x^4 + 0x^3 + 0x^2 + 0x + 7}$$
$$\underline{-2x^4 + 2x^2}$$
$$2x^2 + 7$$
$$\underline{-2x^2 + 2}$$
$$9$$

22.
$$x + 8 - \frac{8x + 24}{x^2 + 5}$$
$$x^2 + 5 \overline{) x^3 + 8x^2 - 3x + 16}$$
$$\underline{-x^3 - 5x}$$
$$8x^2 - 8x + 16$$
$$\underline{-8x^2 - 40}$$
$$-8x - 24$$

23.
$$3x - 4 + \frac{5}{2x + 3}$$
$$2x + 3 \overline{) 6x^2 + x - 7}$$
$$\underline{-6x^2 - 9x}$$
$$-8x - 7$$
$$\underline{8x + 12}$$
$$5$$

24.
$$10x + 7 + \frac{5}{x^2 + 2x}$$
$$x^2 + 2x \overline{) 10x^3 + 27x^2 + 14x + 5}$$
$$\underline{-10x^3 - 20x^2}$$
$$7x^2 + 14x$$
$$\underline{-7x^2 - 14x}$$
$$0 + 5$$

25.
$$5x^2 - x + 3$$
$$x^2 + 3x \overline{) 5x^4 + 14x^3 + 0x^2 + 9x}$$
$$\underline{-5x^4 - 15x^3}$$
$$-x^3 + 0x^2$$
$$\underline{x^3 + 3x^2}$$
$$3x^2 + 9x$$
$$\underline{-3x^2 - 9x}$$
$$0$$

26.
$$2x - \frac{9}{x^3 + x^2 - 5}$$
$$x^3 + x^2 - 5 \overline{) 2x^4 + 2x^3 + 0x^2 - 10x - 9}$$
$$\underline{-2x^4 - 2x^3 + 10x}$$
$$0 - 9$$

27.
$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & -6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & -12 \end{array}$$
$$x^2 + 2x - 3 - \frac{12}{x - 2}$$

Left column

```
     1     0    -14     8
                -4     16    -8
    ─────────────────────────
     1    -4      2     0
```
$-4x + 2$

```
     4     5    -4
          -4    -1
    ─────────────────
     4     1    -5
```
$-1 - \dfrac{5}{x+1}$

30.
```
2 | 1    -4     3
         2    -4
   ────────────────
   1    -2    -1
```
$x - 2 - \dfrac{1}{x-2}$

```
     2     7     8
           4    22
    ─────────────────
     2    11    30
```
$2 - 11 + \dfrac{30}{x-2}$

32.
```
6 | 3   -10     0
         18    48
   ────────────────
   3     8    48
```
$3x + 8 + \dfrac{48}{x-6}$

```
     1     0    10
          -4    16
    ─────────────────
     1    -4    26
```
$-\dfrac{26}{x+4}$

34.
```
-3 | 1     0     3
         -3     9
    ────────────────
    1    -3    12
```
$x - 3 + \dfrac{12}{x+3}$

```
    10     5     4     0    -9
         -10     5    -9     9
    ───────────────────────────
    10    -5     9    -9     0
```
$5x^2 + 9x - 9$

```
          -6     0   -40    33
           7     7    49    63
    ───────────────────────────
           1     7     9    96
```
$x^2 + 7x + 9 + \dfrac{96}{x-7}$

```
     2    -6     1    -3    -3
           6     0     3     0
    ───────────────────────────
     2     0     1     0    -3
```
$-x - \dfrac{3}{x-3}$

```
     4     5     2     0    -1
          -4    -1    -1     1
    ───────────────────────────
     4     1     1    -1     0
```
$-x^2 - x - 1$

```
          -5    -2    24
          -2    14   -24
    ───────────────────────
          -7    12     0
```
$= (x - 2)(x^2 - 7x + 12)$
$= (x - 2)(x - 3)(x - 4)$

```
          -3   -16   -12
           6    18    12
    ───────────────────────
     3     2     0
```
$= (x - 6)(x^2 + 3x + 2)$
$= (x - 6)(x + 1)(x + 2)$

Middle column

41.
```
10 | 1   -12    12    80
          10   -20   -80
    ────────────────────────
    1    -2    -8     0
```
$f(x) = (x^2 - 2x - 8)(x - 10)$
$\quad = (x - 4)(x + 2)(x - 10)$

42.
```
9 | 1   -18    95   -126
         9    -81    126
   ─────────────────────────
   1    -9    14      0
```
$f(x) = (x^2 - 9x + 14)(x - 9)$
$\quad = (x - 7)(x - 2)(x - 9)$

43.
```
-5 | 1    -1   -21    45
          -5    30   -45
    ────────────────────────
    1    -6     9     0
```
$f(x) = (x + 5)(x^2 - 6x + 9)$
$\quad = (x + 5)(x - 3)(x - 3)$

44.
```
8 | 1   -11    14    80
         8    -24   -80
   ─────────────────────────
   1    -3   -10     0
```
$f(x) = (x - 8)(x^2 - 3x - 10)$
$\quad = (x - 8)(x - 5)(x + 2)$

45.
```
1 | 4    -4    -9     9
         4     0    -9
   ─────────────────────
   4     0    -9     0
```
$f(x) = (x - 1)(4x^2 - 9)$
$\quad = (x - 1)(2x - 3)(2x + 3)$

46.
```
-6 | 2     7   -33   -18
          72    30    18
    ────────────────────────
    2    -5    -3     0
```
$f(x) = (x + 6)(2x^2 - 5x - 3)$
$\quad = (x + 6)(2x + 1)(x - 3)$

47.
```
-2 | 9    10   -17    -2
         -18    16     2
    ────────────────────────
    9    -8    -1     0
```
$f(x) = (x + 2)(9x^2 - 8x - 1)$
$\quad = (x + 2)(9x + 1)(x - 1)$
$-2, -\frac{1}{9}, 1$

48.
```
-14 | 1    11   -150   -1512
           -14    42    1512
     ───────────────────────────
     1    -3   -108      0
```
$f(x) = (x + 14)(x^2 - 3x - 108)$
$\quad = (x + 14)(x - 12)(x + 9)$
$-14, 12, -9$

Right column (cut off)

49. 4

$f(x$

$f(x$

4.

50. 8

$f(x$

$f(x$

51. 9

$f(x$

9.

52. -5

$f(x$

-5

53. -3

$f(x$

-3

54. 6

$f(x$

6.

49. $4 \underline{\smash{\big|}\ 2 \quad 3 \quad -39 \quad -20}$

$\ \cdot \quad 8 \quad 44 \quad 20$

$\ \overline{2 \quad 11 \quad 5 \quad 0}$

$f(x) = (x - 4)(2x^2 + 11x + 5)$

$f(x) = (x - 4)(2x + 1)(x + 5)$

$4, -\frac{1}{2}, -5$

50. $8 \underline{\smash{\big|}\ 15 \quad -119 \quad -10 \quad 16}$

$\ 120 \quad 8 \quad -16$

$\ \overline{15 \quad 1 \quad -2 \quad 0}$

$f(x) = (x - 8)(15x^2 + x - 2)$

$f(x) = (x - 8)(5x + 2)(3x - 1)$

$8, -\frac{2}{5}, \frac{1}{3}$

51. $9 \underline{\smash{\big|}\ 1 \quad -14 \quad 47 \quad -18}$

$\ 9 \quad -45 \quad 18$

$\ \overline{1 \quad -5 \quad 2 \quad 0}$

$f(x) = (x - 9)(x^2 - 5x + 2)$

$9, \dfrac{5 \pm \sqrt{17}}{2}$

52. $-5 \underline{\smash{\big|}\ 4 \quad 9 \quad -52 \quad 15}$

$\ -20 \quad 55 \quad -15$

$\ \overline{4 \quad -11 \quad 3 \quad 0}$

$f(x) = (x + 5)(x^2 - 11x + 3)$

$-5, \dfrac{11 \pm \sqrt{73}}{8}$

53. $-3 \underline{\smash{\big|}\ 1 \quad 1 \quad 2 \quad 24}$

$\ -3 \quad 6 \quad -24$

$\ \overline{1 \quad -2 \quad 8 \quad 0}$

$f(x) = (x + 3)(x^2 - 2x + 8)$

$-3, 1 \pm \sqrt{7}i$

54. $6 \underline{\smash{\big|}\ 5 \quad -27 \quad -17 \quad -6}$

$\ 30 \quad 18 \quad 6$

$\ \overline{5 \quad 3 \quad 1 \quad 0}$

$f(x) = (x - 6)(5x^2 + 3x + 1)$

$6, \dfrac{-3 \pm i\sqrt{11}}{10}$

55.

$$\begin{array}{r} 3x^2 + 5x - 50 \\ x + 1 \overline{\smash{)}\ 3x^3 + 8x^2 - 45x - 50} \\ \underline{-3x^3 - 3x^2} \\ 5x^2 - 45x \\ \underline{-5x - 5x} \\ -50x - 50 \\ \underline{50x + 50} \\ 0 \end{array}$$

$$\begin{array}{r} 3x - 10 \\ x + 5 \overline{\smash{)}\ 3x^2 + 5x - 50} \\ \underline{-3x^2 - 15x} \\ -10x - 50 \\ \underline{10x + 50} \\ 0 \end{array}$$

$f(x) = (x + 1)(x + 5)(3x - 10)$

56.

$$\begin{array}{r} 2x^2 + 15x + 25 \\ x + 1 \overline{\smash{)}\ 2x^3 + 17x^2 + 40x + 25} \\ \underline{-2x^3 - 2x^2} \\ 15x^2 + 40x \\ \underline{-15x^2 - 15x} \\ 25x + 25 \\ \underline{-25x - 25} \\ 0 \end{array}$$

$$\begin{array}{r} 2x + 5 \\ x + 5 \overline{\smash{)}\ 2x^2 + 15x + 25} \\ \underline{-2x^2 - 10x} \\ 5x + 25 \\ \underline{-5x - 25} \\ 0 \end{array}$$

$f(x) = (x + 1)(2x + 5)(x + 5)$

57. $x^3 + x^2 - 5x = -x^2 - 4x + 2$

$x^3 + 2x^2 - x - 2 = 0$

$1 \underline{\smash{\big|}\ 1 \quad 2 \quad -1 \quad -2}$

$\ 1 \quad 3 \quad 2$

$\ \overline{1 \quad 3 \quad 2 \quad 0}$

$(x - 1)(x^2 + 3x + 2) = 0$

$(x - 1)(x + 2)(x + 1) = 0$

x at $1, -2, -1$

$(1, -3), (-1, 5), (-2, 6)$

58. $-6x^2 + 6x + 3 = -x^2 + 7x - 2$

$x^3 - 5x^2 - x + 5 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & -1 & 5 \\ & & 1 & -4 & -5 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$(x - 1)(x^2 - 4x - 5) = 0$

$(x - 1)(x - 5)(x + 1) = 0$

x at $1, 5, -1$

$(1, 4), (5, 8), (-1, -10)$

59. $5x^3 - 3x^2 + 21x - 8$; I multiplied $5x^2 - 13x + 47$ by $x - 2$ and added -102.

60. $62.5 = (100 - 8x^2)x - 25x$

$62.5 = 100x - 8x^3 - 25x$

$8x^3 - 75x + 62.5 = 0$

$$\begin{array}{r|rrrr} 2.5 & 8 & 0 & -75 & 62.5 \\ & & 20 & 50 & -62.5 \\ \hline & 8 & 20 & -25 & 0 \end{array}$$

$(x - 2.5)(8x^2 + 20x - 25) = 0$

$x = 2.5$ or $x = 0.92$

about 0.92 million cameras

61.
$$-8.4x + 580 \overline{)\begin{array}{l} 0.0031x^2 + 0.158x + 11.155 \\ -0.026x^3 + 0.47x^2 - 2.2x + 72 \end{array}}$$

$$\begin{array}{l} \underline{0.026x^3 - 1.80x^2} \\ -1.33x^2 - 2.2x \\ \underline{+1.33x^2 - 91.5x} \\ -93.7x + 72 \\ \underline{93.7x - 6469.76} \\ -6398 \end{array}$$

$C = 0.0031x^2 + 0.158x + 11.155 - \dfrac{6398}{-8.4x + 580}$

$C = 0.0031(10)^2 + 1.58 + 11.155 - \dfrac{6398}{496}$

$C = 0.31 + 1.58 + 11.155 - 12.90$

$C = 0.145$

about 145 million cars

62. *Sample answer:* $\quad -1.1686x^2 + 137.4713x - 13,097.384 + \dfrac{3,240,124}{2.61x + 247}$

$$2.61x + 247 \overline{)\; -3.05x^3 + 70.2x^2 - 225x + 5070}$$

$$\begin{array}{l} \underline{3.05x^3 + 288.6x^2} \\ 358.8x^2 - 225x \\ \underline{-358.8x^2 - 33959x} \\ -34184x + 5070 \\ \underline{34184x + 3235054} \\ 3,240,124 \end{array}$$

about $20.50

63.
$$\begin{array}{r|rrrr} 3 & 1 & 0 & -9 & 5 \\ & & 3 & 9 & 0 \\ \hline & 1 & 3 & 0 & 5 \end{array}$$

$x^2 + 3x + \dfrac{5}{x - 3}$

C

64. $2x^3 - 19x^2 - 20x + 100$

$$2x + 5 \overline{)\begin{array}{l} x^2 - 12x + 20 \\ 2x^3 - 19x^2 - 20x + 100 \end{array}}$$

$$\begin{array}{l} \underline{-2x^3 - 5x^2} \\ -24x^2 - 20x \\ \underline{24x^2 + 60x} \\ 40x + 100 \\ \underline{-40x - 100} \\ 0 \end{array}$$

E

65.
$$2x + 1 \overline{)\begin{array}{l} 6x^2 - 7x + 6 - \dfrac{4}{2x + 1} \\ 12x^3 - 8x^2 + 5x + 2 \end{array}}$$

$$\begin{array}{l} \underline{-12x^3 - 6x^2} \\ -14x^2 + 5x \\ \underline{14x^2 + 7x} \\ 12x + 2 \\ \underline{-12x - 6} \\ -4 \end{array}$$

$$3x + 1 \overline{)\begin{array}{l} 4x^2 - 4x + 3 - \dfrac{1}{3x + 1} \\ 12x^3 - 8x^2 + 5x + 2 \end{array}}$$

$$\begin{array}{l} \underline{-12x^3 - 4x^2} \\ -12x^2 + 5x \\ \underline{12x^2 + 4x} \\ 9x + 2 \\ \underline{-9x - 3} \\ -1 \end{array}$$

—CONTINUED—

Chapter 6 *continued*

65. —CONTINUED—

$$3x^2 - \frac{11}{4}x + \frac{31}{16} + \frac{1}{16(4x+1)}$$

$$4x + 1 \overline{) \, 12x^3 - 8x^2 + 5x + 2}$$

$$\underline{-12x^3 - 3x^2}$$

$$-11x^2 + 5x$$

$$\underline{11x^2 + \tfrac{11}{4}x}$$

$$\tfrac{31}{4}x + 2$$

$$\underline{-\tfrac{31}{4}x - \tfrac{31}{16}}$$

$$\tfrac{1}{16}$$

$$-\tfrac{1}{2} \begin{array}{|rrrr} 12 & -8 & 5 & 2 \\ & -6 & 7 & -6 \\ \hline 12 & -14 & 12 & -4 \end{array} \qquad -\tfrac{1}{3} \begin{array}{|rrrr} 12 & -8 & 5 & 2 \\ & & -4 & 4 & -3 \\ \hline 12 & -12 & 9 & -1 \end{array}$$

$$-\tfrac{1}{4} \begin{array}{|rrrr} 12 & -8 & 5 & 2 \\ & -3 & \tfrac{11}{4} & -\tfrac{31}{16} \\ \hline 12 & -11 & \tfrac{31}{4} & \tfrac{1}{16} \end{array}$$

The remainders are all the same, but the coefficients are 2, 3, and 4 times larger with synthetic division.

6.5 Mixed Review (p.358)

66. $6 + 7(-2) \le -8$

$-8 \le -8$ yes

$-2 + 7(-3) \le -8$

$-2 - 21 \le -8$

$-23 \le -8$ yes

67. $2(-2) + 5(4) \ge 1$

$-4 + 20 \ge 1$

$16 \ge 1$ yes

$2(8) + 5(-3) \ge 1$

$16 - 15 \ge 1$

$1 \ge 1$ yes

68. $9(-1) - 4(-4) > 7$

$-9 + 16 > 7$

$7 > 7$ no

$9(2) - 4(2) > 7$

$18 - 8 > 7$

$10 > 7$ yes

69. $-3(2) - 2(0) < -6$

$-6 < -6$ no

$-3(1) - 2(4) < -6$

$-3 - 8 < -6$

$-11 < -6$ yes

70. $x = \dfrac{5 \pm \sqrt{25 - 12}}{2}$

$x = \dfrac{5 \pm \sqrt{13}}{2}$

71. $x = \dfrac{8 \pm \sqrt{64 - 12}}{2}$

$x = 4 \pm \sqrt{13}$

72. $x = \dfrac{10 \pm \sqrt{100 - 60}}{2}$

$x = 5 \pm \sqrt{10}$

73. $x = \dfrac{7 \pm \sqrt{49 - 16}}{8}$

$x = \dfrac{7 \pm \sqrt{33}}{8}$

74. $x = \dfrac{9 \pm \sqrt{81 + 48}}{-12}$

$x = -\dfrac{9 \pm \sqrt{129}}{12}$ or

$x = \dfrac{-3}{4} \pm \dfrac{\sqrt{129}}{12}$

75. $x = \dfrac{-1 \pm \sqrt{1 + 40}}{10}$

$x = \dfrac{-1 \pm \sqrt{41}}{10}$

76. $x = \dfrac{-3 \pm \sqrt{9 - 40}}{4}$

$x = \dfrac{-3 \pm i\sqrt{31}}{4}$

77. $x = \dfrac{1 \pm \sqrt{1 - 160}}{-10}$

$x = \dfrac{1 \pm i\sqrt{159}}{-10}$

78. $x = \dfrac{-3 \pm \sqrt{9 - 12}}{6} = -\dfrac{3 \pm i\sqrt{3}}{6}$ or

$x = \dfrac{-1}{2} \pm \dfrac{i\sqrt{3}}{6}$

79. $(x^2 - 3x + 8) - (x^2 + x - 1) = -4x + 9$

80. $(14x^2 - 15x + 3) + (11x - 7) = 14x^2 - 4x - 4$

81. $(8x^3 - 1) - (22x^3 + 2x^2 - x - 5) = -14x^3 - 2x^2 + x + 4$

82. $(x + 5)(x^2 - x + 5) = x^3 + 4x^2 + 25$

83. $c + v = 120$

$24c + 21v = 2766$

$24c + 21(120 - c) = 2766$

$24c - 21c = 246$

$3c = 246$

$c = 82$

Lesson 6.6

6.6 Guided Practice (p. 362)

1. constant term, leading coefficient

2. a. yes; coefficients are all integers

  b. no; coefficients are not all integers

  c. no; coefficients are not all integers

3. Make a graph **4.** $\pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$

5. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm 36, \pm 72$

6. $\pm\tfrac{1}{2}, \pm\tfrac{3}{2}, \pm\tfrac{5}{2}, \pm\tfrac{15}{2}, \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

7. $\pm\tfrac{1}{5}, \pm\tfrac{2}{5}, \pm 1, \pm 2, \pm 5, \pm 10$

8. $1 \begin{array}{|rrr} 1 & -3 & -6 & 8 \\ & 1 & -2 & -8 \\ \hline 1 & -2 & -8 & 0 \end{array}$

$f(x) = (x^2 - 2x - 8)(x - 1)$

$\quad = (x - 4)(x + 2)(x - 1)$

$x = 4, -2, 1$

9. $1 \begin{array}{|rrr} 1 & 4 & -1 & -4 \\ & 1 & 5 & 4 \\ \hline 1 & 5 & 4 & 0 \end{array}$

$f(x) = (x - 1)(x^2 + 5x + 4)$

$\quad = (x - 1)(x + 1)(x + 4)$

$x = 1, -1, -4$

2 -5 -2 5
 2 -3 -5
----
2 -3 -5 0

$f(x) = (x - 1)(2x^2 - 3x - 5)$

$\quad = (x - 1)(2x - 5)(x + 1)$

$x = 1, \frac{5}{2}, -1$

2 -1 -15 18
 4   6 -18
----
2 3 -9 0

$f(x) = (x - 2)(2x^2 + 3x - 9)$

$\quad = (x - 2)(x + 3)(2x - 3)$

$x = 2, -3, \frac{3}{2}$

12. 1 | 1 4 1 -6
 1 5 6
----
1 5 6 0

$f(x) = (x - 1)(x^2 + 5x + 6)$

$\quad = (x - 1)(x + 3)(x + 2)$

$x = 1, -2, -3$

13. 1 | 1 5 -1 -5
 1 6 5
----
1 6 5 0

$f(x) = (x - 1)(x^2 + 6x + 5)$

$\quad = (x - 1)(x + 5)(x + 1)$

$x = 1, -5, -1$

14. $18 = \frac{1}{3}x^2(x + 3)$

$54 = x^3 + 3x^2$

$0 = x^3 + 3x^2 - 54$

3 | 1 3 0 -54
 3 18 54
----
1 6 18 0

$0 = (x - 3)(x^2 + 6x + 18)$

$x = 3$

3 in. by 3 in. by 6 in.

6.6 Practice and Applications (pp. 362–364)

15. $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$ **16.** $\pm1, \pm\frac{1}{2}$

17. $\pm1, \pm2, \pm4, \pm8, \pm16, \pm\frac{1}{2}$

18. $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm10, \pm12, \pm15, \pm20, \pm30, \pm60$
$= \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

19. $\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

20. $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

21. $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$

22. $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{3}, \pm\frac{5}{3}$

23. -2 | 1 7 -4 -28
 -2 -10 28
----
1 5 -14 0

$f(x) = (x + 2)(x^2 + 5x - 14)$

$\quad = (x + 2)(x - 2)(x + 7)$

$x = -2, 2$

24. 1 | 1 5 2 -8
 1 6 8
----
1 6 8 0

$f(x) = (x - 1)(x^2 + 6x + 8)$

$\quad = (x - 1)(x + 2)(x + 4)$

$x = 1, -2$

25. -1 | 1 3 -7 -27 -18
 -1 -2 9 18
----
1 2 -9 -18 0

-2 | 1 2 -9 -18
 -2 0 18
----
1 0 -9 0

$f(x) = (x + 1)(x - 2)(x^2 - 9)$

$\quad = (x + 1)(x + 2)(x - 3)(x + 3)$

$x = -1, -2$

26. 1 | 2 -9 8 9 -10
 2 -7 1 10
----
2 -7 1 10 0

1 | 2 -7 1 10
 -2 9 -10
----
2 -9 10 0

2 | 2 -9 10
 4 -10
----
2 -5 0

$f(x) = (x - 1)(x + 1)(x - 2)(2x - 5)$

$x = 1, -1, 2$

27. 1 | 1 3 3 -3 -4
 1 4 7 4
----
1 4 7 4 0

-1 | 1 4 7 4
 -1 -3 -4
----
1 3 4 0

$f(x) = (x - 1)(x + 1)(x^2 + 3x + 4)$

$x = 1, -1$

28. none   **29.** none

30. 2 | 1 1 -11 10
 2 6 -10
----
1 3 -5 0

$f(x) = (x - 2)(x^2 + 3x - 5)$

$x = 2$

Chapter 6 *continued*

31.
$$
\begin{array}{r|rrrrrrr}
-2 & 1 & 0 & -2 & 0 & -11 & 0 & 12 \\
   &   & -2 & 4 & -4 & 8 & 6 & -12 \\
\hline
   & 1 & -2 & 2 & -4 & -3 & 6 & 0
\end{array}
$$

$$
\begin{array}{r|rrrrrr}
2 & 1 & -2 & 2 & -4 & -3 & 6 \\
  &   & 2 & 0 & 4 & 0 & -6 \\
\hline
  & 1 & 0 & 2 & 0 & -3 & 0
\end{array}
$$

$$
\begin{array}{r|rrrrr}
1 & 1 & 0 & 2 & 0 & -3 \\
  &   & 1 & 1 & 3 & 3 \\
\hline
  & 1 & 1 & 3 & 3 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
-1 & 1 & 1 & 3 & 3 \\
   &   & -1 & 0 & -3 \\
\hline
   & 1 & 0 & 3 & 0
\end{array}
$$

$x = \pm 2, \pm 1$

32.
$$
\begin{array}{r|rrrrrr}
2 & 1 & -1 & -2 & -1 & 1 & 2 \\
  &   & 2 & 2 & 0 & -2 & -2 \\
\hline
  & 1 & 1 & 0 & -1 & -1 & 0
\end{array}
$$

$$
\begin{array}{r|rrrrr}
-1 & 1 & 1 & 0 & -1 & -1 \\
   &   & -1 & 0 & 0 & 1 \\
\hline
   & 1 & 0 & 0 & -1 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
1 & 1 & 0 & 0 & -1 \\
  &   & 1 & 1 & 1 \\
\hline
  & 1 & 1 & 1 & 0
\end{array}
$$

$f(x) = (x - 1)(x + 1)(x - 2)(x^2 + x + 1)$
$x = \pm 1, 2$

33.
$$
\begin{array}{r|rrrr}
10 & 1 & -8 & -23 & 30 \\
   &   & 10 & 20 & -30 \\
\hline
   & 1 & 2 & -3 & 0
\end{array}
$$

$f(x) = (x - 10)(x^2 + 2x - 3)$
$ = (x - 10)(x + 3)(x - 1)$
$x = 10, -3, 1$

34.
$$
\begin{array}{r|rrrr}
-1 & 1 & 2 & -11 & -12 \\
   &   & -1 & -1 & 12 \\
\hline
   & 1 & 1 & -12 & 0
\end{array}
$$

$f(x) = (x + 1)(x^2 + x - 12)$
$ = (x + 1)(x + 4)(x - 3)$
$x = -1, -4, 3$

35.
$$
\begin{array}{r|rrrr}
-2 & 1 & -7 & 2 & 40 \\
   &   & -2 & 18 & -40 \\
\hline
   & 1 & -9 & 20 & 0
\end{array}
$$

$f(x) = (x + 2)(x^2 - 9x + 20)$
$ = (x + 2)(x - 5)(x - 4)$
$x = -2, 5, 4$

36.
$$
\begin{array}{r|rrrr}
-1 & 1 & 1 & -2 & -2 \\
   &   & -1 & 0 & 2 \\
\hline
   & 1 & 0 & -2 & 0
\end{array}
$$

$f(x) = (x + 1)(x^2 - 2)$
$x = -1, \pm\sqrt{2}$

37.
$$
\begin{array}{r|rrrr}
3 & 1 & -5 & -18 & 72 \\
  &   & 3 & -6 & -72 \\
\hline
  & 1 & -2 & -24 & 0
\end{array}
$$

$f(x) = (x - 3)(x^2 - 2x - 24)$
$ = (x - 3)(x - 6)(x + 4)$
$x = 3, 6, -4$

38.
$$
\begin{array}{r|rrrr}
2 & 1 & 9 & -4 & -36 \\
  &   & 2 & 22 & 36 \\
\hline
  & 1 & 11 & 18 & 0
\end{array}
$$

$f(x) = (x - 2)(x^2 + 11x + 18)$
$ = (x - 2)(x + 9)(x + 2)$
$x = 2, -2, -9$

39.
$$
\begin{array}{r|rrrrr}
2 & 1 & -5 & 7 & 3 & -10 \\
  &   & 2 & -6 & 2 & 10 \\
\hline
  & 1 & -3 & 1 & 5 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
-1 & 1 & -3 & 1 & 5 \\
   &   & -1 & 4 & -5 \\
\hline
   & 1 & -4 & 5 & 0
\end{array}
$$

$f(x) = (x - 2)(x + 1)(x^2 - 4x + 5)$
$x = 2, -1$

40.
$$
\begin{array}{r|rrrrr}
2 & 1 & 1 & 1 & -9 & -10 \\
  &   & 2 & 6 & 14 & 10 \\
\hline
  & 1 & 3 & 7 & 5 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
-1 & 1 & 3 & 7 & 5 \\
   &   & -1 & -2 & -5 \\
\hline
   & 1 & 2 & 5 & 0
\end{array}
$$

$f(x) = (x - 2)(x + 1)(x^2 + 2x + 5)$
$x = 2, -1$

41.
$$
\begin{array}{r|rrrrr}
1 & 1 & 1 & -11 & -9 & 18 \\
  &   & 1 & 2 & -9 & -18 \\
\hline
  & 1 & 2 & -9 & -18 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
-2 & 1 & 2 & -9 & -18 \\
   &   & -2 & 0 & 18 \\
\hline
   & 1 & 0 & -9 & 0
\end{array}
$$

$f(x) = (x - 1)(x + 2)(x^2 - 9)$
$ = (x - 1)(x + 2)(x - 3)(x + 3)$
$x = 1, -2, 3, -3$

```
        -3    6   -2  -12
         2   -2    8   12
       ───────────────────
    1   -1    4    6    0
```

```
-   1   -1    4    6
        -1    2   -6
      ───────────────
    1   -2    6    0
```

$f(x) = (x-2)(x+1)(x^2-2x+6)$

$x = 2, -1$

43.
```
-2  1    1   -9   -5    0  -36
        -2    2   14  -18   36
       ────────────────────────
    1   -1   -7    9  -18    0
```

```
3   1   -1   -7    9  -18
         3    6   -3   18
       ───────────────────
    1    2   -1    6    0
```

```
-3  1    2   -1    6
        -3    3   -6
       ──────────────
    1   -1    2    0
```

$f(x) = (x+2)(x+3)(x-3)(x^2-x+2)$

$x = -2, 3, -3$

44.
```
-3  1   -1   -7   11   -8   12
        -3   12  -15   12  -12
       ────────────────────────
    1   -4    5   -4    4    0
```

```
2   1   -4    5   -4    4
         2   -4    2   -4
       ───────────────────
    1   -2    1   -2    0
```

$f(x) = (x+3)(x-2)(x^3-2x^2+x-2)$

$x = -3, 2$

45.
```
        -12   -1   15
          6   -9  -15
       ───────────────
        -6  -10    0
```

```
         4   -6  -10
             10   10
       ──────────────
         4    4    0
```

$x = \frac{5}{2}, \frac{5}{2}, -1$

46.
```
-    -3   20  -36   16
        -12   32  -16
       ───────────────
     -3    8   -4    0
```

```
2    -3    8   -4
         -6    4
       ──────────
     -3    2    0
```

$x = -4, 2, \frac{2}{3}$

47.
```
-2 |  2    4   -2   -4
          -4    0    4
      ─────────────────
      2    0   -2    0
```

$f(x) = (x+2)(2x^2-2)$

$x = -2, \pm1$

48.
```
4 |  2   -5  -14    8
          8   12   -8
     ─────────────────
     2    3   -2    0
```

$f(x) = (x-4)(2x^2+3x-2)$

$= (x-4)(2x-1)(x+2)$

$x = 4, \frac{1}{2}, -2$

49.
```
2 |  2   -5   -1    6
          4   -2   -6
     ─────────────────
     2   -1   -3    0
```

$f(x) = (x-2)(2x^2-x-3)$

$= (x-2)(2x-3)(x+1)$

$x = 2, \frac{3}{2}, -1$

50.
```
-1/2 | 2    1  -50  -25
           -1    0   25
       ────────────────
       2    0  -50    0
```

$f(x) = \left(x+\frac{1}{2}\right)(2x^2-50)$

$x = -\frac{1}{2}, +5, -5$

51.
```
1/2 | 2   -1  -32   16
           1    0  -16
      ─────────────────
      2    0  -32    0
```

$f(x) = \left(x-\frac{1}{2}\right)(2x^2-32)$

$x = \frac{1}{2}, 4, -4$

52.
```
-3 | 3   12    3  -18
         -9   -9   18
     ─────────────────
     3    3   -6    0
```

$f(x) = (x+3)(3x^2+3x-6)$

$= (x+3)(x+2)(3x-3)$

$x = -3, -2, 1$

53.
```
1 | 2    3   -3    3   -5
         2    5    2    5
    ──────────────────────
    2    5    2    5    0
```

```
-5/2 | 2    5    2    5
          -5    0   -5
       ────────────────
       2    0    2    0
```

$f(x) = (x-1)\left(x+\frac{5}{2}\right)(2x^2+2)$

$x = 1, -\frac{5}{2}$

54. $x = \dfrac{-1 \pm \sqrt{61}}{6}$

$x = \dfrac{3 \pm \sqrt{5}}{2}$

55.

56.

57.

58.

59.

55. -1 | 2   1 -1 -1 -1

$$\begin{array}{r|rrrrr} -1 & 2 & 1 & -1 & -1 & -1 \\ & & -2 & 1 & 0 & 1 \\ \hline & 2 & -1 & 0 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & 0 & -1 \\ & & 2 & 1 & 1 \\ \hline & 2 & 1 & 1 & 0 \end{array}$$

$f(x) = (x - 1)(x + 1)(2x^2 + x + 1)$

$x = 1, -1$

56.

$$\begin{array}{r|rrrrr} -2 & 3 & 11 & 11 & 1 & -2 \\ & & -6 & -10 & -2 & 2 \\ \hline & 3 & 5 & 1 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 3 & 5 & 1 & -1 \\ & & -3 & -2 & 1 \\ \hline & 3 & 2 & -1 & 0 \end{array}$$

$f(x) = (x + 2)(x + 1)(3x^2 + 2x - 1)$

$\qquad = (x + 2)(x + 1)(3x - 1)(x + 1)$

$x = -2, -1, \frac{1}{3}$

57.

$$\begin{array}{r|rrrrrr} -\frac{1}{2} & 2 & 1 & 0 & 0 & -32 & -16 \\ & & -1 & 0 & 0 & 0 & 16 \\ \hline & 2 & 0 & 0 & 0 & -32 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 2 & 0 & 0 & 0 & -32 \\ & & 4 & 8 & 16 & 32 \\ \hline & 2 & 4 & 8 & 16 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 2 & 4 & 8 & 16 \\ & & -4 & 0 & -16 \\ \hline & 2 & 0 & 8 & 0 \end{array}$$

$f(x) = \left(x + \frac{1}{2}\right)(x - 2)(x + 2)(2x^2 + 8)$

$x = -\frac{1}{2}, 2, -2$

58.

$$\begin{array}{r|rrrrrr} -3 & 3 & 1 & 0 & 0 & -243 & -81 \\ & & -9 & 24 & -72 & 216 & 81 \\ \hline & 3 & -8 & 24 & -72 & -27 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 3 & -8 & 24 & -72 & -27 \\ & & 9 & 3 & 81 & 27 \\ \hline & 3 & 1 & 27 & 9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 1 & 27 & 9 \\ & & -1 & 0 & -9 \\ \hline & 3 & 0 & 27 & 0 \end{array}$$

$f(x) = (x - 3)(x + 3)\left(x + \frac{1}{3}\right)(3x^2 + 27)$

$x = 3, -3, -\frac{1}{3}$

59. $777 = 2t^3 + 23t^2 + 5t + 501$

$2t^3 + 23t^2 + 5t - 276 = 0$

$$\begin{array}{r|rrrr} 3 & 2 & 23 & 5 & -276 \\ & & 6 & 87 & 276 \\ \hline & 2 & 29 & 92 & 0 \end{array}$$

$0 = (t - 3)(2t^2 + 29 + 92)$

$t = 3 \qquad 1993$

60. $90 = 3x^3 + x^2$

$3x^3 + x^2 - 90 = 0$

$$\begin{array}{r|rrrr} 3 & 3 & 1 & 0 & -90 \\ & & 9 & 30 & 90 \\ \hline & 3 & 10 & 30 & 0 \end{array}$$

$0 = (x - 3)(3x^2 + 10x + 30)$

$x = 3$

3 ft

61. $20 = x^2(x + 3)$

$x^3 + 3x^2 - 20 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & 0 & -20 \\ & & 2 & 10 & 20 \\ \hline & 1 & 5 & 10 & 0 \end{array}$$

$f(x) = (x - 2)(x^2 + 5x + 10)$

$x = 2$

2 in. by 2 in. by 5 in

62. $48\pi = \frac{1}{3}\pi(x^2)(x + 5)$

$144 = x^3 + 5x^2$

$x^3 + 5x^2 - 144 = 0$

$$\begin{array}{r|rrrr} 4 & 1 & 5 & 0 & -144 \\ & & 4 & 36 & 144 \\ \hline & 1 & 9 & 36 & 0 \end{array}$$

$(x - 4)(x^2 + 9x + 36) = 0$

$x = 4$

4 in. radius, 9 in. height

63. $2000 = (x^2 + 5x)(x + 35)$

$x^3 + 40x^2 + 175x - 2000 = 0$

$$\begin{array}{r|rrrr} 5 & 1 & 40 & 175 & -2000 \\ & & 5 & 225 & 2000 \\ \hline & 1 & 45 & 400 & 0 \end{array}$$

$(x - 5)(x^2 + 45x + 400) = 0$

$x = 5$

5 ft deep, 10 ft wide, 40 ft long

64. $150 = \frac{1}{2}15x^2(2x + 1)$

$150 = 15x^3 + \dfrac{15x^2}{2}$

$15x^3 + \dfrac{15}{2}x^2 - 150 = 0$

$$\begin{array}{r|rrrr} 2 & 15 & \frac{15}{2} & 0 & -150 \\ & & 30 & 75 & 150 \\ \hline & 15 & \frac{75}{2} & 75 & 0 \end{array}$$

$(x - 2)\left(15x^2 + \dfrac{75}{2}x + 75\right) = 0$

$x = 2$

2 ft by 6 ft by 25 ft

66. A   **67.** $-2, -1, 1$; B   **68.** $-2, 1$; A

C

no; If a cubic polynomial had 4 or more distinct real zeros, then there would be 4 or more binomials of the form $x - a$ that divide the polynomial to give a zero remainder. This would imply that the ploynomial has degree 4 or greater. However, this is impossible since the polynomial is a cubic polynomial. So a cubic polynomial has at most 3 real zeros. As $x \to -\infty$ and $x \to +\infty$, the values of a cubic polynomial approach $-\infty$ and $+\infty$, respectively, or else $+\infty$ and $-\infty$. At some value of x, therefore, the graph is below the x-axis, and at some other values of x, the graph is above the x-axis. This means that the graph crosses the x-axis somewhere between these two values, and the x-coordinate of the point where the graph crosses the x-axis is a zero.

Mixed Review (p. 365)

$x^2 - 6x + 9 = 0$
$(x - 3)^2 = 0$
$x - 3 = 0$
$x = 3$

72. $x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x - 5 = 0$
$x = 5$

73. $x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0$
$x - 1 = 0$
$x = 1$

74. $2x^2 - 12x - 18 = 0$
$2(x - 3)(x - 3) = 0$
$x - 3 = 0$
$x = 3$

75. $x^2 - 20x + 100 = 0$
$(x - 10)^2 = 0$
$x - 10 = 0$
$x = 10$

76. $x^2 - 18x + 81 = 0$
$(x - 9)^2 = 0$
$x - 9 = 0$
$x = 9$

77. $= a(x + 3)(x - 3)$
$5 = a(-9)$
$= -\frac{5}{9}(x + 3)(x - 3)$

78. $y = a(x + 5)(x - 1)$
$-6 = a(-2 + 5)(-2 - 1)$
$-6 = a - 9$
$\frac{2}{3} = a$
$y = \frac{2}{3}(x + 5)(x - 1)$

79. $= a(x + 1)(x - 5)$
$10 = a(-5)$
$-2 = a$
$= -2(x + 1)(x - 5)$

80. $y = a(x - 12)(x - 7)$
$7 = a(-23)(-18)$
$y = \frac{7}{414}(x - 12)(x - 7)$

81. $= a(x + 12)(x + 6)$
$-5 = a(21)(15)$
$a = -\frac{1}{63}$
$= -\frac{1}{63}(x + 12)(x + 6)$

82. $y = a(x - 2)(x - 8)$
$-4 = a(-5)$
$\frac{4}{5} = a$
$y = \frac{4}{5}(x - 2)(x - 8)$

83. $y = a(x - 4)(x - 10)$
$3 = a(3)(-3)$
$-\frac{1}{3} = a$
$y = -\frac{1}{3}(x - 4)(x - 10)$

84. $y = a(x + 6)(x + 0)$
$16 = a(16)$
$1 = a$
$y = (x + 6)(x + 0)$

85. $y = a(x + 9)(x + 1)$
$20 = a(20)$
$a = (1)$
$y = (x + 9)(x + 1)$

86. $(16 + 2x)(12 + 2x) = 204 + 192$
$(8 + x)(6 + x) = 99$
$48 + 14x + x^2 = 99$
$x^2 + 14x - 51 = 0$
$(x - 3)(x + 17) = 0$
$x = 3$ or $x = -17$
width of mat: 3 in.
overall: 18 in. by 22 in.

Quiz 2 (p. 365)

1. $5x^3 + 135 = 5(x^3 + 27) = 5(x + 3)(x^2 - 3x + 9)$

2. $6x^3 + 12x^2 + 12x + 24 = 6(x^3 + 2x^2 + 2x + 4)$
$= 6(x + 2)(x^2 + 2)$

3. $4x^5 - 16x = 4x(x^4 - 4) = 4x(x^2 + 2)(x^2 - 2)$

4. $3x^3 - x^2 - 15x + 5 = x^2(3x - 1) - 5(3x - 1)$
$= (3x - 1)(x^2 - 5)$

5. $7x^4 = 252x^2$
$x^2 = 36$
$x = \pm 6, 0$

6. $16x^6 = 54x^3$
$x^3 = \frac{27}{8}$
$x = \frac{3}{2}, 0$

7. $6x^5 - 18x^4 + 12x^3 - 36x^2 = 0$
$6x^4(x - 3) + 12x^2(x - 3) = 0$
$6x^2(x^2 + 2)(x - 3) = 0$
$x = 0, 3$

8. $2x^3 + 5x^2 - 8x - 20 = 0$
$x^2(2x + 5) - 4(2x + 5) = 0$
$(x + 2)(x - 2)(2x + 5) = 0$
$x = 2, -2, -\frac{5}{2}$

9.
$$
\begin{array}{r|rrr}
4 & 1 & 7 & -44 \\
  &   & 4 & 44 \\
\hline
  & 1 & 11 & 0
\end{array}
$$
$(x - 4)(x + 11)$

10.
$$
\begin{array}{r}
x - \dfrac{10}{3} + \dfrac{80}{3(3x + 2)} \\
3x + 2 \overline{) 3x^2 - 8x + 20} \\
\underline{-3x^2 - 2x} \\
-10x + 20 \\
\underline{10x + \frac{20}{3}} \\
\frac{80}{3}
\end{array}
$$

11

12.

13.

14.

15. f

$x =$

16. 2

$f(x$

$x =$

17. $\frac{1}{2}$

$f(x$

$x =$

Chapter 6 *continued*

11.

$$4x - 7 + \frac{11x - 11}{x^2 - 3}$$

$$x^2 - 3 \overline{)\,4x^3 - 7x^2 - x + 10}$$
$$\underline{-4x^3 + 12x}$$
$$-7x^2 + 11x$$
$$\underline{7x^2 - 21}$$
$$11x - 11$$

12.

$$12x^3 - 7x^2 + 10x - 10 + \frac{5}{x + 1}$$

$$x + 1 \overline{)\,12x^4 + 5x^3 + 3x^2 + 0x - 5}$$
$$\underline{-12x^4 - 12x^3}$$
$$-7x^3 + 3x^2$$
$$\underline{7x^3 + 7x^2}$$
$$10x^2 + 0x$$
$$\underline{-10x^2 - 10x}$$
$$-10x - 5$$
$$\underline{10x + 10}$$
$$5$$

13.

$$x + \frac{2x^2 + 6x + 6}{x^3 - 3}$$

$$x^3 - 3 \overline{)\,x^4 + 0x^3 + 2x^2 + 3x + 6}$$
$$\underline{-x^4 + 3x}$$
$$2x^2 + 6x + 6$$

14.

$$5x^3 - 23x^2 + 115x - 576 + \frac{2875}{x + 5}$$

$$x + 5 \overline{)\,5x^4 + 2x^3 + 0x^2 - x - 5}$$
$$\underline{-5x^4 - 25x^3}$$
$$-23x^3 + 0x^2$$
$$\underline{23x^2 + 115x^2}$$
$$115x^2 - x$$
$$\underline{-115x^2 - 575x}$$
$$-576x - 5$$
$$\underline{576x + 2880}$$
$$2875$$

15. $f(x) = x^3 - 4x^2 - 7x + 28$

$$= x^2(x - 4) - 7(x - 4)$$

$$= (x - 4)(x^2 - 7)$$

$$x = 4, \pm\sqrt{7}$$

16.
$$2 \,\big|\; 1 \quad -6 \quad 21 \quad -26$$
$$\quad \underline{2 \quad -8 \quad 26}$$
$$1 \quad -4 \quad 13 \quad 0$$

$$f(x) = (x - 2)(x^2 - 4x + 13)$$

$$x = 2$$

17.
$$\tfrac{1}{2} \,\big|\; 2 \quad 15 \quad 22 \quad -15$$
$$\phantom{\tfrac{1}{2}\,\big|\;2}\quad \underline{1 \quad 8 \quad 15}$$
$$\phantom{\tfrac{1}{2}\,\big|\;}2 \quad 16 \quad 30 \quad 0$$

$$f(x) = \left(x - \tfrac{1}{2}\right)2(x^2 + 8x + 15)$$

$$x = \tfrac{1}{2}, -3, -5$$

18.
$$2 \,\big|\; 2 \quad 7 \quad -28 \quad 12$$
$$\quad \underline{4 \quad 22 \quad -12}$$
$$2 \quad 11 \quad -6 \quad 0$$

$$f(x) = (x - 2)(2x^2 + 11 - 6)$$

$$x = 2, \tfrac{1}{2}, -6$$

19. $128 = x^2(x - 15.5)$

$$x^3 - 15.5x^2 - 128 = 0$$

$$16 \,\big|\; 1 \quad -15.5 \quad 0 \quad -128$$
$$\quad \underline{16 \quad 8 \quad 128}$$
$$1 \quad 0.5 \quad 8 \quad 0$$

$$0 = (x - 16)(x^2 + 0.5x + 8)$$

$$x = 16$$

16 ft × 16 ft × 0.5 ft

Lesson 6.7

Activity (p. 366)

1. a. $2x - 1 = 0$

$$x = \tfrac{1}{2};$$

1; rational

b. $x^2 - 2 = 0$

$$x = \pm\sqrt{2};$$

2; irrational

c. $x = 1, \dfrac{-1 \pm i\sqrt{3}}{2};$

3; 1 is rational, $\dfrac{-1 \pm i\sqrt{3}}{2}$ are imaginary

Sample answer: If $f(x)$ has a degree $n > 1$, then $f(x) = 0$ has n solutions.

2. $x^3 + x^2 - x - 1 = 0$

$$x^2(x + 1) - (x + 1) = 0$$

$$(x + 1)(x^2 - 1) = 0$$

$$(x + 1)(x + 1)(x - 1) = 0$$

$$x = 1, -1;$$

2 different solutions; -1 is a solution twice

6.7 Guided Practice (p. 369)

1. *Sample answer:* If $f(x)$ is a polynomial of positive degree, then $f(x) = 0$ has at least one root in the set of complex numbers.

2. *Sample answer:* The existence of an imaginary zero would imply that there are two distinct imaginary zeros which is not consistent with the fact that $f(x)$ is degree 3.

3. *Sample answer:* 2 real zeros; no imaginary zeros; the existence of an imaginary zero would imply the existence of two distinct imaginary zeros, which would not be consistent with the fact that $f(x)$ has degree 3. The real number 2 is a repeated zero.