

## Chapter 6 continued

$$11. \quad \begin{array}{r} 4x - 7 + \frac{11x - 11}{x^2 - 3} \\ x^2 - 3 \overline{) 4x^3 - 7x^2 - x + 10} \\ \underline{-4x^3 + 12x} \phantom{+ 10} \\ -7x^2 + 11x \phantom{+ 10} \\ \underline{7x^2 - 21} \phantom{+ 10} \\ 11x - 11 \end{array}$$

$$12. \quad \begin{array}{r} 12x^3 - 7x^2 + 10x - 10 + \frac{5}{x + 1} \\ x + 1 \overline{) 12x^4 + 5x^3 + 3x^2 + 0x - 5} \\ \underline{-12x^4 - 12x^3} \phantom{+ 3x^2 + 0x - 5} \\ -7x^3 + 3x^2 \phantom{+ 0x - 5} \\ \underline{7x^3 + 7x^2} \phantom{+ 0x - 5} \\ 10x^2 + 0x \phantom{+ 0x - 5} \\ \underline{-10x^2 - 10x} \phantom{+ 0x - 5} \\ -10x - 5 \phantom{+ 0x - 5} \\ \underline{10x + 10} \phantom{+ 0x - 5} \\ 5 \end{array}$$

$$13. \quad \begin{array}{r} x + \frac{2x^2 + 6x + 6}{x^3 - 3} \\ x^3 - 3 \overline{) x^4 + 0x^3 + 2x^2 + 3x + 6} \\ \underline{-x^4} \phantom{+ 0x^3 + 2x^2 + 3x + 6} \\ 2x^2 + 6x + 6 \end{array}$$

$$14. \quad \begin{array}{r} 5x^3 - 23x^2 + 115x - 576 + \frac{2875}{x + 5} \\ x + 5 \overline{) 5x^4 + 2x^3 + 0x^2 - x - 5} \\ \underline{-5x^4 - 25x^3} \phantom{+ 0x^2 - x - 5} \\ -23x^3 + 0x^2 \phantom{+ 0x^2 - x - 5} \\ \underline{23x^3 + 115x^2} \phantom{+ 0x^2 - x - 5} \\ 115x^2 - x \phantom{+ 0x^2 - x - 5} \\ \underline{-115x^2 - 575x} \phantom{+ 0x^2 - x - 5} \\ -576x - 5 \phantom{+ 0x^2 - x - 5} \\ \underline{576x + 2880} \phantom{+ 0x^2 - x - 5} \\ 2875 \end{array}$$

$$15. \quad \begin{aligned} f(x) &= x^3 - 4x^2 - 7x + 28 \\ &= x^2(x - 4) - 7(x - 4) \\ &= (x - 4)(x^2 - 7) \\ x &= 4, \pm\sqrt{7} \end{aligned}$$

$$16. \quad \begin{array}{r} 2 \overline{) 1 \quad -6 \quad 21 \quad -26} \\ \phantom{2} \underline{2 \quad -8 \quad 26} \\ 1 \quad -4 \quad 13 \quad 0 \\ f(x) = (x - 2)(x^2 - 4x + 13) \\ x = 2 \end{array}$$

$$17. \quad \begin{array}{r} \frac{1}{2} \overline{) 2 \quad 15 \quad 22 \quad -15} \\ \phantom{\frac{1}{2}} \underline{1 \quad 8 \quad 15} \\ 2 \quad 16 \quad 30 \quad 0 \\ f(x) = \left(x - \frac{1}{2}\right)2(x^2 + 8x + 15) \\ x = \frac{1}{2}, -3, -5 \end{array}$$

$$18. \quad \begin{array}{r} 2 \overline{) 2 \quad 7 \quad -28 \quad 12} \\ \phantom{2} \underline{4 \quad 22 \quad -12} \\ 2 \quad 11 \quad -6 \quad 0 \\ f(x) = (x - 2)(2x^2 + 11 - 6) \\ x = 2, \frac{1}{2}, -6 \end{array}$$

$$19. \quad \begin{array}{r} 128 = x^2(x - 15.5) \\ x^3 - 15.5x^2 - 128 = 0 \\ 16 \overline{) 1 \quad -15.5 \quad 0 \quad -128} \\ \phantom{16} \underline{16 \quad 8 \quad 128} \\ 1 \quad 0.5 \quad 8 \quad 0 \\ 0 = (x - 16)(x^2 + 0.5x + 8) \\ x = 16 \\ 16 \text{ ft} \times 16 \text{ ft} \times 0.5 \text{ ft} \end{array}$$

### Lesson 6.7

#### Activity (p. 366)

1. a.  $2x - 1 = 0$                       b.  $x^2 - 2 = 0$   
 $x = \frac{1}{2}$ ;                                       $x = \pm\sqrt{2}$ ;  
 1; rational                                2; irrational
- c.  $x = 1, \frac{-1 \pm i\sqrt{3}}{2}$ ;  
 3; 1 is rational,  $\frac{-1 \pm i\sqrt{3}}{2}$  are imaginary

Sample answer: If  $f(x)$  has a degree  $n > 1$ , then  $f(x) = 0$  has  $n$  solutions.

2.  $x^3 + x^2 - x - 1 = 0$   
 $x^2(x + 1) - (x + 1) = 0$   
 $(x + 1)(x^2 - 1) = 0$   
 $(x + 1)(x + 1)(x - 1) = 0$   
 $x = 1, -1$ ;  
 2 different solutions;  $-1$  is a solution twice

#### 6.7 Guided Practice (p. 369)

1. Sample answer: If  $f(x)$  is a polynomial of positive degree, then  $f(x) = 0$  has at least one root in the set of complex numbers.
2. Sample answer: The existence of an imaginary zero would imply that there are two distinct imaginary zeros which is not consistent with the fact that  $f(x)$  is degree 3.
3. Sample answer: 2 real zeros; no imaginary zeros; the existence of an imaginary zero would imply the existence of two distinct imaginary zeros, which would not be consistent with the fact that  $f(x)$  has degree 3. The real number 2 is a repeated zero.

## Chapter 6 continued

4.  $f(x) = x^3 - x^2 - 2x$   
 $= x(x^2 - x - 2)$   
 $= x(x - 2)(x + 1)$   
 $x = 0, 2, -1$
5.  $f(x) = x^4 + x^2 - 12$   
 $= (x^2 - 3)(x^2 + 4)$   
 $= (x - \sqrt{3})(x + \sqrt{3})(x^2 + 4)$   
 $x = \pm\sqrt{3}, \pm 2i$
6.  $f(x) = x^3 + 5x^2 - 9x - 45$   
 $= (x + 5)(x^2 - 9)$   
 $= (x + 5)(x + 3)(x - 3)$   
 $x = -5, -3, 3$
7.  $f(x) = x^4 - x^3 + 2x^2 - 4x - 8$   
 $= (x + 1)(x^3 - 2x^2 + 4x - 8)$   
 $= (x + 1)(x - 2)(x^2 + 4)$   
 $x = -1, 2, \pm 2i$
8.  $x = 3, 0, -2$   
 $f(x) = (x - 3)(x + 2)x$   
 $= (x^2 - x - 6)x$   
 $= x^3 - x^2 - 6x$
9.  $x = 1, 1, i, -i$   
 $f(x) = (x - 1)(x - 1)(x - i)(x + i)$   
 $= (x^2 - 2x + 1)(x^2 + 1)$   
 $= x^4 - 2x^3 + 2x^2 - 2x + 1$
10.  $x = 5, 2 + 3i, 2 - 3i$   
 $f(x) = (x - 5)[x - (2 + 3i)][x - (2 - 3i)]$   
 $= (x - 5)[(x - 2) - 3i][(x - 2) + 3i]$   
 $= (x - 5)[(x - 2)^2 + 9]$   
 $= x^3 - 9x^2 + 33x - 65$
11.  $x = 1, -1, 2, -2, 3$   
 $f(x) = (x^2 - 1)(x^2 - 4)(x - 3)$   
 $= (x^4 - 5x^2 + 4)(x - 3)$   
 $= x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$
12.  $x = 3, -2, -1 + i, -1 - i$   
 $f(x) = (x - 3)(x + 2)[x - (-1 + i)][x - (-1 - i)]$   
 $= (x^2 - x - 6)[(x + 1) - i][(x + 1) + i]$   
 $= (x^2 - x - 6)[(x + 1)^2 - i^2]$   
 $= (x^2 - x - 6)[x^2 + 2x + 2]$   
 $= x^4 + 2x^3 + 2x^2 - x^3 - 2x^2 - 2x - 6x^2 - 12x - 12$   
 $= x^4 + x^3 - 6x^2 - 14x - 12$

13.  $x = 4i, 4i, -4i, -4i$   
 $f(x) = (x^2 + 16)(x^2 + 16)$   
 $= x^4 + 32x + 256$
14.  $1.5 = \frac{1}{10,000}(-t^4 + 12t^3 - 77t^2 + 600t + 13,650)$   
 $t^4 - 12t^3 + 77t^2 - 600t + 1350 = 0$
- $$\begin{array}{r|rrrrr} 3 & 1 & -12 & 77 & -600 & 1350 \\ & & 3 & -27 & 150 & -1350 \\ \hline & 1 & -9 & 50 & -450 & 0 \end{array}$$
- $$\begin{array}{r|rrrr} 9 & 1 & -9 & 50 & -450 \\ & & 9 & 0 & 450 \\ \hline & 1 & 0 & 50 & 0 \end{array}$$
- $t = 3, t = 9$

### 6.7 Practice and Applications (pp. 369–371)

15.  $f(x) = x^3 - x^2 + 4x - 4$   
 $f(1) = (1)^3 - (1)^2 + 4(1) - 4 = 0$   
 yes
16.  $f(x) = x^3 + 3x^2 - 5x + 8$   
 $f(4) = (4)^3 + 3(4)^2 - 5(4) + 8 = 100$   
 no
17.  $f(x) = x^4 - x^2 - 3x + 3$   
 $f(0) = 0^4 - 0^2 - 3(0) + 3 = 3$   
 no
18.  $f(x) = x^3 + 5x^2 + x + 5$   
 $f(-5) = (-5)^3 + 5(-5)^2 - 5 + 5 = 0$   
 yes
19.  $f(x) = x^3 - 4x^2 + 16x - 64$   
 $f(4i) = (4i)^3 - 4(4i)^2 + 16(4i) - 64 = 0$   
 yes
20.  $f(x) = x^3 - 3x^2 + x - 3$   
 $f(-i) = (-i)^3 - 3(i)^2 - i - 3 = 0$   
 yes
21.  $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$   
 $= (x - 1)(x^3 + 6x^2 + 11x + 6)$   
 $= (x - 1)(x + 1)(x^2 + 5x + 6)$   
 $= (x - 1)(x + 1)(x + 3)(x + 2)$   
 $x = 1, -1, -3, -2$
22.  $f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$   
 $= (x - 3)(x^3 + 7x^2 + 15x + 9)$   
 $= (x - 3)(x + 3)(x^2 + 4x + 3)$   
 $= (x - 3)(x + 3)(x + 3)(x + 1)$   
 $x = 3, -3, -3, -1$

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$$23. f(x) = x^3 - 4x^2 + 3x$$

$$= x(x^2 - 4x + 3)$$

$$= x(x - 3)(x - 1)$$

$$x = 0, 3, 1$$

$$24. f(x) = x^3 + 5x^2 - 4x - 20$$

$$= (x + 5)(x^2 - 4)$$

$$= (x + 5)(x - 2)(x + 2)$$

$$x = -5, 2, -2$$

$$25. f(x) = x^4 + 7x^3 - x^2 - 67x - 60$$

$$= (x + 5)(x^3 + 2x^2 - 11x - 12)$$

$$= (x + 5)(x + 4)(x^2 - 2x - 3)$$

$$x = -5, -4, 3, -1$$

$$26. f(x) = x^4 - 5x^2 - 36$$

$$= (x^2 + 4)(x^2 - 9)$$

$$= (x + 3)(x - 3)(x^2 + 4)$$

$$x = 3, -3, \pm 2i$$

$$27. f(x) = x^3 - x^2 + 49x - 49$$

$$= (x^2 + 49)(x - 1)$$

$$x = 1, \pm 7i$$

$$28. f(x) = x^3 - x^2 + 25x - 25$$

$$= (x^2 + 25)(x - 1)$$

$$x = 1, \pm 5i$$

$$29. f(x) = x^4 + 6x^3 + 14x^2 + 54x + 45$$

$$= (x + 5)(x^3 + x^2 + 9x + 9)$$

$$= (x + 5)(x + 1)(x^2 + 9)$$

$$x = -5, -1, \pm 3i$$

$$30. f(x) = x^3 + 3x^2 + 25x + 75$$

$$= (x^2 + 25)(x + 3)$$

$$x = \pm 5i, -3$$

$$31. f(x) = x^4 - x^3 - 5x^2 - x - 6$$

$$= (x + 2)(x^3 - 3x^2 + x - 3)$$

$$= (x + 2)(x - 3)(x^2 + 1)$$

$$x = -2, 3, \pm i$$

$$32. f(x) = x^4 + x^3 + 2x^2 + 4x - 8$$

$$= (x + 2)(x^3 - x^2 + 4x - 4)$$

$$= (x + 2)(x - 1)(x^2 + 4)$$

$$x = -2, 1, \pm 2i$$

$$33. f(x) = 2x^4 - 7x^3 - 27x^2 + 63x + 81$$

$$= (x^2 - 9)(2x^2 - 7x - 9)$$

$$= (x^2 - 9)(x + 1)(2x - 9)$$

$$x = 3, -3, -1, \frac{9}{2}$$

$$34. f(x) = 2x^4 - x^3 - 42x^2 + 16x + 160$$

$$= (x^2 - 16)(2x^2 - x - 10)$$

$$x = \pm 4, x \approx 2.5, x \approx -2$$

$$35. x = 2, 1, 4$$

$$f(x) = (x - 2)(x - 1)(x - 4)$$

$$= (x - 2)(x^2 - 5x + 4)$$

$$= x^3 - 7x^2 + 14x - 8$$

$$36. x = 1, -4, 5$$

$$f(x) = (x - 1)(x + 4)(x - 5)$$

$$= (x^2 + 3x - 4)(x - 5)$$

$$= x^3 - 2x^2 - 19x + 20$$

$$37. x = -6, 3, 5$$

$$f(x) = (x + 6)(x - 3)(x - 5)$$

$$= (x^2 + 3x - 18)(x - 5)$$

$$= x^3 - 2x^2 - 33x + 90$$

$$38. x = -5, 2, -2$$

$$f(x) = (x + 5)(x^2 - 4)$$

$$= x^3 + 5x^2 - 4x - 20$$

$$39. x = -2, -4, -7$$

$$f(x) = (x + 2)(x + 4)(x + 7)$$

$$= (x^2 + 6x + 8)(x + 7)$$

$$= x^3 + 13x^2 + 50x + 56$$

$$40. x = 8, -i, i$$

$$f(x) = (x - 8)(x^2 + 1)$$

$$= x^3 - 8x^2 + x - 8$$

$$41. x = 5, 3i, -3i$$

$$f(x) = (x - 5)(x - 3i)(x + 3i)$$

$$= (x - 5)(x^2 + 9)$$

$$= x^3 - 5x^2 + 9x - 45$$

$$42. x = 2, -2, -6i, 6i$$

$$f(x) = (x^2 - 4)(x^2 + 36)$$

$$= x^4 + 32x^2 - 144$$

$$43. x = i, -i, -3i, 3i$$

$$f(x) = (x^2 + 1)(x^2 + 9)$$

$$= x^4 + 10x^2 + 9$$

$$44. x = 3 - i, 3 + i, 5i, -5i$$

$$f(x) = [x - (3 - i)][x - (3 + i)](x - 5i)(x + 5i)$$

$$= [(x - 3) + i][(x - 3) - i](x^2 - 25i^2)$$

$$= [(x - 3)^2 - i^2](x^2 + 25)$$

$$= (x^2 - 6x + 10)(x^2 + 25)$$

$$= x^4 - 6x^3 + 35x^2 - 150x + 250$$

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$$\begin{aligned} &= 4, 4, 2 + i, 2 - i \\ &= (x - 4)(x - 4)[x - (2 + i)][x - (2 - i)] \\ &= (x^2 - 8x + 16)[(x - 2) - i][(x - 2) + i] \\ &= (x^2 - 8x + 16)(x - 2)^2 - i^2 \\ &= (x^2 - 8x + 16)(x^2 - 4x + 5) \\ &= x^4 - 12x^3 + 53x^2 - 104x + 80 \end{aligned}$$

$$\begin{aligned} &= -2, -2, 3, -4i, 4i \\ &= (x + 2)^2(x - 3)(x^2 + 16) \\ &= (x^2 + 4x + 4)(x - 3)(x^2 + 16) \\ &= x^5 + x^4 + 8x^3 + 4x^2 - 128x - 192 \end{aligned}$$

$$x^3 - x^2 - 5x + 3$$

$$\approx -2.09, 0.57, 2.51$$

$$2x^3 - x^2 - 3x - 1$$

$$\approx -0.62, -0.5, 1.62$$

$$x^3 - 2x^2 + x + 1$$

$$\approx -0.47$$

$$x^4 - 2x - 1$$

$$\approx -0.47, 1.40$$

$$x^4 - x^3 - 4x^2 - 3x - 2$$

$$\approx -1.27, 2.86$$

$$x^4 - x^3 - 3x^2 - x + 1$$

$$\approx 0.42, 2.37$$

$$x^4 + 3x^2 - 2$$

$$\approx -0.75, 0.75$$

$$x^4 - x^3 - 20x^2 + 10x + 27$$

$$\approx -4.09, -0.98, 1.47, 4.60$$

$$S = -0.131t^3 + 5.033t^2 - 23.2t + 233$$

$$312.76 = -0.131t^3 + 5.033t^2 - 23.2t + 233$$

$$-0.131t^3 + 5.033t^2 - 23.2t - 79.76 = 0$$

$$t \approx 8.3$$

$$1988$$

$$D = 1.78t^3 - 6.02t^2 + 752t + 6701$$

$$4300 = 1.78t^3 - 6.02t^2 + 752t + 6701$$

$$1.78t^3 - 6.02t^2 + 752t - 7599 = 0$$

$$t \approx 9.02$$

$$1992$$

$$S = -0.982t^5 + 24.6t^4 - 211t^3 + 661t^2 - 318t + 1520$$

$$2000 = -0.982t^5 + 24.6t^4 - 211t^3 + 661t^2$$

$$- 318t + 1520$$

$$= -0.982t^5 + 24.6t^4 - 211t^3 + 661t^2 - 318t - 480$$

$$= 0$$

$$t \approx 1.62, 6.3$$

$$\text{late 1988, 1993}$$

$$58. S = -0.213t^3 + 3.96t^2 + 10.2t + 366$$

$$455 = -0.213t^3 + 3.96t^2 + 10.2t + 366$$

$$-0.213t^3 + 3.96t^2 + 10.2t - 89 = 0$$

$$t \approx 3.95$$

$$\text{late 1993}$$

$$59. P = 0.00496t^3 - 0.432t^2 + 11.3t + 212$$

$$0.00496t^3 - 0.432t^2 + 11.3t - 510 = 0$$

$$t = 75$$

$$1965$$

$$60. \text{ a. } 1000g, 1000g^2, 1000g$$

$$\text{ b. } 1000g^3 + 1000g^2 + 1000g + 1000$$

$$\text{ c. } 1.05, 5\%, \text{ Sample answer: I graphed}$$

$x^3 + x^2 + x + 1 = y$  and  $y = 4.3$  and found the  $x$ -coordinate of the intersection point.

61. a.

Zeros	Sum of zeros	Product of zeros
2, 3	5	6
-3, 1, 2	0	-6
-3, 1, $\pm 2i$	-2	-12
-3, 2, 0, $2 \pm \sqrt{3}$	3	0

b. *Sample answer:* If  $f(x)$  is a polynomial with leading coefficient 1 and degree  $n$ , where  $n > 0$ , then the sum of the roots is the opposite of the coefficient of the  $x^{n-1}$  term.

c. *Sample answer:* If  $f(x)$  is a polynomial of degree  $n$ , where  $n > 0$ , then the product of the zeros is the constant term if  $n$  is even and the opposite of the constant term if  $n$  is odd.

$$62. (a + bi) + (a - bi) = (a + a) + (bi - bi) = 2a;$$

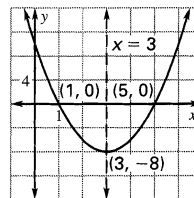
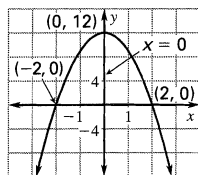
Since  $a$  is real,  $2a$  must be real.

$$63. (a + bi)(a - bi) = [a^2 + abi - abi + (bi)^2] = a^2 - b^2;$$

Since  $a$  and  $b$  are real,  $a^2 - b^2$  is real.

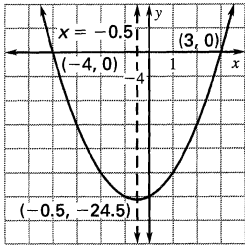
### 6.7 Mixed Review (p. 371)

$$64. y = -3(x - 2)(x + 2) \quad 65. y = 2(x - 1)(x - 5)$$

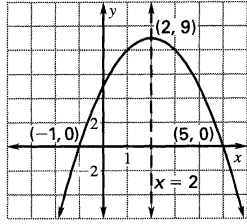


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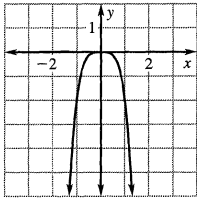
66.  $y = 2(x + 4)(x - 3)$



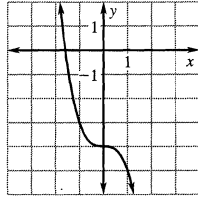
67.  $y = -(x + 1)(x - 5)$



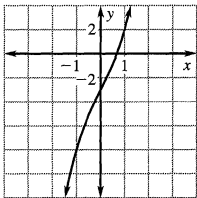
68.  $f(x) = -2x^4$



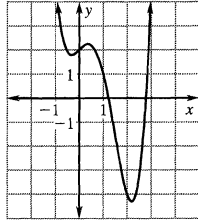
69.  $y = -x^3 - 4$



70.  $f(x) = x^3 + 4x - 3$



71.  $f(x) = x^4 - 3x^3 + x + 2$



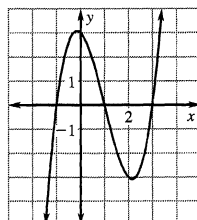
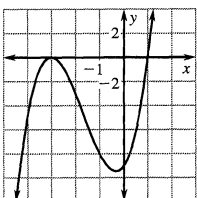
### Developing Concepts Activity 6.8 (p. 372)

1. -0.640, 1.135, 5.505    2. -0.640    3. 5
4. -0.219, 2.047, 14.839
5. -2.334, -0.742, 0.742, 2.334
6. -3.629, -0.629, 1.085, 18.173
7. -1.088, -0.668, 1.191    8. -0.735, 0.722, 1.326
9. -7.349, 16.429, 30.921; yes

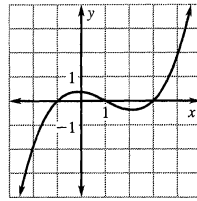
### Lesson 6.8

#### 6.8 Guided Practice (p. 376)

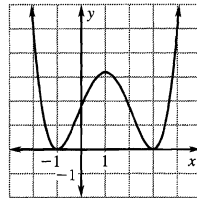
1. The y-coordinate of a point of the graph that is higher than all nearby points.
2. a. 4    b. 4    c. -2, 6    3. 4
4.  $f(x) = (x - 1)(x + 3)^2$     5.  $f(x) = (x - 1)(x + 1)(x - 3)$



6.  $f(x) = \frac{1}{8}(x + 1)(x - 1)(x - 3)$



7.  $f(x) = \frac{1}{5}(x - 3)^2(x + 1)^2$



8. x-intercepts: -1.40, -0.29

local max: (0.21, 1.21)

local min: (-1, -3), (0.79, 0.63)

9. x-intercepts: -0.41, 1, 2.41

local max: (0.18, 1.09)

local min: (1.82, -1.09)

10. x-intercepts: -1.19, 0, 1.69

local max: (1, 3)

local min: (-0.67, -1.63)

11. x-intercepts: 0, 1, 1.51

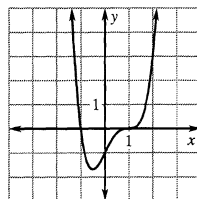
local max: (-1.59, -3.23), (0.49, 1.35)

local min: (-1, -4), (1.30, -0.79)

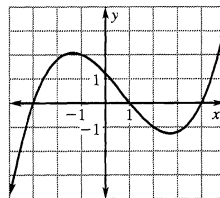
12. a.  $0 < x < 9$ ; the flaps can't be more than 9 in.

b. 3 in.    c. 432 in.<sup>3</sup>

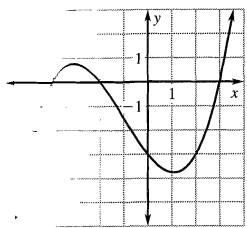
13.  $f(x) = (x - 1)^3(x + 1)$



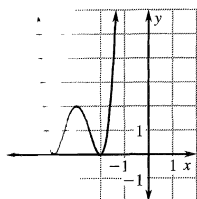
14.  $f(x) = \frac{1}{10}(x + 3)(x - 1)(x - 4)$



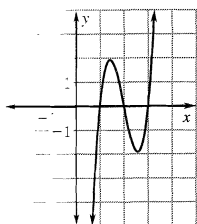
$$= \frac{1}{5}(x+4)(x+2)(x-3)$$



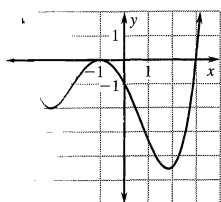
$$= 2(x+2)^2(x+4)^2$$



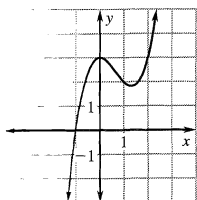
$$= 5(x-1)(x-2)(x-3)$$



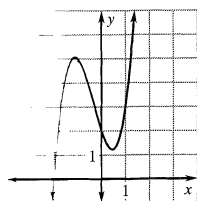
$$= \frac{1}{12}(x+4)(x-3)(x+1)^2$$



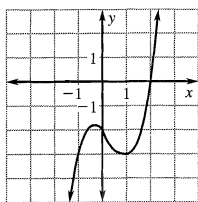
$$= (x+1)(x^2-3x+3)$$



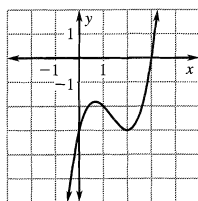
$$= (x+2)(2x^2-2x+1)$$



21.  $f(x) = (x-2)(x^2+x+1)$



22.  $f(x) = (x-3)(x^2-x+1)$



23. local max:  $(-\frac{1}{2}, \frac{2}{3})$   
 local min:  $(\frac{1}{2}, -\frac{1}{3})$   
 real zeros:  $-1, 0, 1$   
 degree: 3

24. local max:  $(-\frac{1}{2}, 5)$   
 local min:  $(-2, 0), (1, 0)$   
 real zeros:  $-2, 1$   
 degree: 4

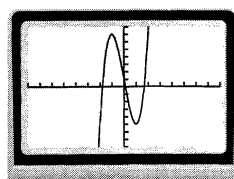
25. local max:  $(0, 2)$   
 local min:  $(-2, 1)$   
 real zeros:  $\frac{4}{3}$   
 degree: 3

26. local max:  $(-2, 2\frac{1}{2}), (1, -1)$   
 local min:  $(0, -1\frac{1}{4}), (2\frac{1}{2}, -2)$   
 real zeros:  $-2.5, -1, 3$   
 degree: 5

27. local max:  $(-2, -1), (1, -2)$   
 local min:  $(0, -2)$   
 real zeros: 0  
 degree: 4

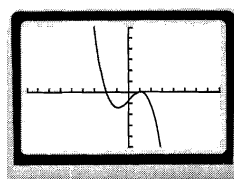
28. local max:  $(-2\frac{1}{2}, 2), (2, 1)$   
 local min:  $(0, \frac{1}{3})$   
 real zeros:  $-3\frac{2}{3}, 2\frac{2}{3}$   
 degree: 4

29.  $f(x) = 3x^3 - 9x + 1$



- x-intercepts:  
 $-1.79, 0.11, 1.67$   
 local max:  $(-1, 7)$   
 local min:  $(1, -5)$

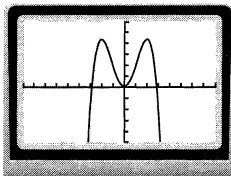
30.  $f(x) = -\frac{1}{3}x^3 + x - \frac{2}{3}$



- x-intercepts:  $-2, 1$   
 local max:  $(1, 0)$   
 local min:  $(-1, -\frac{4}{3})$

## Chapter 6 continued

31.  $f(x) = -\frac{1}{4}x^4 + 2x^2$

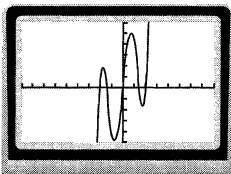


$x$ -intercepts:  $-2.83, 0, 2.83$

local max:  $(-2, 4), (2, 4)$

local min:  $(0, 0)$

32.  $f(x) = x^5 - 6x^3 + 9x$

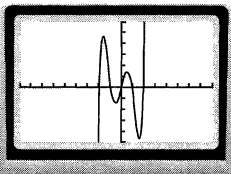


$x$ -intercepts:  $-1.73, 0, 1.73$

local max:  $(-1.73, 0), (0.77, 4.46)$

local min:  $(-0.77, -4.46), (1.73, 0)$

33.  $f(x) = x^5 - 5x^3 + 4x$

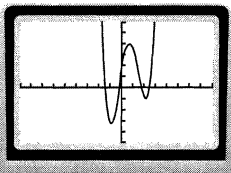


$x$ -intercepts:  $-2, -1, 0, 1, 2$

local max:  $(-1.64, 3.63), (0.54, 1.42)$

local min:  $(-0.54, -1.42), (1.64, -3.63)$

34.  $f(x) = x^4 - 2x^3 - 3x^2 + 5x + 2$



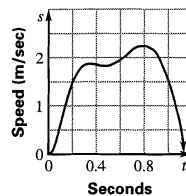
$x$ -intercepts:  $-1.53, -0.35, 1.88, 2$

local max:  $(0.61, 3.62)$

local min:  $(-1.05, -3.03), (1.94, -0.03)$

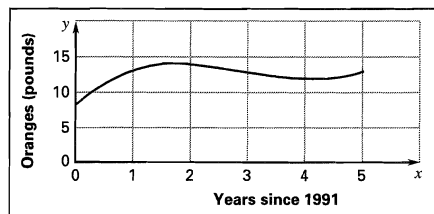
35.  $S = -241t^7 + 1062t^6 - 1871t^5 + 1647t^4 - 737t^3 + 144t^2 - 2.432t$

Speed of Swimmer



at about 0.8 seconds

36.  $f(x) = 0.298x^3 - 2.73x^2 + 7.05x + 8.45$



The points are the average of oranges in pounds eaten in a given year since 1991.

37.  $600 = \pi r^2 + \pi r l$   
 $\pi r l = 600 - \pi r^2$   
 $l = \frac{600 - \pi r^2}{\pi r}$

38.  $V = \frac{1}{2} \pi r^2 \left( \frac{600 - \pi r^2}{\pi r} \right)$

$V = \frac{1}{2} r (600 - \pi r^2)$

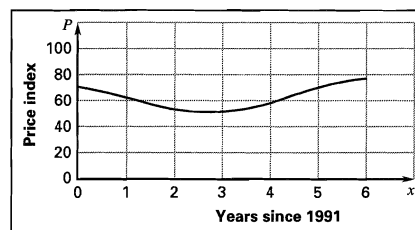
$V = 300r - \frac{1}{2} \pi r^3$

39. about  $1600 \text{ ft}^3$

$r \approx 8 \text{ ft}$

$l \approx 16 \text{ ft}$

40.  $P = -0.233x^4 + 2.64x^3 - 6.59x^2 - 3.93x + 69.1$



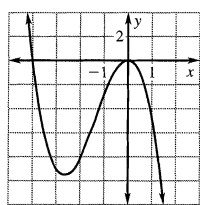
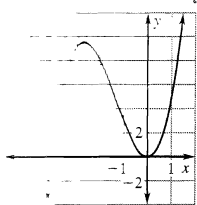
reaches a local min at  $(2.71, 50.03)$ ; the producer price index declined from 1991 to a low of about 50.03 around September 1993, after which it began to increase.

41. A polynomial with 3 turning points must be of degree four or higher.

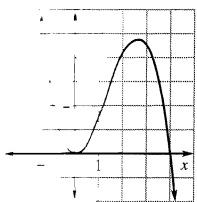
42. A 43. B

$$= x^2 - 4x^2$$

$$y = -f(x)$$



$$= -x^2$$



Guided Review (p. 378)

46.  $y = -\frac{3}{2}x$     47.  $y = \frac{1}{4}x$     48.  $y = -\frac{5}{2}x$   
 49.  $y = -\frac{1}{2}x$     50.  $y = \frac{5}{2}x$     51. yes;  $4 \times 1$     52. yes;  $2 \times 5$

54. yes;  $6 \times 5$   
 $-4 = a(x - 1)^2$     56.  $y - 6 = a(x + 2)^2$

$-5 - 4 = a(4 - 1)^2$      $-4 = 4a$   
 $-9 = 9a$      $-1 = a$

$-1 = a$      $y = -(x + 2)^2 + 6$   
 $= -(x - 1)^2 + 4$

$= a(x + 5)(x - 5)$     58.  $y = a(x + 2)(x - 4)$   
 $-4 = a(1 + 2)(1 - 4)$

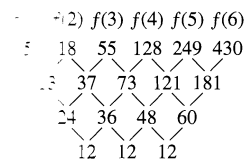
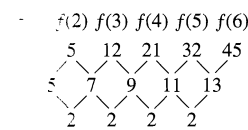
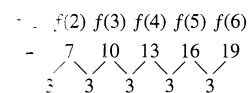
$\frac{4}{9} = a$   
 $= \frac{4}{9}(x + 5)(x - 5)$      $y = \frac{4}{9}(x + 2)(x - 4)$

$\frac{-30}{3} = \frac{30}{3} = 10 \text{ in./day}$

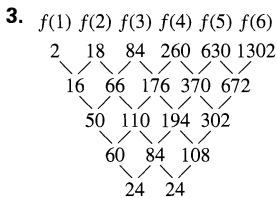
Lesson 6.9

Developing Concepts Activity (p. 379)

Drawing Conclusions



2. degrees: 1, 2, 3; 1, 2, 3; number of times differences were calculated before arriving at a row of constant, nonzero differences: 1, 2, 3; 1, 2, 3; the degree equals the number of times differences were calculated.



6.9 Guided Practice (p. 383)

1. the differences between  $f(n)$  and  $f(n + 1)$ ; the differences of adjacent first-order differences

2. 5

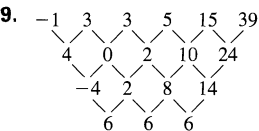
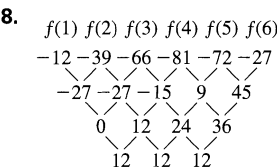
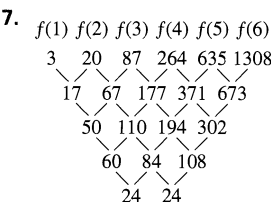
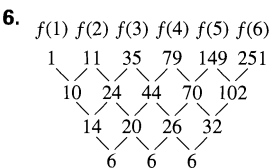
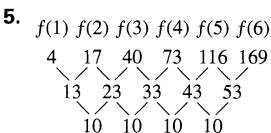
3. because the points will not lie exactly on the curve generated by the model

4.  $2 = a(1 - 3)(1 + 1)(1 + 2)$

$2 = a(-12)$

$a = -\frac{1}{6}$

$f(x) = -\frac{1}{6}(x - 3)(x + 1)(x + 2)$



3<sup>rd</sup> degree



## Chapter 6 continued

10. 
$$\begin{array}{cccccc} 0 & 8 & 12 & 12 & 8 & 0 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 8 & 4 & 0 & -4 & -8 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 4 & 4 & 4 & 4 \end{array}$$

2<sup>nd</sup> degree

11.  $f(x) = -x^3 + 5x^2 + x + 1$     12.  $f(x) = x^3 - 4x^2 + 2x$

13.  $d = \frac{1}{2}n^2 - \frac{3}{2}n$

### 6.9 Practice and Applications (pp. 383–385)

14.  $f(x) = a(x + 1)(x - 1)(x - 3)$

$$3 = a(3)$$

$$1 = a$$

$$f(x) = (x + 1)(x - 1)(x - 3)$$

15.  $f(x) = a(x - 3)(x - 2)(x + 1)$

$$-2 = a(-2)(-1)(2)$$

$$-\frac{1}{2} = a$$

$$f(x) = -\frac{1}{2}(x + 1)(x - 2)(x - 3)$$

16.  $f(x) = a(x + 3)(x + 1)(x - 3)$

$$1 = a(1)(-1)(-5)$$

$$\frac{1}{5} = a$$

$$f(x) = \frac{1}{5}(x + 3)(x + 1)(x - 3)$$

17.  $f(x) = a(x + 1)(x + 2)(x + 0)$

$$-3 = a(2)(3)(1)$$

$$-\frac{1}{2} = a$$

$$f(x) = -\frac{1}{2}(x + 1)(x + 2)(x + 0)$$

18.  $f(x) = a(x - 3)(x - 2)(x + 3)$

$$-1 = a(-2)(-1)(4)$$

$$-\frac{1}{8} = a$$

$$f(x) = -\frac{1}{8}(x - 3)(x - 2)(x + 3)$$

19.  $f(x) = a(x - 1)(x - 3)(x + 2)$

$$1 = a(1)(-1)(4)$$

$$-\frac{1}{4} = a$$

$$f(x) = -\frac{1}{4}(x - 1)(x - 3)(x + 2)$$

20.  $f(x) = a(x + 1)(x + 4)(x - 4)$

$$3 = a(1)(4)(-4)$$

$$-\frac{3}{16} = a$$

$$f(x) = -\frac{3}{16}(x + 1)(x + 4)(x - 4)$$

21.  $f(x) = a(x - 3)(x - 2)(x + 1)$

$$4 = a(-2)(-1)(2)$$

$$1 = a$$

$$f(x) = (x - 3)(x - 2)(x + 1)$$

22.  $f(x) = a(x + 0)(x + 3)(x - 5)$     23.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$3 = a(-2)(1)(-7)$$

$$\frac{3}{14} = a$$

$$f(x) = \frac{3}{14}(x + 0)(x + 3)(x - 5)$$

$$\begin{array}{cccccc} 5 & 5 & 7 & 11 & 17 & 25 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 0 & 2 & 4 & 6 & 8 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 2 & 2 & 2 & 2 \end{array}$$

24.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} -4 & -6 & 6 & 44 & 120 & 246 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & -2 & 12 & 38 & 76 & 126 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 14 & 26 & 38 & 50 \\ & & & \swarrow & \downarrow & \swarrow \\ & & & 12 & 12 & 12 \end{array}$$

25.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} -3 & -3 & -9 & -27 & -63 & -123 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 0 & -6 & -18 & -36 & -60 \\ & & \swarrow & \downarrow & \swarrow & \\ & & -6 & -12 & -18 & -24 \\ & & & \swarrow & \downarrow & \swarrow \\ & & & -6 & -6 & -6 \end{array}$$

26.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} -2 & -8 & 0 & 64 & 250 & 648 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & -6 & 8 & 64 & 186 & 398 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 14 & 56 & 122 & 212 \\ & & & \swarrow & \downarrow & \swarrow \\ & & & 42 & 66 & 90 \\ & & & & \swarrow & \downarrow \\ & & & & 24 & 24 \end{array}$$

27.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} -18 & -8 & 102 & 432 & 1150 & 2472 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 10 & 110 & 330 & 718 & 1322 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 100 & 220 & 388 & 604 \\ & & & \swarrow & \downarrow & \swarrow \\ & & & 120 & 168 & 216 \\ & & & & \swarrow & \downarrow \\ & & & & 48 & 48 \end{array}$$

28.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} 3 & -8 & -27 & -54 & -89 & -132 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & -11 & -19 & -27 & -35 & -43 \\ & & \swarrow & \downarrow & \swarrow & \\ & & -8 & -8 & -8 & -8 \end{array}$$

29.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} 4 & 4 & -36 & -176 & -500 & -1116 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 0 & -40 & -140 & -324 & -616 \\ & & \swarrow & \downarrow & \swarrow & \\ & & -40 & -100 & -184 & -292 \\ & & & \swarrow & \downarrow & \swarrow \\ & & & -60 & -84 & -108 \\ & & & & \swarrow & \downarrow \\ & & & & -24 & -24 \end{array}$$

30.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} -4 & 2 & 34 & 110 & 248 & 466 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 6 & 32 & 76 & 138 & 218 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 26 & 44 & 62 & 80 \\ & & & \swarrow & \downarrow & \swarrow \\ & & & 18 & 18 & 18 \end{array}$$

31.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} 3 & -2 & -13 & -30 & -53 & -82 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & -5 & -11 & -17 & -23 & -29 \\ & & \swarrow & \downarrow & \swarrow & \\ & & -6 & -6 & -6 & -6 \end{array}$$

32.  $f(x) = 3x^2 - 5x - 2$     33.  $f(x) = -3x^2 + 20x$

34.  $f(x) = x^3 - 3x^2 - 2$     35.  $f(x) = x^3 - 4x^2 + x$

36.  $f(x) = 0.5x^3 - 4x^2 + 3.5x - 3$

37.  $f(x) = x^3 + 4x^2 - x - 2$

38.  $f(x) = -x^3 + 8x^2 - 12x$

39.  $f(x) = 2x^3 - 16x^2 + 37x - 25$

40.  $f(x) = -2x^3 + 22x^2 - 74x + 74$

41.  $f(x) = -x^3 + 10x^2 - 30x + 23$

42.  $f(x) = x^4 - 15x^3 + 81x^2 - 183x + 142$

43.  $f(x) = -x^4 + 13x^3 - 58x^2 + 104x - 58$

44.  $f(1) f(2) f(3) f(4) f(5) f(6)$     45.  $f(1) f(2) f(3) f(4) f(5) f(6)$

$$\begin{array}{cccccc} 1 & 5 & 12 & 22 & 35 & 51 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 4 & 7 & 10 & 13 & 16 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 3 & 3 & 3 & 3 \end{array}$$

$$\begin{array}{cccccc} 1 & 6 & 15 & 28 & 45 & 66 \\ & \swarrow & \downarrow & \downarrow & \swarrow & \\ & 5 & 9 & 13 & 17 & 21 \\ & & \swarrow & \downarrow & \swarrow & \\ & & 4 & 4 & 4 & 4 \end{array}$$

46.  $f(n) = \frac{n}{6}(2n + 1)(n + 1)$

47.  $f(t) = 0.641t^3 - 4.93t^2 + 25.8t + 232$  where  $t$  is the number of years since 1989.

$$f(11) = 0.641(11)^3 - 4.93(11)^2 + 25.8(11) + 232$$

$$f(11) \approx 772.4; \text{ about } 772,000 \text{ Girl Scouts}$$

## Chapter 6 continued

48.  $y = 0.242t^3 - 3.00t^2 + 13.5t + 140$  where  $t$  is the number of years since 1987.

$$y = 0.242(13)^3 - 3.00(13)^2 + 13.5(13) + 140$$

$$y \approx 340.2$$

about \$340,000

49.  $y = 0.007t^3 + 0.740t^2 + 49t - 236$

$$4400 = 0.007(t)^3 + 0.740(t)^2 + 49(t) - 236$$

$$0.007t^3 + 0.74t^2 + 49t - 4636 = 0$$

$$t \approx 101$$

about 101 seconds

50 a. Dog-walking:  $y = 7.5x^2 - 20.5x + 16$

Lawn-care:  $y = 0.833x^3 - 4x^2 + 24.17x - 18$

- b. *Sample answer:* Solve for  $y$  in both equations when  $x = 12$ . Dog walking profits in December equal \$850 while lawn care profits are \$1135.

51 a.  $f(x) = ax^3 + bx^2 + cx + d$

$f(x+1)$	$ax^3 + 3ax^2 + 3ax + a + bx^2 + 2bx + b + cx + c + d$	}	$3ax^2 + 3ax + a + 2bx + b + c$	}	$6ax + 6a + 2b$	}	$6a$
$f(x+2)$	$ax^3 + 6ax^2 + 12ax + 8a + bx^2 + 4bx + 4b + cx + 2c + d$	}	$3ax^2 + 9ax + 7a + 2bx + 3b + c$	}	$6ax + 12a + 2b$		
$f(x+3)$	$ax^3 + 9ax^2 + 27ax + 27a + bx^2 + 6bx + 9b + cx + 3c + d$	}	$3ax^2 + 15a + 19a + 2bx + 5b + c$	}	$6ax + 18a + 2b$		
$f(x+4)$	$ax^3 + 12ax^2 + 48a + 64a + bx^2 + 8b + 16b + cx + 4c + d$	}	$3ax^2 + 21ax + 37a + 2bx + 7b + c$	}	$6ax + 24a + 2b$		
$f(x+5)$	$ax^3 + 15ax^2 + 75ax + 125a + bx^2 + 10bx + 25b + cx + 5c + d$	}	$3ax^2 + 27ax + 61a + 2bx + 9b + c$	}	$6ax + 30a + 2b$		

b.  $f(x) = x^3 - 9x^2 + 22x - 15$

### Mixed Review (p. 386)

52.  $x^2 = 6$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

53.  $16x^2 = 4$

$$x^2 = \frac{1}{4}$$

$$x = \pm\frac{1}{2}$$

54.  $4x^2 = 14$

$$x^2 = \frac{14}{4}$$

$$x = \pm\frac{\sqrt{14}}{2}$$

55.  $x^2 = 13$

$$x = \pm\frac{13}{6}$$

$$x = \pm\frac{\sqrt{78}}{6}$$

56.  $3x^2 = 15$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

57.  $2x^2 = 1$

$$x^2 = \frac{1}{2}$$

$$x = \pm\frac{\sqrt{2}}{2}$$

58.  $-12x + 36 = -27 + 36$

$$(x+6)^2 = 9$$

$$(x+6) = \pm 3$$

$$x = -9, -3$$

59.  $x^2 + 6x + 9 = 24 + 9$

$$(x+3)^2 = 33$$

$$x+3 = \pm\sqrt{33}$$

$$x = -3 \pm \sqrt{33}$$

60.  $x^2 - 3x + \frac{9}{4} = 18 + \frac{9}{4}$

$$\left(x - \frac{3}{2}\right)^2 = \frac{81}{4}$$

$$x - \frac{3}{2} = \pm\frac{9}{2}$$

$$x = 6, -3$$

61.  $x^2 + 4x + 4 = -\frac{11}{2} + 4$

$$(x+2)^2 = -\frac{3}{2}$$

$$x+2 = \pm\frac{i\sqrt{6}}{2}$$

$$x = -2 \pm \frac{i\sqrt{6}}{2}$$

## Chapter 6 continued

62.  $x^2 - 14x + 49 = 15 + 49$

$$(x - 7)^2 = 64$$

$$x - 7 = \pm 8$$

$$x = 15, -1$$

63.  $x^2 - 6x + 9 = -\frac{32}{3} + \frac{27}{3}$

$$(x - 3)^2 = -\frac{5}{3}$$

$$x - 3 = \pm \frac{i\sqrt{15}}{3}$$

$$x = 3 \pm \frac{i\sqrt{15}}{3}$$

64.  $(2x - 1)(4x^2 + 2x + 1)$     65.  $(3x + 2)(9x^2 - 6x + 4)$

66.  $8(3x + 2)(9x^2 - 6x + 4)$

67.  $(2x - 5)(4x^2 + 10x + 25)$     68.  $3(x - 2)(x^2 + 2x + 4)$

69.  $8(x + 3)(x^2 - 3x + 9)$

70.  $(3x + 10)(9x^2 - 30x + 100)$

71.  $3(x + 3)(x^2 - 3x + 9)$

### Quiz 3 (p. 386)

1.  $f(x) = 2x^3 - x^2 - 22x - 15$

$$x \approx -2.61, -0.74, 3.86$$

2.  $f(x) = x^3 + 3x^2 + 3x + 2$

$$x = -2, \frac{-1 \pm i\sqrt{3}}{2}$$

3.  $f(x) = x^4 - 3x^3 - 2x^2 - 6x - 8$

$$x = -1, 4, \pm i\sqrt{2}$$

4.  $f(x) = 2x^4 - x^3 - 8x^2 + x + 6$

$$x = -\frac{3}{2}, -1, 1, 2$$

5.  $y = (x - 2)(x + 2)(x + 2)$

$$= (x^2 - 4)(x + 2)$$

$$= x^3 + 2x^2 - 4x - 8$$

6.  $y = (x + 0)(x - 1)(x + 3)$

$$= (x + 0)(x^2 + 2x - 3)$$

$$= x^3 + 2x^2 - 3x$$

7.  $y = (x - 4)(x - 2 - i)(x - 2 + i)$

$$= (x - 4)(x - 2)^2 + 1$$

$$= x^3 - 8x^2 + 21x - 20$$

8.  $y = (x - 2)(x - 5)(x - i)(x + i)$

$$= (x^2 - 7x + 10)(x^2 + 1)$$

$$= x^4 - 7x^3 + 11x^2 - 7x + 10$$

9.  $y = (x - 4)(x - 2 + 3i)(x - 2 - 3i)$

$$= (x - 4)(x^2 - 4x + 13)$$

$$= x^3 - 8x^2 + 29x - 52$$

10.  $y = (x - 1 + i)(x - 1 - i)(x - 2 - 2i)(x - 2 + 2i)$

$$= (x^2 - 2x + 2)(x^2 - 4x + 8)$$

$$= x^4 - 6x^3 + 18x^2 - 24x + 16$$

11. local max: (0.79, 8.21)

local min: (-2.12, -4.06)

12. local max: (-0.5, 0.56)

local min: (-1.62, -1), (0.62, -1)

13. local max: (2.42, 0.77)

local min: (3.58, -0.77)

14. local max: (-3, 0)

local min: (-1.67, -1.19)

15.  $y = a(x + 2)(x - 2)(x + 4)$

$$3 = a(1)(-3)(3)$$

$$-\frac{1}{3} = a$$

$$y = -\frac{1}{3}(x + 2)(x - 2)(x + 4)$$

16.  $y = a(x + 1)(x - 4)(x - 2)$

$$1 = a(-2)(-7)(-5)$$

$$-\frac{1}{70} = a$$

$$y = -\frac{1}{70}(x + 1)(x - 4)(x - 2)$$

17.  $y = a(x + 0)(x - 3)(x - 5)$

$$6 = a(2)(-1)(-3)$$

$$1 = a$$

$$y = (x + 0)(x - 3)(x - 5)$$

18.  $y = a(x - 1)(x + 3)(x + 5)$

$$10 = a(-5)(-1)(1)$$

$$2 = a$$

$$y = 2(x - 1)(x + 3)(x + 5)$$

19.  $f(x) = x^3 - 3x^2 + x - 4$     20.  $f(x) = x^3 - 4x^2 + 2x$

21.  $N = -3.75x^3 + 50.9x^2 - 97.3x + 3210$  where  $x$  is the number of years since 1988.

### Chapter 6 Review (pp. 388-390)

1.  $\frac{4}{9} \cdot \frac{216x^3}{y^3} = \frac{96x^3}{y^3}$ ; negative exponent, power of a quotient, power of a product, and power of a power properties

2.  $\frac{x^4}{x^4} = 1$ ; negative exponent, product of powers, power of a power, and zero exponent properties

3.  $\frac{-63xy^9}{18x^{-2}y^3} = -\frac{7}{2}x^3y^6$ ; quotient of powers property

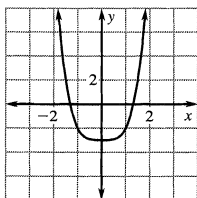
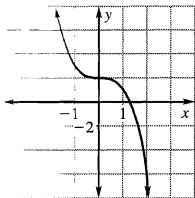
4.  $5x^2y^2 \cdot \frac{1}{25x^2y} = \frac{y}{5}$ ; negative exponent, quotient of powers, and zero exponent properties

# Chapter 6 continued

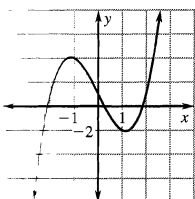
$$\begin{array}{r} 3 \phantom{00} -12 \phantom{00} 7 \\ 3 \phantom{00} 18 \phantom{00} -18 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \phantom{00} 6 \phantom{00} 25 \\ 1 \phantom{00} -5 \phantom{00} -3 \phantom{00} 1 \phantom{00} -5 \\ -1 \phantom{00} 6 \phantom{00} -3 \phantom{00} 2 \\ \hline 1 \phantom{00} -6 \phantom{00} 3 \phantom{00} -2 \phantom{00} -3 \end{array}$$

$= -x^3 + 2$       8.  $f(x) = x^4 - 3$



$x = x^3 - 4x + 1$



$3x^2 - x^2 + 1) - (x^3 + 3) = 2x^2 + x^2 - 2$

$-3)(x^2 + x - 7) = x^3 - 2x^2 - 10x + 21$

$-3)(x - 5)(2x + 1) = (x^2 - 2x - 15)(2x + 1)$   
 $= 2x^3 - 3x^2 - 32x - 15$

$= -64$

$x = -4$

$x^4 - 6x^2 - 27 = 0$

$-3)(x^3 - 3x^2 + 3x - 9) = 0$

$(x + 3)(x - 3)(x^2 + 3) = 0$

$= -3, 3$

$x^2(x + 3) - (x + 3) = 0$

$(x + 3)(x^2 - 1) = 0$

$-3)(x - 1)(x + 1) = 0$

$= -3, -1, 1$

$x^3 + 6x^2 + 5x + 2 + \frac{1}{x - 1}$

$-1) x^4 + 5x^3 - x^2 - 3x - 1$

$-x^4 + x^3$

$6x^3 - x^2$

$-6x^3 + 6x^2$

$5x^2 - 3x$

$-5x^2 + 5x$

$2x - 1$

$-2x + 2$

$1$

17.  $x^2 + \frac{5}{2} + \frac{33}{2(2x - 5)}$

$2x - 5 \overline{) 2x^3 - 5x^2 + 5x + 4}$   
 $\underline{-2x^3 + 5x^2}$

$5x$   
 $\underline{-5x + \frac{25}{2}}$   
 $\frac{33}{2}$

18.  $f(x) = x^3 + 12x^2 + 21x + 10$

$= (x + 1)(x^2 + 11 + 10)$

$= (x + 1)^2(x + 10)$

$x = -1, -10$

19.  $f(x) = x^4 + x^3 - x^2 + x - 2$

$= (x - 1)(x^3 + 2x^2 + x + 2)$

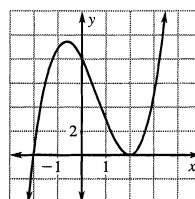
$= (x - 1)(x + 2)(x^2 + 1)$

$x = 1, -2$

20. x-intercepts: 2, -2

local max: (-0.68, 9.5)

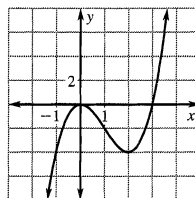
local min: (2, 0)



21. x-intercepts: 0, 3

local max: (0, 0)

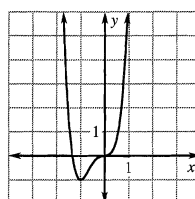
local min: (2, -4)



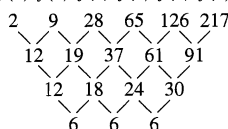
22. x-intercepts: 0, -1.34

local max: none

local min: (-1, -1)



23.  $f(1) f(2) f(3) f(4) f(5) f(6)$



## Chapter 6 continued

$$24. \quad y = a(x-1)(x+1)(x-4)$$

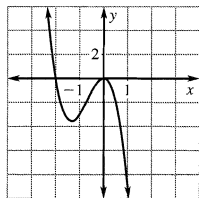
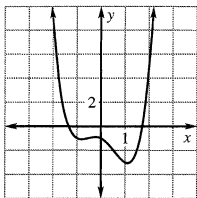
$$-12 = a(1)(3)(-2)$$

$$2 = a$$

$$y = 2(x-1)(x+1)(x-4)$$

### Chapter 6 Test (p. 391)

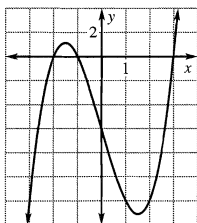
- $x^5$ ; quotient of powers property
- $729x^{18}$ ; power of a product and power of a power properties
- $x^{11}$ ; quotient of powers property
- $\frac{1}{512x^9y^6}$ ; power of a power, power of a product, and negative exponent properties
- $\frac{3}{y^3}$ ; product of a power, quotient of a power, zero exponent, and negative exponent properties
- $y = x^4 - 2x^2 - x - 1$     7.  $y = -3x^3 - 6x^2$



$$f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty, \quad f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty,$$

$$f(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty, \quad f(x) \rightarrow -\infty \text{ as } x \rightarrow +\infty,$$

- $y = (x-3)(x+1)(x+2)$   $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$ ,  
 $f(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$



- $x^2 - 14x + 8$
- $10x^3 - 17x^2 + 15x - 18$     11.  $x^3 - 13x - 12$
- $64x^3 + 343 = (4x+7)(16x^2 - 28x + 49)$
- $400x^2 - 25 = 25(4x-1)(4x+1)$
- $x^4 + 8x^2 - 9 = (x^2+9)(x-1)(x+1)$
- $2x^3 - 3x^2 + 4x - 6 = (2x-3)(x^2+2)$

$$16. \quad 3x^4 - 11x^2 - 20 = 0$$

$$(3x^2 + 4)(x^2 - 5) = 0$$

$$x = \pm\sqrt{5}, \pm\frac{2i\sqrt{3}}{3}$$

$$17. \quad 81x^4 - 16 = 0$$

$$(9x^2 - 4)(9x^2 + 4) = 0$$

$$(3x - 2)(3x + 2)(9x^2 + 4) = 0$$

$$x = \pm\frac{2}{3}, \pm\frac{2}{3}i$$

$$18. \quad 4x^3 - 8x^2 - x + 2 = 0$$

$$(x-2)(4x^2 - 1) = 0$$

$$(x-2)(2x-1)(2x+1) = 0$$

$$x = 2, \frac{1}{2}, -\frac{1}{2}$$

$$19. \quad -1 \left| \begin{array}{cccc} 8 & 5 & 4 & -1 & 7 \\ & -8 & 3 & -7 & 8 \end{array} \right.$$

$$8x^3 - 3x^2 + 7x - 8 + \frac{15}{x+1}$$

$$20. \quad -3 \left| \begin{array}{cccc} 12 & 31 & -17 & -6 \\ & -36 & 15 & 6 \end{array} \right.$$

$$12x^2 - 5x - 2$$

$$21. \quad 0, \pm 1, \pm 2, \pm 7, \pm 14;$$

$$f(x) = x^3 - 5x^2 - 14$$

$$= (x+0)(x^2 - 5x - 14)$$

$$= (x+0)(x-7)(x+2)$$

$$x = 0, 7, -2$$

$$22. \quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$$

$$f(x) = x^3 + 4x^2 + 9x + 36$$

$$= (x+4)(x^2 + 9)$$

$$x = -4, \pm 3i$$

$$23. \quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24;$$

$$f(x) = x^4 + x^3 - 2x^2 + 4x - 24$$

$$= (x+3)(x^3 - 2x^2 + 4x - 8)$$

$$= (x+3)(x-2)(x^2 + 4)$$

$$x = -3, 2, \pm 2i$$

$$24. \quad f(x) = (x-1)(x+3)(x-4)$$

$$= (x^2 + 2x - 3)(x-4)$$

$$= x^3 - 2x^2 - 11x + 12$$

$$25. \quad f(x) = (x-2)^2(x+1)x$$

$$= (x^2 - 4x + 4)(x^2 + x)$$

$$= x^4 - 3x^3 + 4x$$

$$26. \quad f(x) = (x-5)(x^2 + 4)$$

$$= x^3 - 5x^2 + 4x - 20$$

## Chapter 6 continued

$$27. f(x) = (x^2 - 9)(x - 2 + i)(x - 2 - i)$$

$$= (x^2 - 9)(x^2 - 4x + 5)$$

$$= x^4 - 4x^3 - 4x^2 + 36x - 45$$

$$28. f(x) = 0.25x^3 - 7x^2 + 15$$

$$x = -1.428, 1.505, 27.923$$

$$29. f(x) = \frac{1}{9}(x - 3)^2(x + 3)^2$$

x-intercepts:  $\pm 3$

local max:  $(0, 9)$

local min:  $(-3, 0), (3, 0)$

$f(x) \rightarrow +\infty$  as  $x \rightarrow -\infty$

$f(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$

$$30. \begin{array}{cccccc} f(1) & f(2) & f(3) & f(4) & f(5) & f(6) \\ 7 & 20 & 83 & 256 & 623 & 1292 \\ & \swarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 13 & 63 & 173 & 367 & 669 \\ & & \swarrow & \downarrow & \downarrow & \downarrow \\ & & 50 & 110 & 194 & 302 \\ & & & \swarrow & \downarrow & \downarrow \\ & & & 60 & 84 & 108 \\ & & & & \swarrow & \downarrow \\ & & & & 24 & 24 \end{array}$$

$$31. f(n) = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$$

$$32. (7.5 \times 10^{13})(1.0 \times 10^{-3}) = 7.5 \times 10^{10} \text{ in.} \times \frac{1 \text{ mi}}{12 \text{ in.}} \times \frac{1 \text{ mi}}{5280 \text{ ft}}$$

$$= 1.1837 \times 10^6 \text{ mi}$$

### Chapter 6 Standardized Test (pp. 392-393)

$$1. -4^0 = -1 \quad 2. f(x) = 7x^4 - 3x^3 + 8x^2 + x - 9$$

D

$$f(-1) = 7 + 3 + 8 - 1 - 9$$

$$f(-1) = 8$$

A

$$3. f(x) = x^4 + 1$$

$$f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty$$

A

$$4. f(t) = 0.141t^4 - 5.577t^3 + 790.32t^2 + 5382.6t + 343,539$$

$$f(2) \approx 3.8 \times 10^5$$

C

$$5. (2x + 1)(4x^2 - 2x + 1) = 8x^3 + 1$$

E

$$6. x^5 = 246x$$

$$x = 0, \pm 4$$

A

$$7. \begin{array}{cccc} 4 & -11 & -9 & -5 \\ & 16 & 20 & 44 \\ \hline 4 & 5 & 11 & 39 \end{array}$$

$$x^2 + 5x + 11 + \frac{39}{x - 4}$$

B

$$8. f(x) = x^3 - 8x^2 + x + 42 \quad 9. f(x) = -3x^4 + x + 2$$

$$f(x) = (x + 2)(x - 3)(x - 7) \quad 4 \text{ zeros}$$

$$x = -2, 3, 7 \quad \text{E}$$

$$10. y = a(x + 3)(x - 1)(x - 4) \quad 11. D \quad 12. A$$

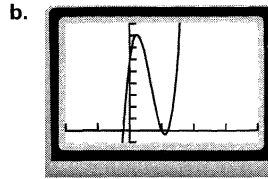
$$7 = a(12)$$

$$\frac{7}{12} = a$$

$$y = \frac{7}{12}(x + 3)(x - 1)(x - 4)$$

D

$$13. a. f(x) = x^3 - 10x^2 + 19x + 30$$



c. no; the local max occurs at about  $(1.15, 40.15)$  and the local min occurs at about  $(5.52, -1.63)$ , but  $x$  must be greater than 6 for the side of length  $x - 6$  to have a positive measure.

$$d. 0 = x^3 - 10x^2 + 19x - 190$$

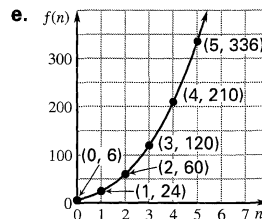
$$x = 10$$

11 ft by 5 ft by 4 ft

$$14. a. 3 \quad b. f(n) = n^3 + 6n^2 + 11n + 6$$

c.  $f(n) = (n + 1)(n + 2)(n + 3)$ ; for prism  $n$ , the dimensions are  $(n + 1)$  by  $(n + 2)$  by  $(n + 3)$

$$d. f(49) = (50)(51)(52) = 132,600$$



The domain is all whole numbers.