Chapter 8

Lesson 8.1

Think & Discuss *(p.463)*

1. Atmospheric pressure decreases as altitude increases.
2. About 7 lb/in^2.

Skills Review *(p. 464)*

1. $4^{-3} = \frac{1}{64}$ **2.** $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ **3.** $\left(\frac{3}{4}\right)^0 = 1$

4. $-5^2 = -(5^2) = -(25) = -25$ **5.** $\left(\frac{5}{2}\right)^{-1} = \frac{2}{5}$

6. $f(x) = 2x^3$ $\quad f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

7. $f(x) = -x^2$ $\quad f(x) \to -\infty$ as $x \to -\infty$

$f(x) = -(x^2)$ $\quad f(x) \to -\infty$ as $x \to +\infty$

8. $f(x) = 4x^4$ $\quad f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

9. $f(x) = -5x^3$ $\quad f(x) \to +\infty$ as $x \to -\infty$

$f(x) = -5(x^3)$ $\quad f(x) \to -\infty$ as $x \to +\infty$

10. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{5.9 - 2.2}{10 - 1}$

$= \dfrac{3.7}{9} = 0.411$

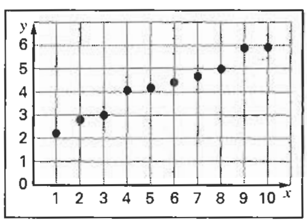
$y = mx + b$

$5.9 = (0.411)(10) + b$

$5.9 = 4.11 + b$

$1.79 = b$

$y = 0.411x + 1.79$



Activity Developing Concepts (p. 465)

1. $y = \frac{1}{3} \cdot 2^x$

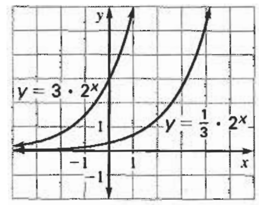
x	y
-3	$\frac{1}{3} \cdot \frac{1}{8} = \frac{1}{24}$
-2	$\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$
-1	$\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$
0	$\frac{1}{3} \cdot 1 = \frac{1}{3}$
1	$\frac{1}{3} \cdot 2 = \frac{2}{3}$
2	$\frac{1}{3} \cdot 4 = \frac{4}{3}$
3	$\frac{1}{3} \cdot 8 = \frac{8}{3}$

The graph passes through the point $\left(0, \frac{1}{3}\right)$. The x-axis is an asymptote of the graph. The domain is all real numbers. The range is $y > 0$.

$y = 3 \cdot 2^x$

x	y
-3	$3 \cdot \frac{1}{8} = \frac{3}{8}$
-2	$3 \cdot \frac{1}{4} = \frac{3}{4}$
-1	$3 \cdot \frac{1}{2} = \frac{3}{2}$
0	$3 \cdot 1 = 3$
1	$3 \cdot 2 = 6$
2	$3 \cdot 4 = 12$
3	$3 \cdot 8 = 24$

The graph passes through the points $(0, 3)$. The x-axis is an asymptote of the graph. The domain is all real numbers. The range is $y > 0$.



$y = \frac{1}{3} \cdot 2x$

This graph is $\frac{1}{3}$ of the graph $y = 2^x$.

$f(x) \to +\infty$ as $x \to +\infty$

$f(x) \to 0$ as $x \to -\infty$.

$y = 3 \cdot 2^x$

This graph is 3 times the graph $y = 2^x$.

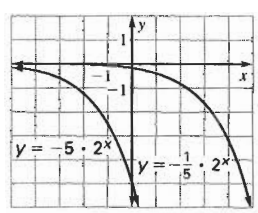
$f(x) \to +\infty$ as $x \to +\infty$

$f(x) \to 0$ as $x \to -\infty$.

2. $y = -\frac{1}{5} \cdot 2^x$

x	y
-3	$-\frac{1}{5} \cdot \frac{1}{8} = -\frac{1}{40}$
-2	$-\frac{1}{5} \cdot \frac{1}{4} = -\frac{1}{20}$
-1	$-\frac{1}{5} \cdot \frac{1}{2} = -\frac{1}{10}$
0	$-\frac{1}{5} \cdot 1 = -\frac{1}{5}$
1	$-\frac{1}{5} \cdot 2 = -\frac{2}{5}$
2	$-\frac{1}{5} \cdot 4 = -\frac{4}{5}$
3	$-\frac{1}{5} \cdot 8 = -\frac{8}{5}$

The graph passes through the point $\left(0, -\frac{1}{5}\right)$. Both graphs have the x-axis as an asymptote. Both graphs' domain is all real numbers.



$y = -5 \cdot 2^x$

x	y
-3	$-5 \cdot \frac{1}{8} = -\frac{5}{8}$
-2	$-5 \cdot \frac{1}{4} = -\frac{5}{4}$
-1	$-5 \cdot \frac{1}{2} = -\frac{5}{2}$
0	$-5 \cdot 1 = -5$
1	$-5 \cdot 2 = -10$
2	$-5 \cdot 4 = -20$
3	$-5 \cdot 8 = -40$

The graph passes through the point $(0, -5)$. Both graphs' range is $y < 0$.

$y = -\frac{1}{5} \cdot 2^x$

This graph is $\frac{1}{5}$ of the graph $y = 2^x$.

$f(x) \to -\infty$ as $x \to +\infty$

$f(x) \to 0$ as $x \to -\infty$

$y = -5 \cdot 2^x$

This graph is 5 times the graph $y = 2^x$.

$f(x) \to -\infty$ as $x \to +\infty$

$f(x) \to 0$ as $x \to -\infty$

3. If $0 < a < 1$, then the graph of $y = a \cdot 2^x$ lies below that of $y = 2^x$, while if $a > 1$, the graph of $y = a \cdot 2^x$ lies above that of $y = 2^x$. In either case, the graph has the same end behavior and general shape. If $-1 < a < 0$, then the graph of $y = a \cdot 2^x$ lies closer to the x-axis than that of $y = 2^x$, but below the axis instead of above it. If $a < -1$, the graph of $y = a \cdot 2^x$ lies below the x-axis, but grows away from the axis more quickly than that of $y = 2^x$. In both cases where a is negative, the graph approaches the x-axis asymptotically as $x \to -\infty$, and $y \to -\infty$ as $x \to +\infty$. In all cases, the y-intercept of the graph is a.

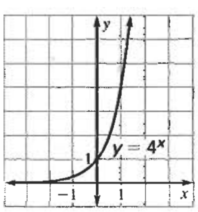
8.1 Guided Practice (p. 469)

1. An asymptote is a line that a graph approaches more and more closely.

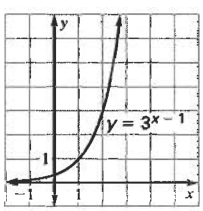
2. If $a < 0$, the graph lies below the line $y = k$, and approaches it asymptotically from below. If $a > 0$, the graph lies above the line $y = k$, and approaches it asymptotically from above. The graph of $y = ab^{x-h} + k$ is the same as that of $y = ab^x$ translated horizontally h units and vertically k units.

3. $b > 1$

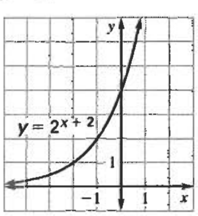
4. $y = 4^x$



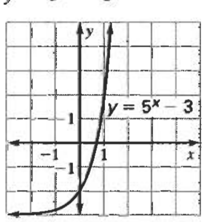
5. $y = 3^{x-1}$



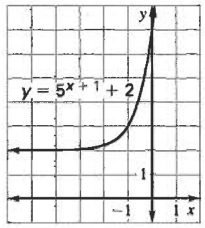
6. $y = 2^{x+2}$



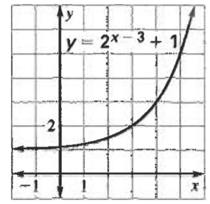
7. $y = 5^x - 3$



8. $y = 5^{x+1} + 2$



9. $y = 2^{x-3} + 1$



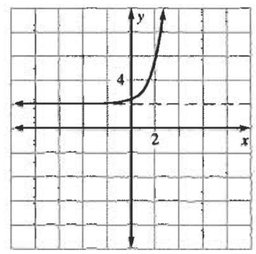
10. $y = 3 \cdot 4^{x-1} + 2$

asymptote: $y = 2$

$y = 3 \cdot 4^{2-1} + 2$

$y = 3 \cdot 4 + 2$

$y = 12 + 2 = 14$



11. $P = 6191(1.04)^t$

t = number of years since 1990

population in 1990 = 6191

population increased by 4% each year

12. a. $A = 500\left(1 + \frac{0.03}{1}\right)^{1 \cdot 2}$

$= 500(1.03)^2$

$= \$530.45$

b. $A = 500\left(1 + \frac{0.03}{4}\right)^{4 \cdot 2}$

$= 500(1.0075)^{4 \cdot 2}$

$= 500(1.0075)^8$

$= \$530.80$

c. $A = 500\left(1 + \frac{0.03}{365}\right)^{365 \cdot 2}$

$= 500(1.000082192)^{730}$

$= \$530.92$

Practice and Applications (p. 469)

13. $y = 5^x \quad x = 0 \quad y = 5^0$

y-intercept is 1.

The asymptote is the x-axis.

14. $y = -2 \cdot 4^x \quad x = 0 \quad y = -2 \cdot 4^0 \quad y = -2 \cdot 1$

y-intercept is -2.

The asymptote is the x-axis.

15. $y = 4 \cdot 2^x \quad x = 0 \quad y = 4 \cdot 2^0 \quad y = 4 \cdot 1$

y-intercept is 4.

The asymptote is the x-axis.

16. $y = 2^x - 1 \quad x = 0 \quad y = 2^0 - 1 \quad y = 1 - 1$

y-intercept is 0.

The asymptote is the line $y = -1$.

17. $y = 3 \cdot 2^{x-1} \quad x = 0 \quad y = 3 \cdot 2^{0-1} \quad y = 3 \cdot \frac{1}{2}$

y-intercept is $\frac{3}{2}$.

The asymptote is the x-axis.

18. $y = 2 \cdot 3^{x-4}$ $x = 0$ $y = 2 \cdot 3^{0-4}$

$y = 2 \cdot 3^{-4}$

$y = 2 \cdot \frac{1}{81}$

y-intercept is $\frac{2}{81}$.

The asymptote is the x-axis.

19. $y = 2 \cdot 5^x$ (C)

20. $y = 3 \cdot 4^x$ (E)

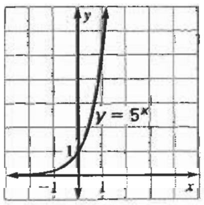
21. $y = -2 \cdot 5^x$ (B)

22. $y = \frac{1}{3} \cdot 4^x$ (A)

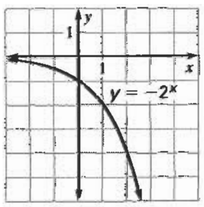
23. $y = 3^{x-2}$ (F)

24. $y = 3^{x-2}$ (D)

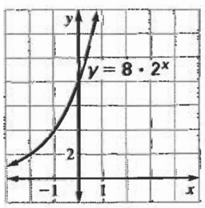
25. $y = 5^x$



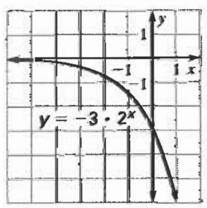
26. $y = -2^x$



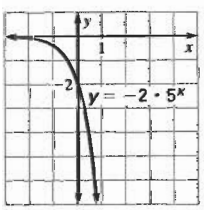
27. $y = 8 \cdot 2^x$



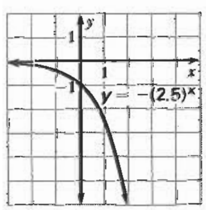
28. $y = -3 \cdot 2^x$



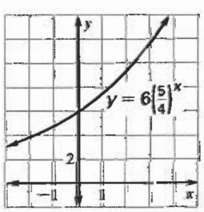
29. $y = -2 \cdot 5^x$



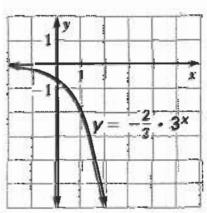
30. $y = -(2.5)^x$



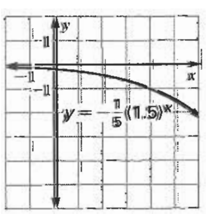
31. $y = 6\left(\frac{5}{4}\right)^x$



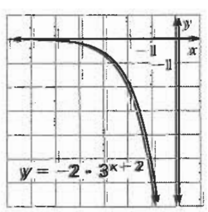
32. $y = -\frac{2}{3} \cdot 3^x$



33. $y = -\frac{1}{5}(1.5)^x$

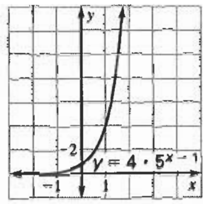


34. $y = -2 \cdot 3^{x+2}$



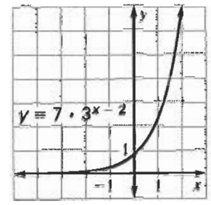
Domain: all real numbers
Range: $y < 0$

35. $y = 4 \cdot 5^{x-1}$



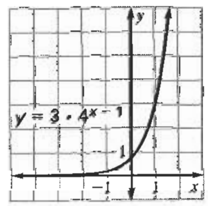
Domain: all real numbers
Range: $y > 0$

36. $y = 7 \cdot 3^{x-2}$



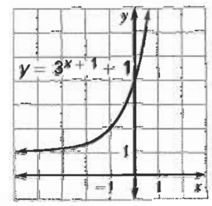
Domain: all real numbers
Range: $y > 0$

37. $y = 3 \cdot 4^{x-1}$



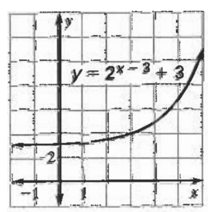
Domain: all real numbers
Range: $y > 0$

38. $y = 3^{x+1} + 1$



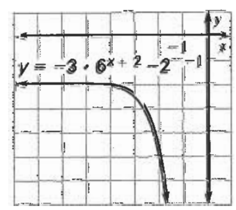
Domain: all real numbers
Range: $y > 1$

39. $y = 2^{x-3} + 3$



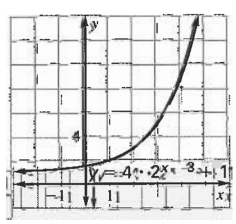
Domain: all real numbers
Range: $y > 3$

40. $y = -3 \cdot 6^{x+2} - 2$



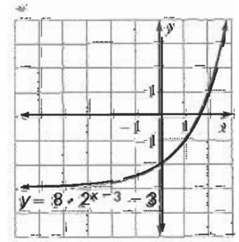
Domain: all real numbers
Range: $y < -2$

41. $y = 4 \cdot 2^{x-3} + 1$



Domain: all real numbers
Range: $y > 1$

42. $y = 8 \cdot 2^{x-3} - 3$



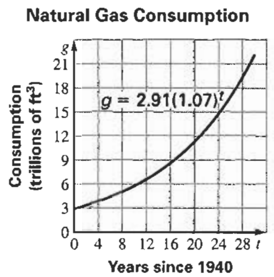
Domain: all real numbers
Range: $y > -3$

43. initial amount: 2.91 trillion ft^3

growth factor: 1.07

annual percent increase: 7%

44. $g = 2.91(1.07)^t$



45. $g = 2.91(1.07)^{15}$

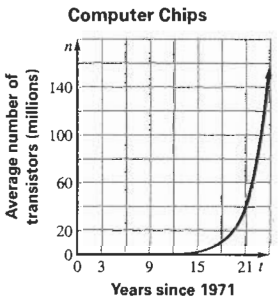
$g = 8.03$ trillion ft³

46. initial amount: 2300

growth factor: 1.59

annual percent increase: 59%

47.



48. $g = 2300(1.59)^{27}$

$g = 630{,}159{,}071.7$

about 630 million transistors

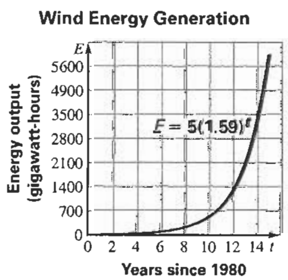
49. $E = 5(1.59)^t$

$E = 5(1.59)^4$

$E \approx 31.9$

about 32 gigawatt-hours

50.



51. Approximately 6 years

1986

$E = 5(1.59)^6$

$E \approx 81$ gigawatt-hours

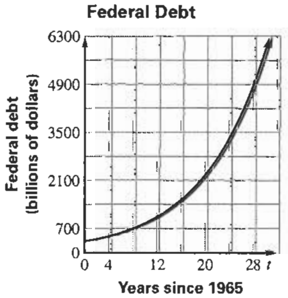
52. $D = 322.3(1.102)^t$

$= 322.3(1.102)^{15}$

$= \$1383.52$ billion

about \$1.384 trillion

53.



54. $D = 322.3(1.102)^t$

approximately 19 years

during 1984

$D = 322.3(1.102)^{19}$

$= \$2{,}040.378$ billion

$= 322.3(1.102)^{19.4}$

$= \$2{,}121.208$ billion

$\approx \$2{,}120$ billion

55. a. $A = 2500(1.04)^1$

$= \$2600$

b. $A = 2500(1.04)^5$

$= \$3041.63$

c. $A = 2500\left(1 + \frac{0.04}{4}\right)^{4.1}$

$A = 2500\left(1 + \frac{0.04}{4}\right)^4$

ANS + ANS × 0.01; push "ENTER" four times.

$A = 2500(1.01)^4$

d. $A = 2500\left(1 + \frac{0.04}{4}\right)^{4 \cdot 5}$

$= 2500(1.01)^{20}$

$= \$3050.48$;

this is \$3050.48 − \$3041.63 = \$8.85 more

56. $V = 110(1.04)^t$

57. $A = 400\left(1 + \frac{0.02}{4}\right)^{4t}$

$= 400(1.005)^{4t}$

where t is the number of years

58. $V = 525(1.05)^t$

59. $A = 1600\left(1 + \frac{0.025}{12}\right)^{12 \cdot 3}$

$= 1600(1.0021)^{36}$

$= \$1724.48$

60. $A = 1600\left(1 + \frac{0.0175}{4}\right)^{4 \cdot 3}$

$= 1600(1.004375)^{12}$

$= \$1686.05$

61. $A = 1600(1.04)^{1 \cdot 3}$

$= \$1799.78$

62. $2500 = P\left(1 + \frac{0.0225}{12}\right)^{24}$

$P = \frac{2500}{(1.001875)^{24}}$

$P = \frac{2500}{1.045983787}$

$P = \$2390.09$

63. $2500 = P\left(1 + \frac{0.02}{4}\right)^8$

$P = \frac{2500}{(1.005)^8}$

$P = \frac{2500}{1.040707044}$

$P = \$2402.21$

64. $2500 = P(1.05)^2$

$$P = \frac{2500}{(1.05)^2}$$

$$P = \$2267.57$$

65. $\text{Juan} = 200\left(1 + \frac{0.03}{12}\right)^{12 \cdot 1}$

$= 200(1.0025)^{12}$

$= \$206.08$ after 1 year

$\text{Juan} = \$200 + \$206.08\left(1 + \frac{0.03}{12}\right)^{12 \cdot 1}$

$= 406.08(1.0025)^{12}$

$= \$418.43$ after 2 years

$\text{Juan} = \$200 + \$418.43\left(1 + \frac{0.03}{12}\right)^{12 \cdot 1}$

$= 618.43(1.0025)^{12}$

$= \$637.24$ after 3 years

$\text{Juan} = \$200 + \$637.24\left(1 + \frac{0.03}{12}\right)^{12 \cdot 1}$

$= 837.24(1.0025)^{12}$

$= \$862.71$ after 4 years

$\text{Michelle} = 800\left(1 + \frac{0.03}{12}\right)^{12 \cdot 4}$

$= 800(1.0025)^{48}$

$= \$901.86$ after 4 years

No. Michelle earned $\$901.86 - \$862.71 = \$39.15$ more than Juan because she had \$800 to earn interest right away, while only part of Juan's \$800 will earn interest each year.

66. a. $V = 30{,}000(1.05)^t$

b. $V = 30{,}000(1.05)^{50}$

$= \$344{,}022$

approximately \$344,000

67. No, $8000(1.06)^t \neq 4000(1.05)^t + 4000(1.07)^t$. The spilt accounts will out perform the single account after the first year.

$A = 8000(1.06)^2$ $\quad A = 4000(1.05)^2 + 4000(1.07)^2$

$= \$8988.80$ $\quad = \$4410 + \$4579.60$

$= \$8989.60$

68. (B) $E = 1240(1.15)^t$

69. (D) $f(x) = 2 \cdot 3^{x-1} + 6$

70. $3^{14/10} = 4.65554$, $3^{141/100} = 4.70697$, $3^{1414/1000} = 4.72770$, $3^{14{,}142/10{,}000} = 4.72873$, $3^{141{,}421/100{,}000} = 4.72879$, $3^{1{,}414{,}213/1{,}000{,}000} = 4.72880$

These successive powers approach a limit, which can be defined to be $3^{\sqrt{2}}$.

71. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ **72.** $\left(\frac{3}{7}\right)^3 = \frac{27}{343}$ **73.** $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

74. $\left(\frac{5}{8}\right)^4 = \frac{625}{4096}$ **75.** $\left(\frac{7}{12}\right)^3 = \frac{343}{1728}$ **76.** $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$

77. $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$ **78.** $\left(\frac{3}{10}\right)^5 = \frac{243}{100{,}000}$ **79.** $8^{3/8} = 2.18$

80. $15{,}625^{1/6} = 5$ **81.** $-243^{1/5} = -3$ **82.** $1024^{1/5} = 4$

83. $10^{1/2} = 3.16$ **84.** $106^{1/3} = 4.73$ **85.** $\sqrt[4]{81} = 3$

86. $\sqrt[7]{100} = 1.93$ **87.** $\sqrt[3]{28} = 3.04$ **88.** $\sqrt[4]{120} = 3.31$

89. $\sqrt[4]{9} = 1.73$ **90.** $\sqrt[6]{180} = 2.38$

91. $f(x) + g(x)$

$4x^2 + 6x - 11$

all real numbers

92. $f(x) - g(x)$

$-4x^2 + 6x - 11$

all real numbers

93. $f(x) \cdot g(x)$

$(6x - 11)4x^2 = 24x^3 - 44x^2$

all real numbers

94. $g(x) - f(x)$

$4x^2 - 6x + 11$

all real numbers

95. $f(g(x))$

$6(4x^2) - 11 = 24x^2 - 11$

all real numbers

96. $g(f(x))$

$4(6x - 11)^2 = 4(6x - 11)(6x - 11)$

$= 4(36x^2 - 132x + 121)$

$= 144x^2 - 528x + 484$

all real numbers

97. $\frac{f(x)}{g(x)} = \frac{6x - 11}{4x^2}$; all non-zero real numbers

98. $\frac{g(x)}{f(x)} = \frac{4x^2}{6x - 11}$; all real numbers except $\frac{11}{6}$

99. $f(f(x)) = 6(6x - 11) - 11 = 36x - 66 - 11$

$= 36x - 77$

all real numbers

100. 40 ft of fencing

$A = 90 = L \cdot W$

$P = 40 = 4S$

$S = 10$

$A = 90 = (10 \cdot 10) - 10 = 10^2 - 10$

$A = 90 = \left(10 + \sqrt{10}\right)\left(10 - \sqrt{10}\right)$

$L = 10 + \sqrt{10} \approx 6.84$ ft

$W = 10 - \sqrt{10} \approx 13.16$ ft

Lesson 8.2

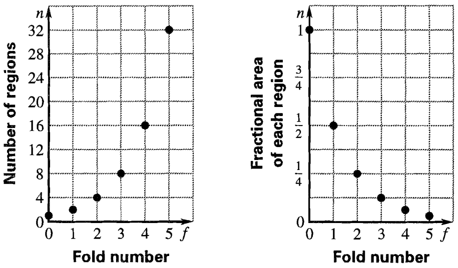
Developing Concepts Activity 8.2 (p. 473)
Exploring the Concept

2. Original piece of paper is folded into 4 regions; $\frac{1}{4}$ of the paper's area is in each region.

3.

Fold Number	0	1	2	3	4	5
Number of Regions	1	2	4	8	16	32
Fractional Area of each Region	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

4.



Drawing Conclusions (p. 473)

1. $y = 2^x$ **2.** $y = 2^8 = 256$ **3.** $y = \left(\frac{1}{2}\right)^x$

4. $y = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$

5. $(2^x)\left(\frac{1}{2}\right)^x = \left(\frac{2}{2}\right)^x = 1$

The original area is one. At each stage the number of regions times the area of each region must continue to equal one whole.

8.2 Guided Practice (p. 477)

1. $y = 1500(0.65)^t$

initial amount $= 1500$

decay factor $= 0.65$

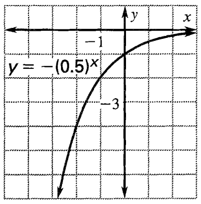
percent decrease is 35% $(1 - 0.65)$

2. $y = 2\left(\frac{1}{5}\right)^{x-2} + 3$

The asymptote is $y = 3$.

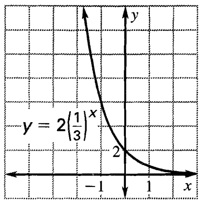
3. $0 < b < 1$

4.



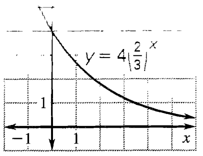
Domain: all real numbers
Range: $y < 0$

5.

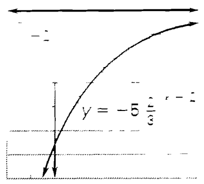


Domain: all real numbers
Range: $y > 0$

6.

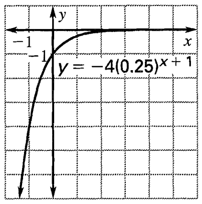


Domain: all real numbers
Range: $y > 0$



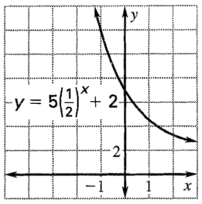
Domain: all real numbers
Range: $y < 0$

8.



Domain: all real numbers
Range: $y < 0$

9.



Domain: all real numbers
Range: $y > 2$

10. $y = 50(0.92)^t$

a. initial amount 50 g **b.** 8% $(1 - 0.08) = (0.92)$

8.2 Practice and Applications (pp. 477–479)

11. $f(x) = 4\left(\frac{3}{8}\right)^x$ exponential decay

12. $f(x) = 10 \cdot 3^x$ exponential growth

13. $f(x) = 8 \cdot 7^{-x} = 8 \cdot \left(\frac{1}{7}\right)^x$ exponential decay

14. $f(x) = 8 \cdot 7^x$ exponential growth

15. $f(x) = 5\left(\frac{1}{8}\right)^x = 5 \cdot (8)^x$ exponential growth

16. $f(x) = 3\left(\frac{4}{3}\right)^x$ exponential growth

17. $f(x) = 8\left(\frac{2}{3}\right)^x$ exponential decay

18. $f(x) = 5(0.25)^{-x} = 5\left(\frac{1}{0.25}\right)^x = 5 \cdot (4)^x$

exponential growth

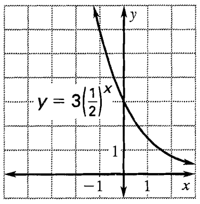
19. $y = (0.25)^x = \left(\frac{1}{4}\right)^x$ (F) **20.** $y = -3^{x-1} + 3$ (E)

21. $y = -\left(\frac{1}{3}\right)^{x-1} + 3$ (D) **22.** $y = \left(\frac{1}{2}\right)^{x-1}$ (B)

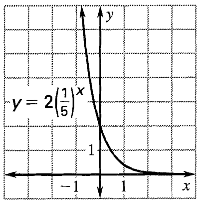
23. $y = -(0.25)^x = -\left(\frac{1}{4}\right)^x$ (C)

24. $y = (0.5)^x - 1$ (A)

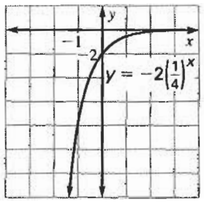
25. $y = 3\left(\frac{1}{2}\right)^x$



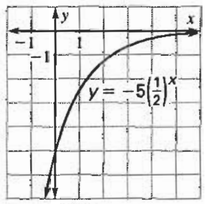
26. $y = 2\left(\frac{1}{5}\right)^x$



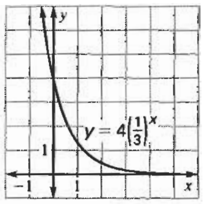
27. $y = -2\left(\frac{1}{4}\right)^x$



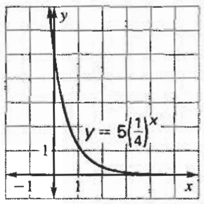
28. $y = -5\left(\frac{1}{2}\right)^x$



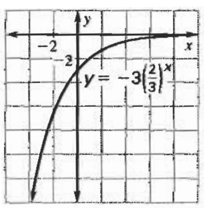
29. $y = 4\left(\frac{1}{3}\right)^x$



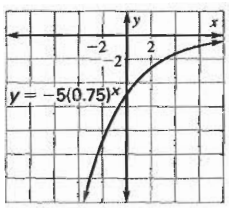
30. $y = 5\left(\frac{1}{4}\right)^x$



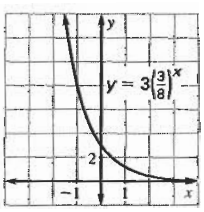
31. $y = -3\left(\frac{2}{3}\right)^x$



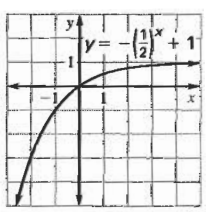
32. $y = -5(0.75)^x$



33. $y = 3\left(\frac{3}{8}\right)^x$

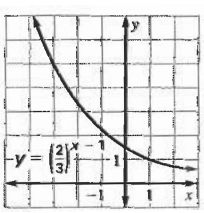


34. $y = -\left(\frac{1}{2}\right)^x + 1$



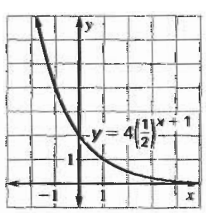
Domain: all real numbers
Range: $y < 1$

35. $y = \left(\frac{2}{3}\right)^{x-1}$



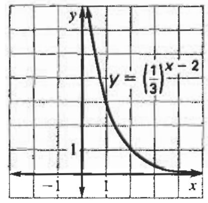
Domain: all real numbers
Range: $y > 0$

36. $y = 4\left(\frac{1}{2}\right)^{x+1}$



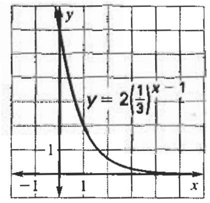
Domain: all real numbers
Range: $y > 0$

37. $y = \left(\frac{1}{3}\right)^{x-2}$



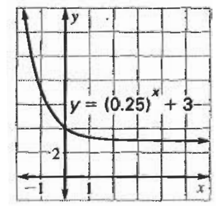
Domain: all real numbers
Range: $y > 0$

38. $y = 2\left(\frac{1}{3}\right)^{x-1}$



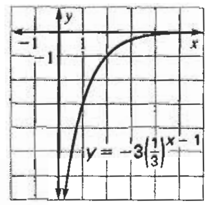
Domain: all real numbers
Range: $y > 0$

39. $y = (0.25)^x + 3$



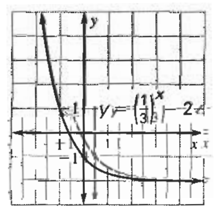
Domain: all real numbers
Range: $y > 3$

40. $y = -3\left(\frac{1}{3}\right)^{x-1}$



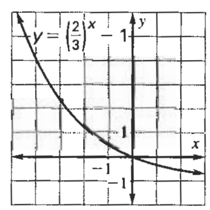
Domain: all real numbers
Range: $y < 0$

41. $y = \left(\frac{1}{3}\right)^x - 2$



Domain: all real numbers
Range: $y > -2$

42. $y = \left(\frac{2}{3}\right)^x - 1$



Domain: all real numbers
Range: $y > -1$

43. $V = 780(0.95)^t$ **44.** $c = 120(0.88)^h$

45. $i = 400(0.71)^h$

46. $P = 100(0.99997)^t$

$P = 100(0.99997)^{20{,}000}$

$P \approx 54.88$ g

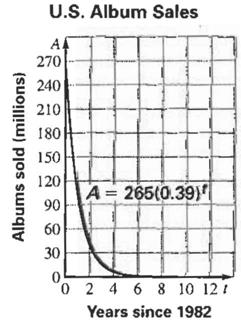
47. $A = 265(0.39)^t$

initial amount: 265

decay factor: 0.39

annual percent decrease: 61%

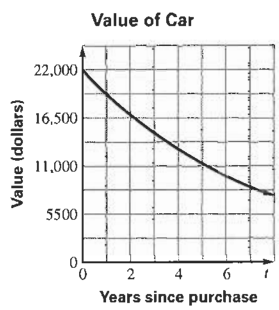
48. U.S. Album Sales



49. The graph shows 1 million albums sold in about 6 years since 1982, or 1988.

50. $V = 22{,}000(0.875)^t$
$= 22{,}000(0.875)^3$
$= \$14{,}738$

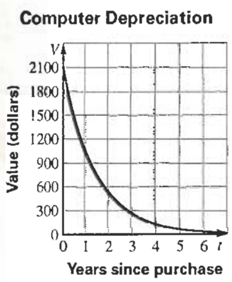
51.



52. Estimate value = \$8,000
≈ 7.6 years

53. $V = \$2{,}100(0.5)^t$
$= \$2{,}100(0.5)^2$
$= \$525$

54.



55. Value \$600 ≈ 22 months

56. $V = 500(0.88)^n$
$= 500(0.88)^{240}$
$= 2.4 \times 10^{-11}$ mL

57. a. $V = 18{,}354(0.83)^t$

b. $$A(n) = \left[18{,}354 - \frac{280}{\left(\frac{0.085}{12}\right)}\right]\left(1 + \frac{0.085}{12}\right)^n + \frac{280}{\left(\frac{0.085}{12}\right)}$$
$$= \left(18{,}354 - \frac{280}{0.0071}\right)(1 + 0.0071)^n + \frac{280}{0.0071}$$
$$= (18{,}354 - 39{,}437)(1.0071)^n + 39{,}437$$
$$= (-21{,}083)(1.007)^n + 39{,}437$$

c.

Value of the car	*Payoff*	*Years after purchase*
\$15,234	\$16,486	1
\$12,644	\$14,452	2
\$10,494	\$12,238	3
\$8,711	\$9,828	4
\$7,230	\$7,205	5

$A_{(12)} = (-21{,}083)(1.0071)^{12} + 39{,}437$
$= (-22{,}951) + 39{,}437$
$= \$16{,}486$

$A_{(24)} = (-21{,}083)(1.0071)^{24} + 39{,}437$
$= (-24{,}985) + 39{,}437$
$= \$14{,}452$

$A_{(36)} = (-21{,}083)(1.0071)^{36} + 39{,}437$
$= (-27{,}199) + 39{,}437$
$= \$12{,}238$

$A_{(48)} = (-21{,}083)(1.0071)^{48} + 39{,}437$
$= (-29{,}609) + 39{,}437$
$= \$9{,}828$

$A_{(60)} = (-21{,}083)(1.0071)^{60} + 39{,}437$
$= (-32{,}232) + 39{,}437$
$= \$7{,}205$

It would make the most sense to sell the car after the fifth year, when the value is more than the amount owed. You could sell the car for enough money to pay off the rest of the loan.

58. The product of two exponential decay functions is always another exponential decay function. Because b is less than one and greater than zero, then the products of two is another exponential decay function.

$$y = ab^x,\ 0 < b < 1 \text{ let } b = \frac{1}{c}$$
$$y = a\left(\frac{1}{c}\right)^x \cdot a\left(\frac{1}{c}\right)^x$$
$$y = a^2\left(\frac{1}{c}\right)^{2x}$$

The quotient of two exponential decay function is not always another exponential decay function.

$$y = ab^x,\ 0 < b < 1$$
$$y = \frac{ab^x}{ab^x} = ab^x \cdot ab^{-x} = a^2b^{x-x} = a^2b^0 = a^2$$

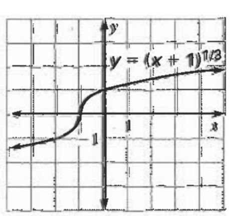
Example:

$$y = \frac{2\left(\frac{1}{3}\right)^x}{2\left(\frac{1}{3}\right)^x} = 2\left(\tfrac{1}{3}\right)^x \cdot 2\left(\tfrac{1}{3}\right)^{-x} = 4\left(\tfrac{1}{3}\right)^{x-x} = 4\left(\tfrac{1}{3}\right)^0 = 4$$

Mixed Review (p. 479)

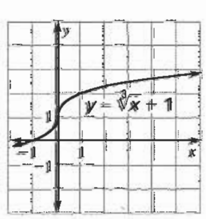
59. $y = (x + 1)^{1/3} = \sqrt[3]{(x + 1)}$

x	*y*
0	1
7	2
−2	−1
−9	−2



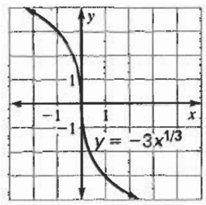
60. $y = \sqrt[3]{x} + 1$

x	*y*
1	2
8	3
0	1
−1	0
−8	−1



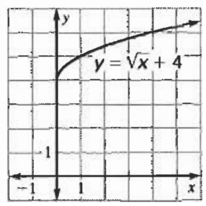
61. $y = -3x^{1/3} = -3\sqrt[3]{x}$

x	y
0	0
1	-3
-1	3
8	-6
-8	6



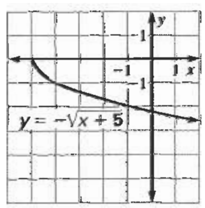
62. $y = \sqrt{x} + 4$

x	y
0	4
1	5
4	6
2	5.41
3	5.73



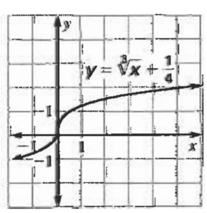
63. $y = -\sqrt{x+5}$

x	y
0	$-\sqrt{5} \approx -2.24$
1	$-\sqrt{6} \approx -2.45$
-1	-2
2	$-\sqrt{7} \approx -2.65$
-2	$-\sqrt{3} \approx -1.73$
3	$-\sqrt{8} \approx -2.83$
-3	$-\sqrt{2} \approx -1.41$
4	-3
-4	-1
-5	0



64. $y = \sqrt[3]{x} + \frac{1}{4}$

x	y
0	$\frac{1}{4}$
1	$\frac{5}{4}$
-1	$-\frac{3}{4}$
8	$\frac{9}{4}$
-8	$-\frac{7}{4}$



65. 11, 18, 13, 15, 17, 15, 23, 20, 12

Mean:

$11 + 18 + 13 + 15 + 17 + 15 + 23 + 20 + 12 = 144$

$144 \div 9 = 16$

Median: 11 12 13 15 <u>15</u> 17 18 20 23; 15

Mode: 15 appears twice; 15

Range: $23 - 11 = 12$

66. 25, 30, 32, 42, 31, 33, 36, 22

Mean:

$25 + 30 + 32 + 42 + 31 + 33 + 36 + 22 = 251$

$251 \div 8 = 31.375$

Median: 22 25 30 31 ↓ 32 33 36 42; 31.5

Mode: none

Range: $42 - 22 = 20$

67. a. $A = 2000\left(1 + \frac{0.07}{4}\right)^{4 \cdot 4}$
$= 2000(1.0175)^{16}$
$= \$2639.86$

b. $A = 2000\left(1 + \frac{0.05}{12}\right)^{12 \cdot 4}$
$= 2000\left(1 + \frac{0.05}{12}\right)^{48}$
$= \$2441.79$

Developing Concepts Activity (p. 480)

1.

n	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \frac{1}{n}\right)^n$	2.594	2.705	2.717	2.718	2.718	2.718

2. Yes; approaching the fixed decimal 2.718

Lesson 8.3

Guided Practice (p. 483)

1. The Euler number, e, is the limit of $\left(1 + \frac{1}{n}\right)^n$ as $n \to +\infty$; 2.718

2. $f(x) = \frac{1}{4}e^{2x}$ is an example of exponential growth, because $\frac{1}{4} > 0$ and $e^2 > 1$.

3. No, since e is irrational.

4. $e^2 \cdot e^6 = e^8$ **5.** $e^{-2} \cdot 3e^7 = 3e^{-2+7} = 3e^5$

6. $(2e^{5x})^2 = 4e^{10x}$ **7.** $(4e^{-2})^3 = 64e^{-6} = \frac{64}{e^6}$

8. $\left(\frac{1}{2}e^{-2}\right)^4 = \frac{1}{16}e^{-8} = \frac{1}{16e^8}$

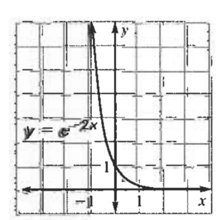
9. $\sqrt{36e^{4x}} = (36e^{4x})^{1/2} = 6e^{2x}$ **10.** $\frac{e^x}{e^{2x}} = \frac{1}{e^x}$

11. $\frac{12e^4}{36e^{-2}} = \frac{1}{3}e^{4+2} = \frac{e^6}{3}$

12. $y = -2$

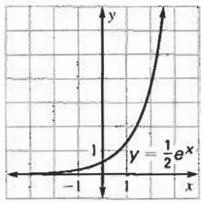
13. $y = e^{-2x}$

x	y
0	1
1	0.135
-1	7.39



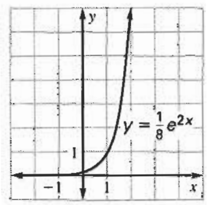
14. $y = \frac{1}{2}e^x$

x	y
-1	0.18
0	$\frac{1}{2}$
1	1.36
2	3.69



15. $y = \frac{1}{8}e^{2x}$

x	y
0	$\frac{1}{8}$
1	0.924
2	6.8



16. $s = 119.6e^{0.0917(26)} \approx 1298$

Practice and Applications (p. 483)

17. $e^2 \cdot e^4 = e^{2+4} = e^6$ **18.** $e^{-3} \cdot e^5 = e^{-3+5} = e^2$

19. $(3e^{-3x})^{-1} = \dfrac{1}{3}e^{3x} = \dfrac{e^{3x}}{3}$ **20.** $(3e^{4x})^2 = 9e^{8x}$

21. $3e^{-2} \cdot e^6 = 3e^{-2+6} = 3e^4$ **22.** $\left(\dfrac{1}{4}e^{-2}\right)^3 = \dfrac{1}{64e^6}$

23. $e^x \cdot e^{-3x} \cdot e^5 = e^{1x+3x+5} = e^{-2x+5}$

24. $\sqrt{4e^{2x}} = (4e^{2x})^{1/2} = 2e^{2x(1/2)} = 2e^x$

25. $(100e^{0.5x})^{-2} = \dfrac{1}{(100)^2}e^{0.5x(-2)} = \dfrac{1}{10{,}000}e^{-x} = \dfrac{1}{10{,}000e^x}$

26. $e^x \cdot 4e^{2x+1} = 4e^{x+2x+1} = 4e^{3x+1}$

27. $\dfrac{e^x}{2e} = \dfrac{e^xe^{-1}}{2} = \dfrac{e^{x-1}}{2}$

28. $\dfrac{5e^x}{e^{5x}} = 5e^x \cdot e^{-5x} = 5e^{x-5x} = 5e^{-4x} = \dfrac{5}{e^{4x}}$

29. $\sqrt[3]{27e^{6x}} = (27e^{6x})^{1/3} = 3e^{6x(1/3)} = 3e^{2x}$

30. $(32e^{-4x})^3 = (32)^3e^{-12x} = 32{,}768e^{-12x} = \dfrac{32{,}768}{e^{12x}}$

31. $\dfrac{6e^{3x}}{4e} = \dfrac{3}{2}e^{3x-1}$ **32.** $\sqrt[3]{64e^{9x}} = (64e^{9x})^{1/3} = 4e^{3x}$

33. $e^3 = 20.086$ **34.** $e^{-2/3} = 0.513$ **35.** $e^{1.7} = 5.474$

36. $e^{1/2} = 1.649$ **37.** $e^{-1/4} = 0.779$ **38.** $e^{3.2} = 24.533$

39. $e^8 = 2980.958$ **40.** $e^{-3} = 0.050$ **41.** $e^{-4} = 0.018$

42. $2e^{1/2} = 3.297$ **43.** $-4e^{-3} = -0.199$

44. $0.5e^{3.2} = 12.266$ **45.** $-1.2e^5 = -178.096$

46. $0.02e^{-0.3} = 0.015$ **47.** $225e^{-50} = 4.34 \times 10^{-20}$

48. $-8.95e^{1/5} = -10.932$

49. $f(x) = 5e^{-3x}$; exponential decay since $5 > 0$ and $-3 < 0$

50. $f(x) = \frac{1}{8}e^{5x}$; exponential growth since $\frac{1}{8} > 0$ and $5 > 0$

51. $f(x) = e^{-4x}$; exponential decay since $1 > 0$ and $-4 < 0$

52. $f(x) = \frac{1}{6}e^{2x}$; exponential growth since $\frac{1}{6} > 0$ and $2 > 0$

53. $f(x) = \frac{1}{4}e^{2x}$; exponential growth since $\frac{1}{4} > 0$ and $2 > 0$

54. $f(x) = e^{-8x}$; exponential decay since $1 > 0$ and $-8 < 0$

55. $f(x) = e^{3x}$; exponential growth since $1 > 0$ and $3 > 0$

56. $f(x) = \frac{1}{4}e^{-x}$; exponential decay since $\frac{1}{4} > 0$ and $-1 < 0$

57. $f(x) = e^{-6x}$; exponential decay since $1 > 0$ and $-6 < 0$

58. $f(x) = \frac{3}{8}e^{7x}$; exponential growth since $\frac{3}{8} > 0$ and $7 > 0$

59. $f(x) = e^{-9x}$; exponential decay since $1 > 0$ and $-9 < 0$

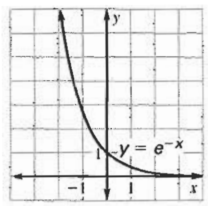
60. $f(x) = e^{8x}$; exponential growth since $1 > 0$ and $8 < 0$

61. $y = 3e^{0.5x}$ (C) **62.** $y = \frac{1}{3}e^{0.5x}$ (E)

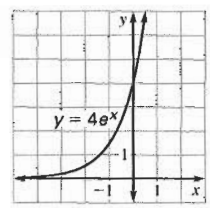
63. $y = \frac{1}{2}e^{-(x-1)}$ (F) **64.** $y = e^{-x} + 1$ (B)

65. $y = 3e^{-x} - 2$ (D) **66.** $y = 3e^x - 2$ (A)

67. $y = e^{-x}$ **68.** $y = 4e^x$

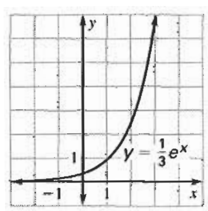


Domain: all real numbers
Range: $y > 0$



Domain: all real numbers
Range: $y > 0$

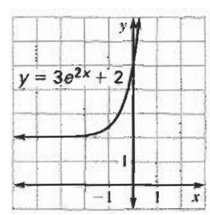
69. $y = \frac{1}{3}e^x$



Domain: all real numbers
Range: $y > 0$

70. $y = 3e^{2x} + 2$

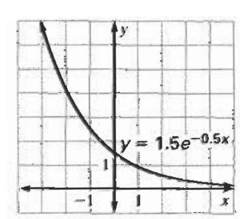
x	y
0	5
1	24.17
-1	2.41



Domain: all real numbers
Range: $y > 2$

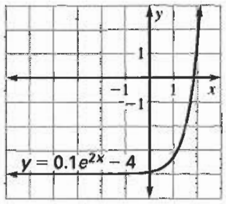
71. $y = 1.5e^{-0.5x}$

x	y
0	1.5
1	0.91
2	0.55
-1	2.47



Domain: all real numbers
Range: $y > 0$

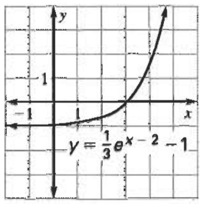
72. $y = 0.1e^{2x} - 4$



x	y
0	−3.9
1	−3.26
−1	−3.99

Domain: all real numbers
Range: $y > -4$

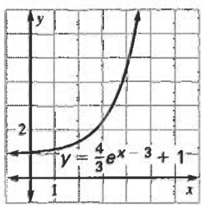
73. $y = \frac{1}{3}e^{x-2} - 1$



x	y
0	−0.95
1	−0.88
2	−0.67
−1	−0.98

Domain: all real numbers
Range: $y > -1$

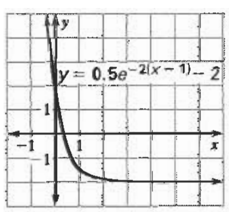
74. $y = \frac{4}{3}e^{x-3} + 1$



x	y
0	1.07
1	1.18
2	1.49
3	2.33
−1	1.02

Domain: all real numbers
Range: $y > 1$

75. $y = 0.5e^{-2(x-1)} - 2$



x	y
0	1.69
1	−1.5
2	−1.93
−1	25.29

Domain: all real numbers
Range: $y > -2$

76. $A = Pe^{rt} \quad P = 975 \quad r = 0.055 \quad t = 6$

$= 975e^{0.055(6)}$

$= \$1356.19$

77. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Annually $A = 2500\left(1 + \frac{0.06}{1}\right)^{1 \cdot 1}$

$= \$2650$

Semiannually $A = 2500\left(1 + \frac{0.06}{2}\right)^{2 \cdot 1}$

$= \$2652.25$

Quarterly $A = 2500\left(1 + \frac{0.06}{4}\right)^{4 \cdot 1}$

$= \$2653.41$

Monthly $A = 2500\left(1 + \frac{0.06}{12}\right)^{12 \cdot 1}$

$= \$2654.19$

Continuously $A = Pe^{rt}$

$= 2500e^{0.06(1)}$

$= \$2654.59$

The extra amount of interest earned with more and more compoundings decreases drastically. The difference between compounding monthly and continuously is only 40¢, 0.016% of the initial amount invested.

78. $A = Pe^{rt}$

$= 2500e^{0.06(1)}$

$= 2654.59$

$A = P\left(1 + \frac{r}{365}\right)^{365 \cdot t}$

$= 2500\left(1 + \frac{0.06}{365}\right)^{365 \cdot 1}$

$= \$2654.58$

The difference is 1¢.

79. $P = 14.7e^{-0.00004(29{,}028)}$

$\approx 4.603 \text{ lb/in.}^2$

80. $A = A_0e^{-0.05t}$

$= 4e^{-0.05(14)}$

$\approx 1.98 \text{ cm}^2$

$\approx 2 \text{ cm}^2$

81. $\sqrt[3]{\frac{8(81e^{11}x)}{3e^5x^{-2}}} = \left(\frac{8(81e^{11}x)}{3e^5x^{-2}}\right)^{1/3} = \left(\frac{648e^{11}x}{3e^5x^{-2}}\right)^{1/3}$

$= (216e^{11-5}x^{1+2})^{1/3}$

$= 6e^2x$ (E)

82. (B) $f(x) = 3e^x - 2$

x	y
0	1
−4	−1.95

83.

n	10^1	10^2	10^3
$\left(1+\frac{1}{n}\right)^n$	2.59374246	2.70481383	2.716923933

n	10^4	10^5	10^6
$\left(1+\frac{1}{n}\right)^n$	2.718145936	2.718268303	2.718281378

$n = 10^{10}$; *Sample answer:* From the table made in the activity on p. 480, I noticed that using a value of $n = 10^k$, the answer is accurate to $k - 1$ decimal places, with an error in the k^{th} decimal place.

8.2 Mixed Review (p. 485)

84.
$$\begin{aligned} f(x) &= -3x \\ y &= -3x \\ x &= -3y \\ y &= -\frac{x}{3} \\ f^{-1}(x) &= -\frac{x}{3} \end{aligned}$$

85.
$$\begin{aligned} f(x) &= 6x + 7 \\ y &= 6x + 7 \\ x &= 6y + 7 \\ 6y &= x - 7 \\ y &= \frac{x-7}{6} \\ f^{-1}(x) &= \frac{x-7}{6} \end{aligned}$$

86.
$$\begin{aligned} f(x) &= -5x - 24 \\ y &= -5x - 24 \\ x &= -5y - 24 \\ -5y &= x + 24 \\ \left(-\frac{1}{5}\right)(-5y) &= (x + 24)\left(-\frac{1}{5}\right) \\ y &= -\left(\frac{x+24}{5}\right) \\ f^{-1}(x) &= -\left(\frac{x+24}{5}\right) \end{aligned}$$

87.
$$\begin{aligned} f(x) &= \tfrac{1}{2}x - 10 \\ y &= \tfrac{1}{2}x - 10 \\ x &= \tfrac{1}{2}y - 10 \\ \tfrac{1}{2}y &= x + 10 \\ \tfrac{1}{2}y(2) &= 2(x + 10) \\ y &= 2x + 20 \\ f^{-1}(x) &= 2x + 20 \end{aligned}$$

88.
$$\begin{aligned} f(x) &= -14x + 7 \\ y &= -14x + 7 \\ x &= -14y + 7 \\ -14y &= x - 7 \\ -14y\left(-\frac{1}{14}\right) &= \left(-\frac{1}{14}\right)(x - 7) \\ y &= -\left(\frac{x-7}{14}\right) \\ f^{-1}(x) &= -\left(\frac{x-7}{14}\right) \end{aligned}$$

89.
$$\begin{aligned} f(x) &= -\tfrac{1}{5}x - 13 \\ y &= -\tfrac{1}{5}x - 13 \\ x &= -\tfrac{1}{5}y - 13 \\ -\tfrac{1}{5}y &= x + 13 \\ (-5)\left(-\tfrac{1}{5}y\right) &= (-5)(x + 13) \\ y &= -5x - 65 \\ f^{-1}(x) &= -5x - 65 \end{aligned}$$

90.
$$\begin{aligned} \sqrt{x} &= 20 \\ \left(\sqrt{x}\right)^2 &= (20)^2 \\ x &= 400 \end{aligned}$$

91.
$$\begin{aligned} \sqrt[3]{5x - 4} + 7 &= 10 \\ (5x - 4)^{1/3} + 7 &= 10 \\ [(5x - 4)^{1/3}]^3 &= (3)^3 \\ 5x - 4 &= 27 \\ 5x &= 27 + 4 \\ 5x &= 31 \\ x &= \tfrac{31}{5} \\ x &= 6.2 \end{aligned}$$

92.
$$\begin{aligned} 2(x + 4)^{2/3} &= 8 \\ (x + 4)^{2/3} &= 4 \\ [(x + 4)^{2/3})]^{3/2} &= (4)^{3/2} \\ x + 4 &= \sqrt{(4)^3} \\ x &= \sqrt{64} - 4 \\ x &= 8 - 4 \\ x &= 4 \end{aligned}$$

-12 is also an answer.

$$\begin{aligned} x &= \sqrt{64} - 4 \\ x &= -8 - 4 \\ x &= -12 \end{aligned}$$

93.
$$\sqrt{x^2-4} = x-2$$
$$(x^2-4)^{1/2} = x-2$$
$$[(x^2-4)^{1/2}]^2 = (x-2)^2$$
$$x^2-4 = (x-2)^2$$
$$x^2-4-[(x-2)(x-2)] = 0$$
$$x^2-4-(x^2-4x+4) = 0$$
$$x^2-4-x^2+4x-4 = 0$$
$$4x-8 = 0$$
$$4x = 8$$
$$x = 2$$

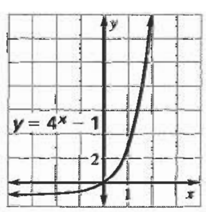
94.
$$\sqrt{x+3} = \sqrt{2x-1}$$
$$[(x+3)^{1/2}]^2 = [(2x-1)^{1/2}]^2$$
$$x+3 = 2x-1$$
$$-x = -4$$
$$x = 4$$

95.
$$\sqrt{3x-5} - 3\sqrt{x} = 0$$
$$\left(\sqrt{3x-5} - 3\sqrt{x}\right)^2 = (0)^2$$
$$3x-5-9x = 0$$
$$-6x = 5$$
$$x = -\tfrac{5}{6} \text{ no solution}$$

Quiz 1 (p. 485)

1. $y = 4^x - 1$

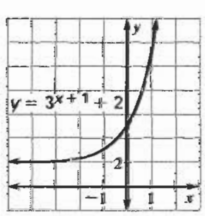
x	y
0	0
1	3
-1	$-\frac{3}{4}$



Domain: all real numbers
Range: $y > -1$

2. $y = 3^{x+1} + 2$

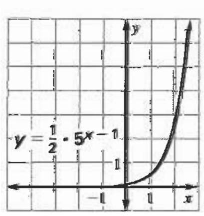
x	y
0	5
1	11
-1	3



Domain: all real numbers
Range: $y > 2$

3. $y = \frac{1}{2} \cdot 5^{x-1}$

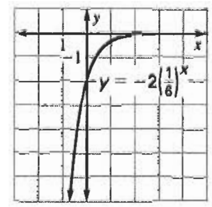
x	y
0	$\frac{1}{10}$
1	$\frac{1}{2}$
-1	$\frac{1}{25}$



Domain: all real numbers
Range: $y > 0$

4. $y = -2\left(\frac{1}{6}\right)^x$

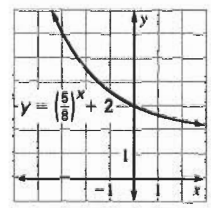
x	y
0	-2
1	$-\frac{1}{3}$
-1	-12



Domain: all real numbers
Range: $y < 0$

5. $y = \left(\frac{5}{8}\right)^x + 2$

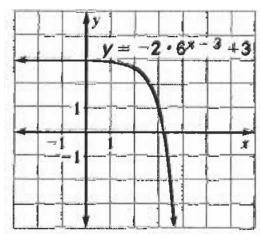
x	y
0	3
1	$2\frac{5}{8}$
-1	$3\frac{3}{8}$



Domain: all real numbers
Range: $y > 2$

6. $y = -2 \cdot 6^{x-3} + 3$

x	y
0	$2\frac{107}{108}$
3	1



Domain: all real numbers
Range: $y < 3$

7. $2e^3 \cdot e^4 = 2e^{3+4} = 2e^7$ **8.** $4e^{-5} \cdot e^7 = 4e^{-5+7} = 4e^2$

9. $(-3e^{2x})^2 = 9e^{4x}$

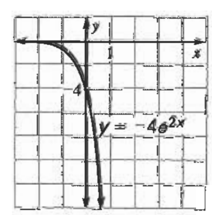
10. $(5e^{-3})^{-4x} = \dfrac{e^{12x}}{5^{4x}}$ **11.** $\dfrac{3e^x}{4e} = \dfrac{3}{4}e^{x-1}$

12. $\dfrac{6e^x}{e^{5x}} = 6e^{x-5x} = \dfrac{6}{e^{4x}}$ **13.** $\sqrt{16e^2x} = 4e\sqrt{x}$

14. $\sqrt[3]{125e^{6x}} = 5e^{2x}$

15. $f(x) = -4e^{2x}$

x	y
0	4
1	-29.56
-1	-0.54



16. $R = 100e^{-0.00043t}$

$R = 100e^{-0.00043(10{,}000)}$

$R = 1.357$ g

x	y
0	100
1	99.96

Amount of Radium Left from a 100 g Sample