Lesson 8.4

8.4 Guided Practice (p. 490)

1. common logarithm

2. **a.** $\log_3(-1)$ is not defined because (-1) is not positive. $(-1) = y$ The logarithm of y with base b is denoted by $\log_b y$, where b and y are positive.

   b. $\log_1 1$ is not defined because $b \neq 1(\log_b y)$

3. The expression $\log_b y$ is read as "log base b of y." Let b and y be positive numbers, $b \neq 1$. The logarithm of y with base b is denoted by $\log_b y$ and is defined as follows:

   $\log_b y = x$ if and only if $b^x = y$.

4. $5^2 = 25$ is true but by definition $\log_b y = x$ if and only if $b^x = y$.

   So, $\log_2 25 = x$ if and only if $2^x = 25$ (not $5^2 = 25$).

   $2^4 = 16$ and $2^5 = 32$ so x is between 4 and 5.

5. $\log_3 9 = 2$ *is* $3^2 = 9$ **6.** $\log_5 5 = 1$ *is* $5^1 = 5$

7. $\log_{1/2} 4 = -2$ *is* $\frac{1}{2}^{-2} = 4$ **8.** $\log_{19} 1 = 0$ *is* $19^0 = 1$

9. $\log_2 64$ $2^6 = 64$ so $\log_2 64 = 6$

10. $\log_{25} 5$ $25^{1/2} = 5$ so $\log_{25} 5 = \frac{1}{2}$

11. $\log_6 1$ $6^0 = 1$ so $\log_6 1 = 0$

12. $10^{\log 4}$

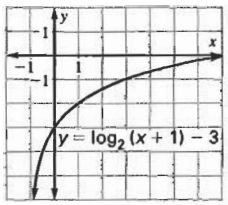
    $g(x) = \log_b x$ $f(g(x)) = b^{\log_b x} = x$

    $f(x) = b^x$

    $f(g(x)) = 10^{\log 4} = 4$

13. $y = \log_2(x + 1) - 3$

x	y
0	-3
3	-1
$-\frac{1}{2}$	-4

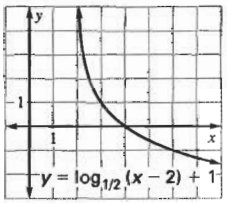


Domain: $x > -1$

Range: all real numbers

14. $y = \log_{1/2}(x - 2) + 1$

x	y
4	0
2.5	2
3	1
6	-1



Domain: $x > 2$

Range: all real numbers

15.
$$s = 0.159 + 0.118 \log d$$
$$0.1 = 0.159 + 0.118 \log d$$
$$0.1 - 0.159 = 0.118 \log d$$
$$\frac{0.1 - 0.159}{0.118} = \log d$$
$$\frac{-0.059}{0.118} = \log d$$
$$-0.5 = \log d$$
$$\log d = -0.5$$
$$10^{-0.5} = d$$
$$0.316 \text{ mm} \approx d$$

8.4 Practice and Applications (pp. 490–492)

16. $\log_4 1024 = 5$ *is* $4^5 = 1024$

17. $\log_5 \frac{1}{5} = -1$ *is* $5^{-1} = \frac{1}{5}$ **18.** $\log_{36} \frac{1}{6} = -\frac{1}{2}$ *is* $36^{-1/2} = \frac{1}{6}$

19. $\log_8 512 = 3$ *is* $8^3 = 512$

20. $\log_{12} 144 = 2$ *is* $12^2 = 144$

21. $\log_{14} 196 = 2$ *is* $14^2 = 196$

22. $\log_8 4096 = 4$ *is* $8^4 = 4096$

23. $\log_{105} 11{,}025 = 2$ *is* $105^2 = 11{,}025$

24. $\log_5 125$ $5^3 = 125$ so $\log_5 125 = 3$

25. $\log_7 343$ $7^3 = 343$ so $\log_7 343 = 3$

26. $\log_8 1$ $8^0 = 1$ so $\log_8 1 = 0$

27. $\log_{12} 12$ $12^1 = 12$ so $\log_{12} 12 = 1$

28. $\log_6 36$ $6^2 = 36$ so $\log_6 36 = 2$

29. $\log_4 16$ $4^2 = 16$ so $\log_4 16 = 2$

30. $\log_9 729$ $9^3 = 729$ so $\log_9 729 = 3$

31. $\log_7 2401$ $7^4 = 2401$ so $\log_7 2401 = 4$

32. $\log_{1/4} \frac{1}{4}$ $\frac{1}{4}^1 = \frac{1}{4}$ so $\log_{1/4} \frac{1}{4} = 1$

33. $\log_4 4^{-0.38}$ $4^{-0.38} = 4^{-0.38}$ so $\log_4 4^{-0.38} = -0.38$

34. $\log_4 \frac{1}{2}$ $4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$ so $\log_4 \frac{1}{2} = -\frac{1}{2}$

35. $\log_{1/5} 25$

$$\left(\frac{1}{5}\right)^{-2} = \frac{1}{\left(\frac{1}{5}\right)^2} = \frac{1}{1} \cdot \frac{(5)^2}{1} = 25$$

so $\log_{1/5} 25 = -2$

36. $\log 8 = 0.903$ **37.** $\ln 10 = 2.303$

38. $\log \sqrt{2} = 0.151$ **39.** $\log 3.724 = 0.571$

40. $\log 2.54 = 0.405$ **41.** $\log 0.3 = -0.523$

42. $\log 4.05 = 0.607$ **43.** $\log 3.5 = 0.544$

44. $\ln 4.6 = 1.526$ **45.** $\ln 150 = 5.011$

46. $\ln 6.9 = 1.932$ **47.** $\ln 22.5 = 3.114$

48. $5^{\log_5 x} = x$ from the definition

49. $\log_2 2^x = x$ from the definition

50. $9^{\log_9 x} = x$ from the definition

51. $35^{\log_{35} x} = x$ from the defintion

52. $\log_4 16^x = \log_4 (4^2)^x = \log_4 (4)^{2x} = 2x$

53. $7^{\log_7 x} = x$ **54.** $\log 100^x = \log (10^2)^x = \log (10)^{2x} = 2x$

55. $\log_{20} 8000^x = \log_{20} (20^3)^x = \log_{20} (20)^{3x} = 3x$

56. $y = \log_9 x \quad y = 9^x$ **57.** $y = \log_{1/4} x \quad y = \frac{1}{4}^x$

58. $y = \log_5 x \quad y = 5^x$ **59.** $y = \log_{1/2} x \quad y = \frac{1}{2}^x$

60. $y = \log_7 49^x$

$y = \log_7 (7^2)^x$

$y = \log_7 (7)^{2x}$

$x = \log_7 (7)^{2y}$ Switch x and y.

$x = 2y$ Write in exponential form.

$\frac{x}{2} = y$ Solve for y.

$y = \frac{x}{2}$

61. $y = \ln 6x$

$x = \ln 6y$ Switch x and y.

$e^x = 6y$ Write in exponential form.

$y = \frac{e^x}{6}$ Solve for y.

62. $y = \ln(x - 1)$

$x = \ln(y - 1)$ Switch x and y.

$e^x = y - 1$ Write in exponential form.

$y = e^x + 1$ Solve for y.

$y = 1 + e^x$

63. $y = \ln(x + 2)$

$x = \ln(y + 2)$ Switch x and y.

$e^x = y + 2$ Write in exponential form.

$y = e^x - 2$ Solve for y.

$y = -2 + e^x$

64. $y = \ln(x - 2)$

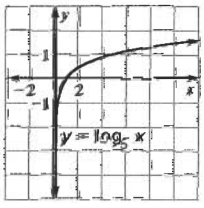
$x = \ln(y - 2)$ Switch x and y.

$e^x = y - 2$ Write in exponential form.

$y = 2 + e^x$ Solve for y.

65. $y = \log_5 x$

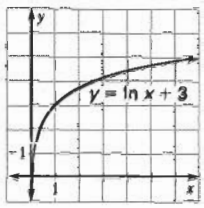
x	y
1	0
$\frac{1}{5}$	-1
25	2



Domain: $x > 0$
Range: all real numbers

66. $y = \ln x + 3$

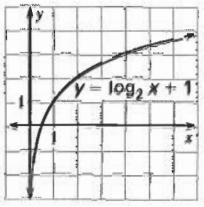
x	y
1	3
2	3.69
0.5	2.31



Domain: $x > 0$
Range: all real numbers

67. $y = \log_2 x + 1$

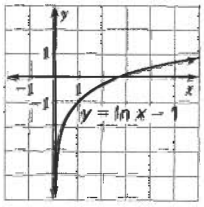
x	y
1	1
2	2
4	3
$\frac{1}{2}$	0



Domain: $x > 0$
Range: all real numbers

68. $y = \ln x - 1$

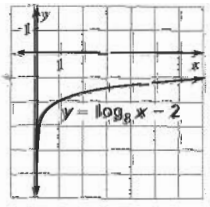
x	y
1	-1
0.5	-1.69
2	-0.31
3	0.10



Domain: $x > 0$
Range: all real numbers

69. $y = \log_8 x - 2$

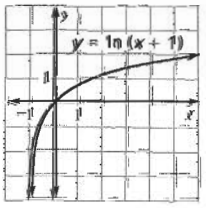
x	y
1	-2
8	-1
$\frac{1}{8}$	-3



Domain: $x > 0$
Range: all real numbers

70. $y = \ln(x + 1)$

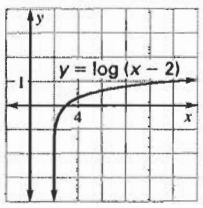
x	y
0	0
1	0.69
2	1.10



Domain: $x > -1$
Range: all real numbers

71. $y = \log(x - 2)$

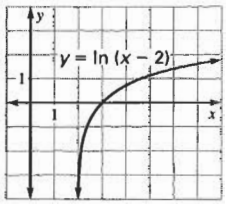
x	y
3	0
2.1	-1
12	1



Domain: $x > 2$
Range: all real numbers

72. $y = \ln(x - 2)$

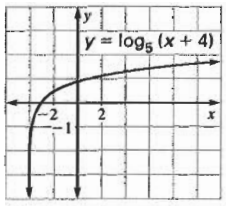
x	y
3	0
2.1	-2.3
4	0.69



Domain: $x > 2$
Range: all real numbers

73. $y = \log_5(x + 4)$

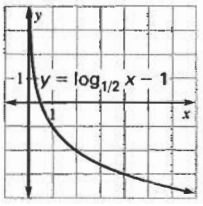
x	y
-3	0
1	1



Domain: $x > -4$
Range: all real numbers

74. $y = \log_{1/2} x - 1$

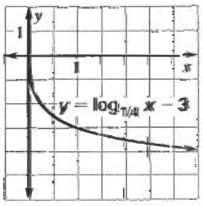
x	y
2	-2
1	-1
$\frac{1}{2}$	0
$\frac{1}{4}$	1



Domain: $x > 0$
Range: all real numbers

75. $y = \log_{1/4} x - 3$

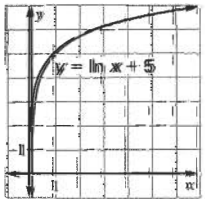
x	y
1	-3
$\frac{1}{4}$	-2
4	-4
$\frac{1}{16}$	-1



Domain: $x > 0$
Range: all real numbers

76. $y = \ln x + 5$

x	y
1	5
0.5	4.3
2	5.69



Domain: $x > 0$
Range: all real numbers

77. a. $\text{pH} = -\log[H^+]$
$= -\log 1 \times 10^{-2.4}$
$= -(10^x = 1 \times 10^{-2.4})$
$= -(x = -2.4)$
$= -(-2.4)$
$= 2.4$

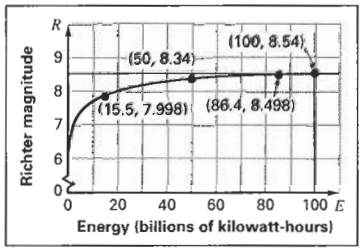
b. $\text{pH} = -\log 1 \times 10^{-3}$
$= -(10^x = 1 \times 10^{-3})$
$= -(x = -3)$
$= -(-3)$
$= 3$

c. $\text{pH} = -\log 1 \times 10^{-3.5}$
$= -(10^x = 1 \times 10^{-3.5})$
$= -(x = -3.5)$
$= -(-3.5)$
$= 3.5$

78. $A = \dfrac{2}{\log e} = \dfrac{2}{0.434294482} = 4.605$

79. $R = 0.67 \log[0.37(15{,}500{,}000{,}000)] + 1.46$
$= 0.67 \log(0.37 \times 1.55 \times 10^{10}) + 1.46$
$= 0.67 \log (5{,}735{,}000{,}000) + 1.46$
$= 0.67(9.758533422) + 1.46$
$= 7.9982$ *or* about 8

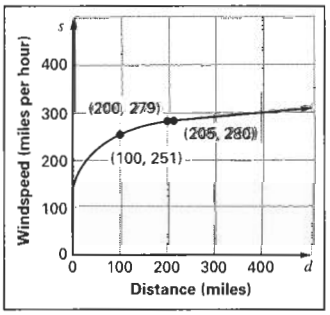
80.



E	R
15.5	7.99
50.0	8.34
75.0	8.46
100.0	8.54
86.4	8.498
86.417	8.5

about 86,000,000,000 kWh

81. $s = 93 \log d + 65$



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81 —CONTINUED—

$s = 93 \log d + 65$

$s - 65 = 93 \log d$

$\log d = \dfrac{s - 65}{93}$

$= \dfrac{280 - 65}{93}$

$= \dfrac{215}{93}$

$= 2.312$

d = about 205 miles

$\log 100 = 2$

$s = 93(2) + 65$

$s = 251$ mph

$\log 200 = 2.301$

$s = 93(2.301) + 65$

$s = 278.993$ mph

$\log 205 = 2.312$

$s = 93(2.312) + 65$

$s = 280.02$

	Column A	*Column B*	
82.	$\log_9 9^{2/3}$	$\log 100$	(B)
83.	$\log_{16} 1$	0	(C)
84.	$\log_4 16$	$\log_8 64$	(C)
85.	$f(8)$ if $f(x) = \log_2 x$	4	(B)
86.	$f(-1)$ if $f(x) = \log_5 5^x$	-1	(C)
87.	$f\left(\frac{1}{2}\right)$ if $f(x) = \log_3 9^x$	$\log_3 81$	(B)

82. $\log_9 9^{2/3} = 9^x = 9^{2/3} \quad x = \frac{2}{3} = 0.67 \quad \log 100 = 2$ (B)

83. $\log_{16} 1 \quad 16^x = 1 \quad x = 0$ (C)

84. $\log_4 16 \quad 4^x = 16 \quad x = 2 \quad \log_8 64 \quad 8^x = 64 \quad x = 2$ (C)

85. $f(8) = \log_2 (8) \quad 2^x = 8 \quad x = 3$ (B)

86. $f(-1) = \log_5 5^{(-1)} \quad 5^x = 5^{(-1)} \quad x = -1$ (C)

87. $f\left(\frac{1}{2}\right) = \log_3 9^{1/2} \quad 3^x = 9^{1/2} \quad \log_3 81 \quad 3^x = 81$ (B)

$3^x = (3^2)^{1/2} \qquad 3^x = 3^4$

$3^x = 3^1 \qquad x = 4$

$x = 1$

88. $\log_{16} 8 \quad 16^x = 8$

$(2^4)^x = 2^3$

$4x = 3$

$x = \frac{3}{4}$

89. $\log_{16} 64 \quad 16^x = 64$

$(2^4)^x = 2^6$

$2^{4x} = 2^6$

$4x = 6$

$x = \frac{3}{2}$

90. $\log_9 27 \quad 9x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$x = \frac{3}{2}$

91. $\log_4 512 \quad 4^x = 512$

$(2^2)^x = 2^9$

$2^{2x} = 2^9$

$2x = 9$

$x = \frac{9}{2}$

92. If $b = c^n$ and $x = c^m$, then $\log_b x = \dfrac{m}{n}$.

8.4 Mixed Review (p. 492)

93. $5^2 \cdot 5^3 = 3125$ **94.** $(3^{-4})^2 = 3^{-8} = \dfrac{1}{3^8} = \dfrac{1}{6561}$

95. $7^0 \cdot 7^3 \cdot 7^{-2} = 7^{0+3+(-2)} = 7^1 = 7$

96. $\left(\frac{3}{7}\right)^{-2} = \left(\frac{7}{3}\right)^2 = \frac{49}{9}$ **97.** $\dfrac{6^3}{6^4} = 6^{3-4} = 6^{-1} = \dfrac{1}{6}$

98. $\left(\dfrac{3}{8}\right)^{-3} = \left(\dfrac{8}{3}\right)^3 = \dfrac{512}{27}$ **99.** $(-2^3)^2 = (-8)^2 = 64$

100. $\left(\dfrac{4}{5}\right)^3 = \dfrac{64}{125}$ **101.** $\left(\dfrac{1}{2}\right)^{-4} = \dfrac{1}{\left(\frac{1}{2}\right)^4} = \dfrac{1}{\frac{1}{16}} = 16$

102. $(-3^2)^{-1} = (-9)^{-1} = -\frac{1}{9}$

103. $\dfrac{2^5}{2^9} = 2^{5-9} = 2^{-4} = \dfrac{1}{16}$ **104.** $\left(\dfrac{7}{9}\right)^{-2} = \left(\dfrac{9}{7}\right)^2 = \dfrac{81}{49}$

105.

$$\begin{array}{r} 2x - 7 \\ x + 4 \overline{)2x^2 + x - 1} \\ \underline{2x^2 + 8x} \\ -7x - 1 \\ \underline{-7x - 28} \\ 27 \end{array}$$

$2x - 7 + \dfrac{27}{x + 4}$

106.

$$\begin{array}{r} x - 4 \\ x - 1 \overline{)x^2 - 5x + 4} \\ \underline{x^2 - 1x} \\ -4x + 4 \\ \underline{-4x + 4} \\ x - 4 \end{array}$$

107.

$$\begin{array}{r} 4x + 3 \\ x^2 + 2 \overline{)4x^3 + 3x^2 + 2x - 3} \\ \underline{4x^3 + 0x^2 + 8x} \\ 3x^2 - 6x - 3 \\ \underline{3x^2 - 0x + 6} \\ -6x - 9 \end{array}$$

$4x + 3 - \dfrac{6x + 9}{x^2 + 2}$

108.

$$\begin{array}{r} 6x^2 + 10x + 30 \\ x - 3 \overline{)6x^3 - 8x^2 + 7} \\ \underline{6x^3 - 18x^2} \\ 10x^2 + 0x + 7 \\ \underline{10x^2 - 30x} \\ 30x + 7 \\ \underline{30x - 90} \\ +97 \end{array}$$

$6x^2 + 10x + 30 + \dfrac{97}{x - 3}$

109. $(2, 0), (-3, 0), (0, 0), (3, -3)$

$y = -\dfrac{1}{6}x(x - 2)(x + 3)$

110. $(3, 0), (2, 0), (-3, 0), (0, -1)$

$y = -\dfrac{1}{18}(x - 3)(x - 2)(x + 3)$

111. $(4, 0), (6, 0), (-4, 0), (1, 1)$

$$y = \frac{1}{75}(x - 4)(x - 6)(x + 4)$$

112. $(-2, 0), (-3, 0), (3, 0), (0, 2)$

$$y = -\frac{1}{9}(x + 2)(x + 3)(x - 3)$$

Lesson 8.5

Developing Concepts Activity 8.5 (p. 493)

1.

$\log_b u$	$\log_b v$	$\log_b uv$
$\log 10 = 1$	$\log 100 = 2$	$\log 1000 = 3$
$\log 0.1 = -1$	$\log 0.01 = -2$	$\log 0.001 = -3$
$\log_2 4 = 2$	$\log_2 8 = 3$	$\log_2 32 = 5$

2. $\log_b uv = \log_b u + \log_b v$

8.5 Guided Practice (p. 496)

1. a. product property example:

$$\log_5 21 = \log_5 (3 \cdot 7)$$
$$= \log_5 3 + \log_5 7$$
$$\approx 0.683 + 1.209 = 1.892$$

b. quotient property example:

$$\log_5 \tfrac{3}{7} = \log_5 3 - \log_5 7$$
$$\approx 0.683 - 1.209 = -0.526$$

c. power property example:

$$\log_5 49 = \log_5 7^2$$
$$= 2 \log_5 7 \approx 2(1.209) = 2.418$$

2. $\log\left(\frac{7}{9}\right)^2 = 2 \log \frac{7}{9}$

$$= 2(\log 7 - \log 9) \text{ (A)}$$

3. $\log_8(5x^2 + 3)$ (C)

None of the properties of logarithms applies to a sum of terms.

4. $\log_6 11 = \dfrac{\ln 11}{\ln 6}$; $\log_6 11 = \dfrac{\log 11}{\log 6}$

5. $\log_3(3 \cdot 9) = \log_3 3 + \log_3 9 = 1 + 2 = 3$

6. $\log_2 4^5 = 5 \log_2 4 = 5(2) = 10$

7. $\log_3 \dfrac{1}{3} = \dfrac{\log \frac{1}{3}}{\log 3} = -1$

8. $\log_5 \left(\dfrac{1}{5}\right)^3 = 3 \log_5 \dfrac{1}{5} = 3 \dfrac{\log \frac{1}{5}}{\log 5} = 3(-1) = -3$

9. $\log_2 3 = \log_2 \frac{21}{7} = \log_2 21 - \log_2 7$

$$\approx 4.39 - 2.81 = 1.58$$

10. $\log_2 49 = \log_2 7^2 = 2 \log_2 7 \approx 2(2.81) = 5.62$

11. $\log_2 147 = \log_2 21(7) = \log_2 21 + \log_2 7$

$$\approx 4.39 + 2.81 = 7.2$$

12. $\log_2 441 = \log_2 21^2 = 2(\log_2 21) = 2(4.39) = 8.78$

13. Difference in loudness $L_2 = L_1$

$$= 10 \log \frac{1.26 \times 10^{-7}}{10^{-12}} - 10 \log \frac{3.16 \times 10^{-10}}{10^{-12}}$$
$$= 10 \log (1.26 \times 10^5) - 10 \log (3.16 \times 10^2)$$
$$= 10(\log 1.26 + \log 10^5) - 10(\log 3.16 + \log 10^2)$$
$$= 10(\log 1.26 + 5 \log 10) - 10(\log 3.16 + 2 \log 10)$$
$$\approx 10(5.1) - 10(2.5)$$
$$\approx 51 - 25$$
$$\approx 26 \text{ decibels}$$

8.5 Practice and Applications (p. 496)

14. $\log_2(4 \cdot 16) = \log_2 64 = \dfrac{\log 64}{\log 2} = 6$

15. $\ln e^{-2} = -2 \ln e = -2(1) = -2$

16. $\log_2 4^3 = 3 \log_2 4 = 3\left(\dfrac{\log 4}{\log 2}\right) = 3(2) = 6$

17. $\log_5 125 = \dfrac{\log 125}{\log 5} = 3$

18. $\log_3 9^4 = 4 \log_3 9 = 4\left(\dfrac{\log 9}{\log 3}\right) = 4(2) = 8$

19. $\log \dfrac{1}{10} = \log 1 - \log 10 = 0 - 1 = -1$

20. $\ln \dfrac{1}{e^3} = \ln 1 - \ln e^3 = 0 - 3 \ln e = 0 - 3(1) = -3$

21. $\log(0.01)^3 = 3 \log(0.01) = 3(-2) = -6$

22. $\log 3 = \log \frac{15}{5} = \log 15 - \log 5$

$$\approx 1.176 - 0.699 = 0.477$$

23. $\log 25 = \log 5 \cdot 5 = \log 5 + \log 5$

$$= 0.699 + 0.699 = 1.398$$

24. $\log 75 = \log 5 \cdot 15 = \log 5 + \log 15$

$$= 0.699 + 1.176 = 1.875$$

25. $\log 125 = \log 5^3 = 3 \log 5 = 3(0.699) = 2.097$

26. $\log \frac{1}{5} = \log 1 - \log 5 = 0 - 0.699 = -0.699$

27. $\log 225 = \log 15^2 = 2 \log 15 = 2(1.176) = 2.352$

28. $\log \frac{1}{15} = \log 1 - \log 15 = 0 - 1.176 = -1.176$

29. $\log \frac{1}{3} = \log 1 - \log 3 = \log 1 - \log \frac{15}{5}$

$$= 0 - (\log 15 - \log 5)$$
$$= -(1.176 - 0.699)$$
$$= -0.477$$

30. $\log_2 9x = \log_2 9 + \log_2 x$ **31.** $\ln 22x = \ln 22 + \ln x$

32. $\log 4x^5 = \log 4 + \log x^5 = \log 4 + 5 \log x$

33. $\log_6 x^6 = 6 \log_6 x$ **34.** $\log_4 \frac{4}{3} = 1 - \log_4 3$

35. $\log_3 25 = \log_3 5^2 = 2 \log_3 5$

36. $\log_6 \frac{10}{3} = \log_6 10 - \log_6 3$

37. $\ln 3xy^3 = \ln 3 + \ln x + \ln y^3 = \ln 3 + \ln x + 3 \ln y$

38. $\log 6x^3yz = \log 6 + \log x^3 + \log y + \log z$
$= \log 6 + 3 \log x + \log y + \log z$

39. $\log_8 64x^2 = \log_8 64 + \log_8 x^2 = \log_8 64 + 2 \log_8 x$
$= \frac{\log 64}{\log 8} + 2 \log_8 x$
$= 2 + 2 \log_8 x$

40. $\ln x^{1/2}y^3 = \ln x^{1/2} + \ln y^3 = \frac{1}{2} \ln x + 3 \ln y$

41. $\log_3 12^{5/6} x^9 = \log_3 12^{5/6} + \log_3 x^9$
$= \frac{5}{6} \log_3 12 + 9 \log_3 x$

42. $\log \sqrt{x} = \log x^{1/2} = \frac{1}{2} \log x$

43. $\ln \frac{3y^4}{x^3} = \ln 3y^4 - \ln x^3$
$= \ln 3 + \ln y^4 - 3 \ln x$
$= \ln 3 + 4 \ln y - 3 \ln x$

44. $\log \sqrt[4]{x^3} = \log(x^3)^{1/4} = \log x^{3/4} = \frac{3}{4} \log x$

45. $\log_2 \sqrt{4x} = \log_2 (4x)^{1/2} = \log_2 4^{1/2} + \log_2 x^{1/2}$
$= \frac{1}{2}\left(\frac{\log 4}{\log 2}\right) + \frac{1}{2} \log_2 x$
$= 1 + \frac{1}{2} \log_2 x$

46. $\log_5 8 - \log_5 12 = \log_5 \frac{8}{12} = \log_5 \frac{2}{3}$

47. $\ln 16 - \ln 4 = \ln \frac{16}{4} = \ln 4$

48. $2 \log x + \log 5 = \log x^2 + \log 5 = \log 5x^2$

49. $4 \log_{16} 12 - 4 \log_{16} 2 = \log_{16} 12^4 - \log_{16} 2^4$
$= \log_{16} 20736 - \log_{16} 16$
$= \frac{\log_{16} 20736}{\log_{16} 16}$
$= \log_{16} 1296$

50. $3 \ln x + 5 \ln y = \ln x^3 + \ln y^5 = \ln x^3y^5$

51. $7 \log_4 2 + 5 \log_4 x + 3 \log_4 y = \log_4 2^7 + \log_4 x^5 + \log_4 y^3$
$= \log_4 128x^5y^3$

52. $\ln 20 + 2 \ln \frac{1}{2} + \ln x = \ln 20 + \ln\left(\frac{1}{2}\right)^2 + \ln x$
$= \ln 20 + \ln \frac{1}{4} + \ln x$
$= \ln 20\left(\frac{1}{4}\right)x$
$= \ln 5x$

53. $\log_3 2 + \frac{1}{2} \log_3 y = \log_3 2 + \log_3 y^{1/2} = \log_3 2y^{1/2}$
$= \log_3 2\sqrt{y}$

54. $10 \log x + 2 \log 10 = \log x^{10} + \log 10^2$
$= \log 100x^{10}$

55. $3(\ln 3 - \ln x) + (\ln x - \ln 9) = (\ln 3^3 - \ln x^3) + \left(\frac{\ln x}{\ln 9}\right)$
$= \left(\frac{\ln 3^3}{\ln x^3}\right)\left(\frac{\ln x}{\ln 9}\right)$
$= (\ln 3^3x^{-3})(\ln 3^{-2}x)$
$= \ln 3^{3-2}x^{1-3}$
$= \ln 3x^{-2} = \ln \frac{3}{x^2}$

56. $2(\log_6 15 - \log_6 5) + \frac{1}{2} \log_6 \frac{1}{25}$
$= \left(\frac{\log_6 15^2}{\log_6 5^2}\right) + \log_6\left(\frac{1}{25}\right)^{1/2}$
$= \left(\frac{\log_6 225}{\log_6 25}\right) + \log_6 \frac{1}{5}$
$= \log_6 9 + \log_6 \frac{1}{5}$
$= \log_6 \frac{9}{5}$

57. $\frac{1}{4} \log_5 81 - \left(2 \log_5 6 - \frac{1}{2} \log_5 4\right)$
$= \log_5 (81)^{1/4} - (\log_5 6^2 - \log_5 4^{1/2})$
$= \log_5 3 - (\log_5 36 - \log_5 2)$
$= \log_5 3 - \log_5 18$
$= \log_5 \frac{3}{18} = \log_5 \frac{1}{6}$

58. $\log_5 7 = \frac{\log 7}{\log 5} = 1.209$ **59.** $\log_7 12 = \frac{\log 12}{\log 7} = 1.277$

60. $\log_3 16 = \frac{\log 16}{\log 3} = 2.524$

61. $\log_9 25 = \frac{\log 25}{\log 9} = 1.465$ **62.** $\log_2 5 = \frac{\log 5}{\log 2} = 2.322$

63. $\log_6 9 = \frac{\log 9}{\log 6} = 1.226$ **64.** $\log_3 17 = \frac{\log 17}{\log 3} = 2.579$

65. $\log_5 32 = \frac{\log 32}{\log 5} = 2.153$

66. $\log_2 125 = \frac{\log 125}{\log 2} = 6.966$

67. $\log_6 24 = \frac{\log 24}{\log 6} = 1.774$

68. $\log_4 19 = \frac{\log 19}{\log 4} = 2.124$

69. $\log_{16} 81 = \frac{\log 81}{\log 16} = 1.585$

70. $\log_8 \frac{22}{7} = \frac{\log \frac{22}{7}}{\log 8} = 0.551$

71. $\log_9 \frac{5}{16} = \frac{\log \frac{5}{16}}{\log 9} = -0.529$

72. $\log_2 \frac{4}{15} = \frac{\log \frac{4}{15}}{\log 2} = -1.907$

73. $\log_5 \frac{32}{3} = \frac{\log \frac{32}{3}}{\log 5} = 1.471$

74. $s = \log_2 f^2 = 2 \log_2 f^2$

75. $s = 2 \log_2 1.414 = 2\left(\frac{\log 1.414}{\log 2}\right) = 1$

$s = 2 \log_2 2.000 = 2\left(\frac{\log 2.000}{\log 2}\right) = 2$

$s = 2 \log_2 2.828 = 2\left(\frac{\log 2.828}{\log 2}\right) = 3$

$s = 2 \log_2 4.000 = 2\left(\frac{\log 4.000}{\log 2}\right) = 4$

$s = 2 \log_2 5.657 = 2\left(\frac{\log 5.657}{\log 2}\right) = 5$

$s = 2 \log_2 8.000 = 2\left(\frac{\log 8.000}{\log 2}\right) = 6$

$s = 2 \log_2 11.314 = 2\left(\frac{\log 11.314}{\log 2}\right) = 7$

$s = 2 \log_2 16.000 = 2\left(\frac{\log 16.000}{\log 2}\right) = 8$

f	1.414	2.000	2.828	4.000
s	1	2	3	4

f	5.657	8.000	11.314	16.000
s	5	6	7	8

As you change f-steps on the 35 mm camera the s increases by 1.

76. $s = 2 \log_2 f$

$9 = 2 \log_2 f$

$9 = 2 \frac{\log f}{\log 2}$

$\frac{9}{2} = \frac{\log f}{\log 2}$

$\log f = \frac{9}{2} \log 2$

$\log f = 1.35463498$

$10^x = f$

$10^{1.35463498} \approx 22.627 = 16\sqrt{2}$

77. $E = 1.4(\log C_2 - \log C_1)$

$E = 1.4 \log \frac{C_2}{C_1}$

78. $E = 1.4 \log \frac{2\,C_2}{C_1}$

$E = 1.4 \log 2 = 0.421$ kcal/g-molecule

79. $E = 1.4 \log \frac{6\,C_2}{C_1}$

$E = 1.4 \log 6$

$E = 1.089$ kcal/g-molecule

80. $L = 10 \log \frac{0.316}{10^{-12}}$

$L \approx 115$ decibels

between rock concert (110 decibels) and riveting machine (120 decibels).

81. $L = 10 \log \frac{0.003}{10^{-12}}$

$L \approx 95$ decibels

between subway train (90 decibels) and boiler shop (100 decibels)

82. $L = 10 \log \frac{3(1.4 \times 10^{-7})}{10^{-12}}$

$= 10(\log 3 + \log 1.4 \times 10^{-7} - \log 10^{-12})$

$\approx 10[0.477 + (-6.854) + (12)] \approx 56.2$ decibels

83. $L = 10 \log \frac{5(3.16 \times 10^{-4})}{10^{-12}}$

$= 10(\log 5 + \log 3.16 \times 10^{-4} - \log 10^{-12})$

$\approx 10[0.699 + (-3.5) + (12)]$

$\approx 10(9.199) \approx 92$ decibels

84. $L = 10 \log \frac{3 \times I}{10^{-12}} - 10 \log \frac{I}{10^{-12}}$

$= 10\left(\log \frac{3I}{10^{-12}} - \log \frac{I}{10^{-12}}\right)$

$= 10\left(\log 3 + \log \frac{I}{10^{-12}} - \log \frac{I}{10^{-12}}\right)$

$= 10 \log 3$ or about 4.8 decibels

85. $L = 10 \log \frac{I}{10^{-12}} - 10 \log \frac{0.5I}{10^{-12}}$

$= 10\left(\log \frac{I}{10^{-12}} - \log \frac{0.5I}{10^{-12}}\right)$

$= 10\left(\log \frac{I}{10^{-12}} - \log \frac{I}{10^{-12}} + \log 0.5\right)$

$= 10 \log 0.5 \approx 3$ decibels less

86. $\log (u + v) = \log u + \log v$ false

Sample answer: $\log(10 + 10) = \log 20 \approx 1.301$

but $\log 10 + \log 10 = 1 + 1 = 2$

87. $\log 1 = \log \frac{2}{2} = \log 2 - \log 2 = 0$

log 2 and log 3 are given.

$\log 4 = \log 2^2 = 2 \log 2 \approx 0.6020$

$\log 5 = \log 10 - \log 2 \approx 0.6990$

$\log 6 = \log (2 \cdot 3) = \log 2 + \log 3 \approx 0.6781$

$\log 8 = \log 2^3 = 3 \log 2 \approx 0.9030$

$\log 9 = \log 3^2 = 2 \log 3 \approx 0.9542$

$\log 10 = 1$

$\log 12 = \log (3 \cdot 4) = \log 3 + 2 \log 2 \approx 1.0791$

$\log 15 = \log (3 \cdot 5) = \log 3 + \log 5 \approx 1.1761$

$\log 16 = \log 2^4 = 4 \log 2 \approx 1.204$

$\log 18 = \log (2 \cdot 9) = \log 2 + 2 \log 3 \approx 1.2552$

$\log 20 = \log (2 \cdot 10) = \log 2 + 1 \approx 1.3010$

log 7, log 11, log 13, log 14, log 17, and log 19 cannot be found. Those numbers with a prime factorization involving only 2, 3, and 5 can be written in terms of these logs.

Conclusion: for the values of n (1 to 20) that I cannot find $\log n$ $\log 1 = 0$, log 5's value is between 0.6020 (log 4) and 0.7781 (log 6), log 7's value is between 0.7781 (log 6) and 0.9030 (log 8), log 10 = 1 and log 11's value is between 1.000 (log 10) and 1.0791 (log 12), log 13's, log 14's, and log 15's values are between 1.0791 (log 12) and 1.204 (log 16), log 17's value is between 1.204 (log 16) and 1.2552 (log 18), and log 19's and log 20's values are slightly higher than 1.2552 (log 18).

88. (C) $\log_2 24 = \log_2 8 + \log_2 16$ incorrect

89. $\log_5 8$ = (E) both (B) and (C)

(B) $\log_5 8 = \dfrac{\log 8}{\log 5}$

(C) $\log_5 8 = \dfrac{\ln 8}{\ln 5}$

90. $4 \log_3 5 = \log_3 5^4 = \log_3 625$ (B)

91. **a.** Product Property

$\log_b uv = \log_b u + \log_b v$

let $x = \log_b u$ then $u = b^x$

let $y = \log_b v$ then $v = b^y$

so that $\log_5 uv = \log_b(b^x \cdot b^y) = \log_b(b^{x+y})$

$= x + y = \log_b u + \log_b v$

b. Quotient Property

$\log_b \dfrac{u}{v} = \log_b u - \log_b v$

let $x = \log_b u$ then $u = b^x$

let $y = \log_b u$ then $v = b^y$

so that $\log_b \dfrac{u}{v} = \log_b \dfrac{b^x}{b^y} = \log_b (b^{x-y})$

$= x - y = \log_b u - \log_b v$

c. Power Property

$\log_b u^n = n \log_b u$

let $x = \log_b u$ then $u = b^x$ and $u^n = b^{nx}$

so that $\log_b u^n = \log_b(b^{nx}) = nx = n \log_b u$

d. Change of base formula

$\log_c u = \dfrac{\log_b u}{\log_b c}$

let $x = \log_b u$ then $u = b^x$

let $y = \log_b c$ then $c = b^y$

let $z = \log_c u$ then $u = c^z$

so that $b^x = c^z$

Thus, $x = \log_b u = \log_b b^x = \log_b c^z$

$= z \log_b c = zy$

Thus, $x = yz$, so $z = \dfrac{x}{y}$, or $\log_c u = \dfrac{\log_b u}{\log_b c}$

8.5 Mixed Review (p. 499)

92. $3 \cdot y^2 \cdot y^2 = 3y^{2+2} = 3y^4$ **93.** $(y^4)^3 = y^{4 \cdot 3} = y^{12}$

94. $(x^3y)^4 = x^{3 \cdot 4}y^4 = x^{12}y^4$ **95.** $(-3x^2)^2 = -3^2x^{2 \cdot 2} = 9x^4$

96. $4x^{-1}y = \dfrac{4y}{x}$ **97.** $xy^{-2}x = \dfrac{x^2}{y^2}$ **98.** $\dfrac{x^3}{x^{-1}} = x^{3-(-1)} = x^4$

99. $\dfrac{4x^2y^7}{8xy^{-1}} = \dfrac{1}{2}x^{2-1}y^{7+1} = \dfrac{xy^8}{2}$

100.
$\sqrt[4]{x + 2} + 9 = 14$
$\sqrt[4]{x + 2} = 5$
$\left(\sqrt[4]{x + 2}\right)^4 = (5)^4$
$x + 2 = 625$
$x = 623$

101.
$\sqrt[3]{3x - 4} = \sqrt[3]{x + 10}$
$\left(\sqrt[3]{3x - 4}\right)^3 = \left(\sqrt[3]{x + 10}\right)^3$
$3x - 4 = x + 10$
$2x = 14$
$x = 7$

102.
$\sqrt{3x + 7} = x + 3$
$\left(\sqrt{3x + 7}\right)^2 = (x + 3)^2$
$3x + 7 = x^2 + 6x + 9$
$x^2 + 6x - 3x + 9 - 7 = 0$
$x^2 + 3x + 2 = 0$
$(x + 2)(x + 1) = 0$
$x = -2$ or $x = -1$

103. $(5x)^{1/2} - 18 = 32$

$(5x)^{1/2} = 50$

$[(5x)^{1/2}]^2 = (50)^2$

$5x = 2500$

$x = 500$

104. $e^9 = 8103.084$

105. $e^{-12} = 6.14 \times 10^{-6}$ **106.** $e^{1.7} = 5.474$

107. $e^{-5.632} = 3.581 \times 10^{-3}$ **108.** $\log 15 = 1.176$

109. $\log 1.729 = 0.238$ **110.** $\ln 16 = 2.773$

111. $\ln 5.89 = 1.773$

Math and History (p. 499)

1. approximate $\log 3$ and $\log 5$

$\log 3 \approx 0.5$ $\log 5 \approx 0.7$

2. $\log 15 = \log(3 \cdot 5) = \log 3 + \log 5$

$= 0.5 + 0.7$

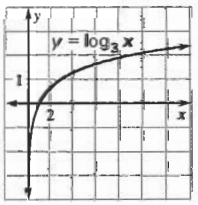
$= 1.2$

8.5 Technology Activity (p. 500)

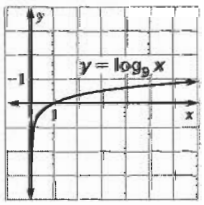
	Point	Vertical asymptote
1. $y = \log_3 x = \frac{\log x}{\log 3}$	$(1, 0)$	$x = 0$
2. $y = \log_9 x = \frac{\log x}{\log 9}$	$(1, 0)$	$x = 0$
3. $y = \log_4 x = \frac{\log x}{\log 4}$	$(1, 0)$	$x = 0$
4. $y = \log_7 x = \frac{\log x}{\log 7}$	$(1, 0)$	$x = 0$
5. $y = \log_5 x = \frac{\log x}{\log 5}$	$(1, 0)$	$x = 0$
6. $y = \log_{11} x = \frac{\log x}{\log 11}$	$(1, 0)$	$x = 0$

Graphs

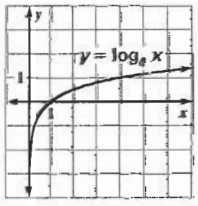
1.



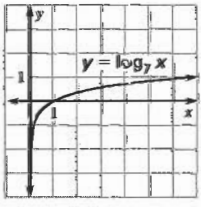
2.



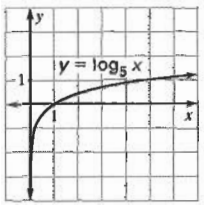
3.



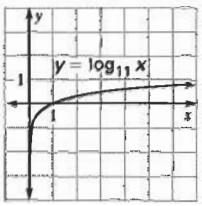
4.



5.



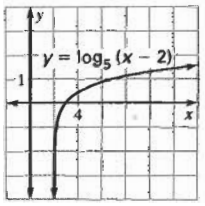
6.



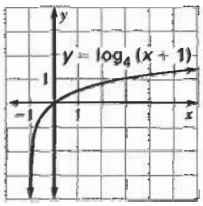
	Point	Vertical asymptote
7. $y = y = \log_5 (x - 2)$	$(3, 0)$	$x = 2$
8. $y = y = \log_4 (x + 1)$	$(0, 0)$	$x = -1$

Graphs

7.



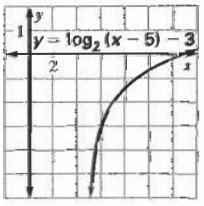
8.



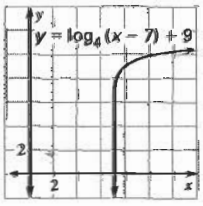
	Point	Vertical asymptote
9. $y = \log_2 (x - 5) - 3$	$(6, -3)$	$x = 5$
10. $y = \log_4 (x - 7) + 9$	$(8, 9)$	$x = 7$
11. $y = \log_5 (x + 2) + 6$	$(-1, 6)$	$x = -2$
12. $y = \log_7 (x - 4) + 4$	$(5, 4)$	$x = 4$

Graphs

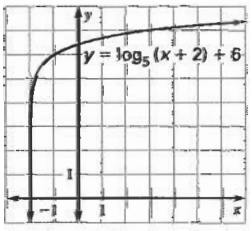
9.



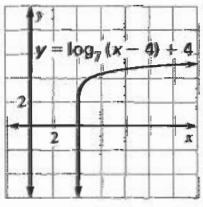
10.



11.



12.



13. The domain of $y = \log x$ is all real numbers greater than 0, while the domain of $y = \log |x|$ is all real numbers except 0. The graph of $y = \log |x|$ is the graph of $y = \log x$ and its reflection is the y-axis.

Lesson 8.6

8.6 Guided Practice (p. 505)

1. $2^{4x} = 8^{x-3}$; $\log (x + 1) + \log (x - 1) = 2.32$

2. Both types of equations can be solved by equating exponents or the expressions whose logarithms you are trying to find; otherwise by using the inverse of the function. In the case of an exponential equation this means taking the logarithm of each side; in the case of a logarithmic equation, raising each side to the same power.

3. Logarithmic equations sometimes have extraneous solutions because the domain of a logarithmic function does not generally include all real numbers.

4. $3^x = 14$

$\log_3 3^x = \log_3 14$

$x = \log_3 14$

$x = \dfrac{\log 14}{\log 3}$

$x = 2.402$

5. $5^x = 8$

$\log_5 5^x = \log_5 8$

$x = \log_5 8$

$x = \dfrac{\log 8}{\log 5}$

$x = 1.292$

6. $9^{2x} = 3^{x-6}$

$(3^2)^{2x} = 3^{x-6}$

$3^{4x} = 3^{x-6}$

$4x = x - 6$

$3x = -6$

$x = -2$

7. $10^{3x-4} = 0.1$

$10^{3x-4} = 10^{-1}$

$3x - 4 = -1$

$3x = 3$

$x = 1$

8. $2^{3x} = 4^{x-1}$

$2^{3x} = (2^2)^{x-1}$

$2^{3x} = 2^{2(x-1)}$

$2^{3x} = 2^{2x-2}$

$3x = 2x - 2$

$x = -2$

9. $10^{3x-1} + 4 = 32$

$10^{3x-1} = 28$

$\log 10^{3x-1} = \log 28$

$3x - 1 = \log 28$

$3x = \log 28 + 1$

$x = \dfrac{\log 28 + 1}{3}$

≈ 0.816

10. $\log x = 2.4$

$10^{2.4} = x$

$x = 251.189$

11. $\log x = 3$

$10^3 = x$

$x = 1000$

12. $\log_3 (2x - 1) = 3$

$3^3 = 2x - 1$

$27 = 2x - 1$

$28 = 2x$

$x = 14$

13. $12 \ln x = 44$

$\ln x = \frac{44}{12}$

$\ln x = 3.67$

$e^{3.67} = x$

$x = 39.121$

14. $\log_2 (x + 2) = \log_2 x^2$

$(x + 2) = x^2$

$x + 2 = x^2$

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1$ or $x = 2$

15. $\log 3x + \log(x + 2) = 1$

$\log [3x(x + 2)] = 1$

$\log (3x^2 + 6x) = 1$

$3x^2 + 6x = 10^1$

$3x^2 + 6x - 10 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(3)(-10)}}{2(3)}$

$x = \dfrac{-6 \pm \sqrt{36 + 120}}{6}$

$x = \dfrac{-6 \pm \sqrt{156}}{6}$

$x = -\dfrac{6}{6} \pm \dfrac{\sqrt{4}\sqrt{39}}{6}$

$x = -1 \pm \dfrac{2\sqrt{39}}{6}$

$x = -1 \pm \dfrac{\sqrt{39}}{3} \approx 1.082$

16. $4^{x+1} = 8^x$

$(2^2)^{x+1} = (2^3)^x$

$2^{2x+2} = 2^{3x}$

$2x + 2 = 3x$

$x = 2$

$\log_4 8 \neq 2$

$4^{x+1} = 8^x$

$\log_4 4^{x+1} = \log_4 8^x$

$x + 1 = x \log + 8$

17. $\log_2 5x = 8$

$e^{\log_2 5x} = e^8$

$e^{\log_2 5x} \neq 5x$, since e^x and $\log_2 x$ are not inverse functions.

18. $M = 0.291 \ln E + 1.17$

$9.2 = 0.291 \ln E + 1.17$

$8.03 = 0.291 \ln E$

$27.595 \approx \ln E$

$e^{27.595} \approx e^{\ln E}$

$9.646 \times 10^{11} \approx E$

$964{,}600{,}000{,}000 \approx E$

about 960 billion ergs

8.6 Practice and Applications (p. 505)

19. $\ln x = 27$, $x = e^{27}$ yes

$e^{27} = x$

20. $5 - \log_4 2x = 3$, $x = 8$ yes

$5 - 3 = \log_4 2x$

$\log_4 2x = 2$

$4^2 = 2x$

$16 = 2x$

$x = 8$

21. $\ln 5x = 4$, $x = \frac{1}{4}e^5$ no

$e^4 = 5x$

$\frac{e^4}{5} = x$

$x = \frac{1}{5}e^4$

22. $\log_5 \frac{1}{2}x = 17$, $x = 2e^{17}$ no

$(5)^{17} = \frac{1}{2}x$

$2(5)^{17} = x$

23. $5e^x = 15$, $x = \ln 3$ yes

$e^x = \frac{15}{5}$

$e^x = 3$

$\ln 3 = x$

24. $e^x + 2 = 18$, $x = \log_2 16$ no

$e^x = 16$

$\ln 16 = x$

25. $10^{x-3} = 100^{4x-5}$

$10^{x-3} = (10^2)^{4x-5}$

$10^{x-3} = 10^{8x-10}$

$x - 3 = 8x - 10$

$7 = 7x$

$x = 1$

26. $25^{x-1} = 125^{4x}$

$(5^2)^{x-1} = (5^3)^{4x}$

$5^{2x-2} = 5^{12x}$

$2x - 2 = 12x$

$-2 = 10x$

$x = -\frac{2}{10} = -\frac{1}{5}$

27. $3^{x-7} = 27^{2x}$

$3^x - 7 = (3^3)^{2x}$

$3^{x-7} = 3^{6x}$

$x - 7 = 6x$

$-7 = 5x$

$x = -\frac{7}{5}$

28. $36^{x-9} = 6^{2x}$

$(6^2)^{x-9} = 6^{2x}$

$6^{2x-18} = 6^{2x}$

$2x - 18 = 2x$

$0 \neq -18$

no solution

29. $8^{5x} = 16^{3x+4}$

$(2^3)^{5x} = (2^4)^{3x+4}$

$2^{15x} = 2^{12x+16}$

$15x = 12x + 16$

$3x = 16$

$x = \frac{16}{3}$

30. $e^{-x} = 6$

$e^x = \frac{1}{6}$

$\ln \frac{1}{6} = x \approx -1.792$

31. $2^x = 15$

$\log_2 2^x = \log_2 15$

$x = \log_2 15$

$x = \frac{\log 15}{\log 2}$

$x = 3.907$

32. $1.2e^{-5x} + 2.6 = 3$

$1.2e^{-5x} = 0.4$

$e^{-5x} = \frac{0.4}{1.2}$

$e^{-5x} = \frac{1}{3}$

$e^{5x} = 3$

$\ln 3 = 5x$

$\frac{\ln 3}{5} = x$

$x = 0.220$

33. $4^x - 5 = 3$

$4^x = 8$

$\log_4 4x = \log_4 8$

$x = \log_4 8$

$x = \frac{\log 8}{\log 4} = 1.5 = \frac{3}{2}$

34. $-5e^{-x} + 9 = 6$

$-5e^{-x} = -3$

$e^{-x} = \frac{3}{5}$

$e^x = \frac{5}{3}$

$\ln \frac{5}{3} = x$

$x = 0.511$

35. $10^{2x} + 3 = 8$

$10^{2x} = 5$

$\log 10^{2x} = \log 5$

$2x = \log 5$

$x = \frac{\log 5}{2}$

$x = 0.349$

36. $0.25^x - 0.5 = 2$

$0.25^x = 2.5$

$\log_{1/4}\left(\frac{1}{4}\right)^x = \log_{1/4} 2.5$

$x = \log_{1/4} 2.5$

$x = \frac{\log 2.5}{\log 0.25}$

$x = -0.661$

37. $\frac{1}{4}(4)^{2x} + 1 = 5$

$\frac{1}{4}(4)^{2x} = 4$

$(4)^{2x} = 16$

$\log_4(4)^{2x} = \log_4 16$

$2x = \log_4 16$

$x = \frac{1}{2}(\log_4 16)$

$x = \frac{1}{2}\left(\frac{\log 16}{\log 4}\right) = \frac{1}{2}(2) = 1$

38. $\frac{2}{3}e^{4x} + \frac{1}{3} = 4$

$\frac{2}{3}e^{4x} = \frac{11}{3}$

$e^{4x} = \frac{11}{3}\left(\frac{3}{2}\right)$

$e^{4x} = \frac{11}{2}$

$\ln e^{4x} = \ln \frac{11}{2}$

$4x = \ln \frac{1}{2}$

$x = \frac{1}{4}\left(\ln \frac{11}{2}\right)$

$x = 0.426$

39. $10^{-12x} + 6 = 100$

$10^{-12x} = 94$

$\log 10^{-12x} = \log 94$

$-12x = \log 94$

$x = -\frac{1}{12}(\log 94)$

$x = -0.164$

40. $4 - 2e^x = -23$

$-2e^x = -23 - 4$

$-2e^x = -27$

$e^x = \frac{27}{2}$

$\ln \frac{27}{2} = x$

$x \approx 2.603$

41. $3^{0.1x} - 4 = 5$

$3^{0.1x} = 9$

$3^{0.1x} = 3^2$

$0.1x = 2$

$x = 20$

42. $-16 + 0.2(10)^x = 35$

$0.2(10)^x = 51$

$\frac{1}{5}(10)^x = 51$

$(10)^x = 255$

$\log 255 = x$

$x \approx 2.407$

43. $\ln(4x + 1) = \ln(2x + 5)$

$4x + 1 = 2x + 5$

$2x = 4$

$x = 2$

44. $\log_2 x = -1$

$2^{-1} = x$

$\frac{1}{2} = x$

45. $4 \log_3 x = 28$

$\log_3 x = \frac{28}{4}$

$\log_3 x = 7$

$3^7 = x$

$x = 2187$

46. $16 \ln x = 30$

$\ln x = \frac{30}{16}$

$\ln x = \frac{15}{8}$

$e^{15/8} = x$

$x \approx 6.521$

47. $\frac{1}{2}\log_6 16x = 3$

$\log_6 16x = 6$

$6^6 = 16x$

$\frac{46{,}656}{16} = x$

$x = 2916$

48. $1 - 2 \ln x = -4$

$-2 \ln x = -5$

$\ln x = \frac{5}{2}$

$e^{5/2} = x$

49. $2 \ln(-x) + 7 = 14$

$2 \ln(-x) = 7$

$\ln(-x) = \frac{7}{2}$

$e^{7/2} = -x$

$x = -e^{7/2}$

50. $\log_5(2x + 15) = \log_5 3x$

$2x + 15 = 3x$

$15 = x$

51. $\ln x + \ln(x - 2) = 1$

$\ln[x(x - 2)] = 1$

$e^{\ln(x^2 - 2x)} = e^1$

$x^2 - 2x = e$

$x^2 - 2x - e = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-e)}}{2(1)} = \frac{2 \pm \sqrt{4 + 4e}}{2}$

$= 1 + \sqrt{1 + e}$

≈ 2.928

52. $\ln x + \ln(x + 3) = 1$

$\ln[x(x + 3)] = 1$

$e^{\ln(x^2 + 3x)} = e^1$

$x^2 + 3x = e$

$x^2 + 3x - e = 0$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-e)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{9 + 4e}}{2} \approx 0.729$

53. $\log_8(11 - 6x) = \log_8(1 - x)$

$11 - 6x = 1 - x$

$11 - 1 = 6x - x$

$10 = 5x$

$2 = x$ extraneous root

$\log_8[11 - 6(2)] = \log_8(1 - 2)$

$\log_8(-1) \neq \log_8(-1)$

no solution

54. $15 + 2 \log_2 x = 31$

$2 \log_2 x = 16$

$\log_2 x = 8$

$2^8 = x$

$x = 256$

55. $-5 + 2 \ln 3x = 5$

$2 \ln 3x = 10$

$\ln 3x = 5$

$e^5 = 3x$

$\frac{e^5}{3} = x$

$x = \frac{1}{3}e^5$

56. $\log(5 - 3x) = \log(4x - 9)$

$5 - 3x = 4x - 9$

$14 = 7x$ no solution

$x = 2$ extraneous root

57. $6.5 \log_5 3x = 20$

$\log_5 3x = \frac{20}{6.5}$

$5^{20/6.5} = 3x$

$\frac{5^{20/6.5}}{3} = x$

$x = 47.158$

58. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

$x + 5 = (x - 1) - (x + 1)$

$x + 5 = x - x - 1 - 1$

$x + 5 = -2$ no solution

$x = -7$ extraneous root

59. $\ln(5.6 - x) = \ln(18.4 - 2.6x)$

$5.6 - x = 18.4 - 2.6x$

$1.6x = 12.8$

$x = \frac{12.8}{1.6}$

$x = 8$ extraneous root

no solution

60. $10 \ln 100x - 3 = 117$

$10 \ln 100x = 120$

$\ln 100x = 12$

$e^{12} = 100x$

$\frac{e^{12}}{100} = x$

$x = 0.01e^{12}$

61. $4^{3x} = 8^{x+1}$

$\log 4^{3x} = \log 8^{x+1}$

$3x \log 4 = (x + 1)\log 8$

$3x \log 4 = x \log 8 + \log 8$

$3x \log 4 - x \log 8 = \log 8$

$x(3 \log 4 - \log 8) = \log 8$

$x = \frac{\log 8}{3 \log 4 - \log 8} = 1$

Sample answer: This method is needlessly complicated.

62. $T = (T_0 - T_R)e^{-rt} + T_R$

$95 = (205 - 68)e^{-0.03t} + 68$

$27 = 137e^{-0.03t}$

$\frac{27}{137} = e^{-0.03t}$

$0.197 \approx e^{-0.03t}$

$\ln 0.197 \approx \ln e^{-0.03t}$

$-1.625 \approx -0.03t$

$t \approx \frac{-1.625}{-0.03}$

$t \approx 54.2$ about 54 minutes

63. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$2400 = 2000\left(1 + \frac{0.02}{4}\right)^{4t}$

$2400 = 2000(1.005)^{4t}$

$\frac{2400}{2000} = (1.005)^{4t}$

$1.2 = (1.005)^{4t}$

$\log 1.2 = \log(1.005)^{4t}$

$0.079 = 4t \log (1.005)$

$\frac{0.079}{4 \log(1.005)} = t$

$t = \frac{0.079}{0.009}$

$t \approx 8.78$ or about 9 years

64. $y = a(1 - r)^t$

$10 = 20(1 - 0.05)^t$

$10 = 20(0.95)^t$

$\frac{1}{2} = (0.95)^t$

$\log \frac{1}{2} = \log(0.95)^t$

$-0.301 = t \log(0.95)$

$\frac{-0.301}{-0.022} \approx t$

$13.5 \approx t$ about 13.5 days

65. $A = Pe^{rt}$

$1000 = 500e^{0.025t}$

$2 = e^{0.025t}$

$\ln 2 = \ln e^{0.025t}$

$\ln 2 = 0.025t \ln e$

$\frac{\ln 2}{0.025 \ln e} = t$

27.7 years $\approx t$

66. $P = 8863(1.04)^t$

$345{,}000 = 8863(1.04)^t$

$\frac{345{,}000}{8863} = (1.04)^t$

$38.926 = (1.04)^t$

$\log 38.926 = t \log(1.04)$

$\frac{\log 38.926}{\log(1.04)} = t$

$93.36 \approx t$

about 93.4 years or in 1713

67. **a.** Subantarctic:

$d = 1.0245 - e^{0.1266T - 7.828}$

$1.0234 = 1.0245 - e^{0.1266T - 7.828}$

$e^{0.1266T - 7.828} = 1.0245 - 1.0234$

$e^{0.1266T - 7.828} = 0.0011$

$\ln e^{0.1266T - 7.828} = \ln 0.0011$

$0.1266T - 7.828 \ln e = \ln 0.0011$

$T = \frac{\ln 0.0011 + 7.828 \ln e}{0.1266}$

$T \approx 8.022°C$

—CONTINUED—

67. —CONTINUED—

b. Antarctic intermediate:

$$1.02384 = 1.0245 - e^{0.1266T - 7.828}$$
$$e^{0.1266T - 7.828} = 1.0245 - 1.02384$$
$$e^{0.1266T - 7.828} = 0.00066$$
$$\ln e^{0.1266T - 7.828} = \ln 0.00066$$
$$0.1266T - 7.828 \ln e = \ln 0.00066$$
$$T = \frac{\ln 0.00066 + 7.828 \ln e}{0.1266}$$
$$T = 3.99°C$$

c. North Atlantic deep:

$$1.02399 = 1.0245 - e^{0.1266T - 7.828}$$
$$e^{0.1266T - 7.828} = 1.0245 - 1.02399$$
$$e^{0.1266T - 7.828} = 0.00051$$
$$\ln e^{0.1266T - 7.828} = \ln 0.00051$$
$$0.1266T - 7.828 \ln e = \ln 0.00051$$
$$T = \frac{\ln 0.00051 + 7.828 \ln e}{0.1266}$$
$$T = 1.95°C$$

d. Antarctic bottom:

$$1.0241 = 1.0245 - e^{0.1266T - 7.828}$$
$$e^{0.1266T - 7.828} = 1.0245 - 1.0241$$
$$e^{0.1266T - 7.828} = 0.0004$$
$$\ln e^{0.1266T - 7.828} = \ln 0.0004$$
$$0.1266T - 7.828 \ln e = \ln 0.0004$$
$$T = \frac{\ln 0.0004 + 7.828 \ln e}{0.1266}$$
$$T = 0.03°C$$

68.
$$m = e^{6.331 - 0.403t}$$
$$204 = e^{6.331 - 0.403t}$$
$$\ln 204 = \ln e^{6.331 - 0.403t}$$
$$\ln 204 = 6.331 - 0.403t \ln e$$
$$\ln 204 - 6.331 = -0.403t \ln e$$
$$\frac{\ln 204 - 6.331}{-0.403 \ln e} = t$$
$$2.51\ \mu s \approx t$$

69.
$$m = 5 \log D + 2$$
$$12 = 5 \log D + 2$$
$$12 - 2 = 5 \log D$$
$$10 = 5 \log D$$
$$2 = \log D$$
$$10^2 = D$$
$$100\ \text{mm} = D$$

70.
$$h = -8005 \ln \frac{P}{101{,}300}$$
$$4000 = -8005 \ln \frac{P}{101{,}300}$$
$$-\frac{4000}{8005} = \ln \frac{P}{101{,}300}$$
$$-0.499 \approx \ln P - \ln 101{,}300$$
$$\ln 101{,}300 - 0.499 \approx \ln P$$
$$11.027 \approx \ln P$$
$$e^{11.027} \approx P$$
$$P \approx 61{,}461 \text{ pascals}$$

71. a.
$$l = 45 - 25.7e^{-0.09a}$$
$$36 = 45 - 25.7e^{-0.09a}$$
$$25.7e^{-0.09a} = 9$$
$$e^{-0.09a} = \frac{9}{25.7}$$
$$\ln e^{-0.09a} \approx \ln 0.350194553$$
$$a(-0.09) \ln e \approx \ln 0.350194553$$
$$a \approx \frac{\ln 0.350194553}{(-0.09) \ln e}$$
$$a \approx 11.66 \text{ years or about 12 years}$$

$$32 = 45 - 25.7e^{-0.09a}$$
$$25.7e^{-0.09a} = 13$$
$$e^{-0.09a} = \frac{13}{25.7}$$
$$\ln e^{-0.09a} \approx \ln 0.506$$
$$a(-0.09) \ln e \approx \ln 0.506$$
$$a \approx \frac{\ln 0.506}{(-0.09) \ln e}$$
$$a \approx 7.57 \text{ years or about 8 years}$$

$$28 = 45 - 25.7e^{-0.09a}$$
$$25.7e^{-0.09a} = 17$$
$$e^{-0.09a} = \frac{17}{25.7}$$
$$\ln e^{-0.09a} \approx \ln 0.661$$
$$a(-0.09) \ln e \approx \ln 0.661$$
$$a \approx \frac{\ln 0.661}{(-0.09) \ln e}$$
$$a \approx 4.59 \text{ years or about 5 years}$$

—CONTINUED—

71. a. —CONTINUED—

$$24 = 45 - 25.7e^{-0.09a}$$

$$25.7e^{-0.09a} = 21$$

$$e^{-0.09a} = \frac{21}{25.7}$$

$$\ln e^{-0.09a} \approx \ln 0.817$$

$$a(-0.09)\ln e \approx \ln 0.817$$

$$a \approx \frac{\ln 0.817}{(-0.09)\ln e}$$

$$a \approx 2.24 \text{ years or about 2 years}$$

b. $I = 45 - 25.7e^{-0.09a}$

$$25.7e^{-0.09a} = 45 - I$$

$$e^{-0.09a} = \frac{45 - I}{25.7}$$

$$\ln e^{-0.09a} = \ln \frac{45 - I}{25.7}$$

$$(-0.09)a(\ln e) = \ln \frac{45 - I}{25.7}$$

$$a = \frac{\ln \frac{45 - I}{25.7}}{-0.09}$$

$$a = \frac{\ln \frac{45 - 36}{25.7}}{-0.09} \approx 11.66 \text{ years or about 12 years}$$

$$a = \frac{\ln \frac{45 - 32}{25.7}}{-0.09} \approx 7.57 \text{ years or about 8 years}$$

$$a = \frac{\ln \frac{45 - 28}{25.7}}{-0.09} \approx 4.59 \text{ years or about 5 years}$$

$$a = \frac{\ln \frac{45 - 24}{25.7}}{-0.09} \approx 2.24 \text{ years or about 2 years}$$

c. It is difficult to solve for a in part 71(b), so I prefer the method in 71(a).

72.

$$2^{x+3} = 5^{3x-1}$$

$$\log_2 2^{x+3} = \log_2 5^{3x-1}$$

$$x + 3 = \log_2 5^{3x-1}$$

$$x + 3 = 3x - 1(\log_2 5)$$

$$x + 3 = 3x(\log_2 5) - (\log_2 5)$$

$$3 + \log_2 5 = 3x(\log_2 5) - x$$

$$3 + \log_2 5 = x(3 \log_2 5 - 1)$$

$$\frac{3 + \log_2 5}{3 \log_2 5 - 1} = x$$

$$\frac{3 + \frac{\log 5}{\log 2}}{3\left(\frac{\log 5}{\log 2}\right) - 1} \approx 0.892 = x$$

73.

$$10^{5x+2} = 5^{4-x}$$

$$\log 10^{5x+2} = \log 5^{4-x}$$

$$5x + 2 = \log 5^{4-x}$$

$$5x + 2 = (4 - x)\log 5$$

$$5x + 2 = 4 \log 5 - x \log 5$$

$$5x + x \log 5 = 4 \log 5 - 2$$

$$x(5 + \log 5) = 4 \log 5 - 2$$

$$x = \frac{4 \log 5 - 2}{5 + \log 5} = \frac{4(0.698970004) - 2}{5 + (0.698970004)}$$

$$= \frac{0.795880017}{5.698970004} \approx 0.14$$

74.

$$\log_3(x - 6) = \log_9 2x$$

$$\log_3(x - 6) = \frac{\log_3 2x}{\log_3 9}$$

$$\log_3(x - 6) = \frac{\log_3 2x}{2}$$

$$2 \log_3(x - 6) = \log_3 2x$$

$$\log_3(x - 6)^2 = \log_3 2x$$

$$(x - 6)^2 = 2x$$

$$x^2 - 14x + 36 = 0$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(36)}}{2(1)}$$

$$x = \frac{14 \pm \sqrt{52}}{2}$$

$$x = \frac{14 \pm 2\sqrt{13}}{2}$$

$$x = 7 - \sqrt{13} \text{ extraneous solution}$$

$$x = 7 + \sqrt{13}$$

75.

$$\log_4 x = \log_8 4x$$

$$\log_4 x = \frac{\log_4 4x}{\log_4 8}$$

$$\log_4 x = \frac{\log_4 4x}{\frac{3}{2}}$$

$$\frac{3}{2}\log_4 x = \log_4 4x$$

$$\log_4 x^{\frac{3}{2}} = \log_4 4x$$

$$x^{\frac{3}{2}} = 4x$$

$$x^{\frac{3}{2}} - 4x = 0$$

$$x(x^{\frac{1}{2}} - 4) = 0$$

$$x = 0 \qquad x^{\frac{1}{2}} - 4 = 0$$

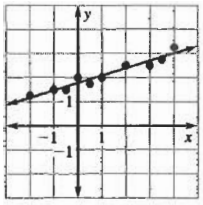
$$\left(x^{\frac{1}{2}}\right)^2 = (4)^2$$

$$x = 16$$

76. *Sample answer:* For an exponential equation, take the logarithm of both sides to one of the bases, use the change of base formula for the other side of the equation or take the common logarithm or natural logarithm of both sides of the equation. For a logarithmic equation, use the change of base formula to rewrite both sides in terms of logarithms with a single base, then solve normally.

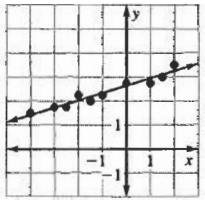
8.6 Mixed Review (p. 508)

77. $m = \dfrac{3.25 - 1.25}{4 + 2} = \dfrac{2}{6} = \dfrac{1}{3}$



$$\frac{y - 1.25}{x + 2} = \frac{1}{3}$$
$$3(y - 1.25) = x + 2$$
$$3y - 3.75 = x + 2$$
$$3y = x + 5.75$$
$$y = \frac{1}{3}x + \frac{5.75}{3}$$
$$y = \frac{1}{3}x + 1.92$$

78. $m = \dfrac{3.5 - 1.5}{2 + 4} = \dfrac{2}{6} = \dfrac{1}{3}$



$$\frac{y - 1.5}{x + 4} = \frac{1}{3}$$
$$3(y - 1.5) = x + 4$$
$$3y - 4.5 = x + 4$$
$$3y = x + 8.5$$
$$y = \frac{1}{3}x + \frac{8.5}{3}$$
$$y = \frac{1}{3}x + 2.83$$

79. $2x - y = 3$
$-y = -2x + 3$
$y = 2x - 3$
$3x - 2y = 2$
$3x - 2(2x - 3) = 2$
$3x - 4x + 6 = 2$
$-x = 2 - 6$
$-x = -4$
$x = 4$

$y = 2x - 3$
$y = 2(4) - 3$
$y = 8 - 3$
$y = 5$
$(4, 5)$

80. $2x + y = 4$
$x + y = 3$
$y = 3 - x$
$y = 3 - 1 = 2$

$2x + (3 - x) = 4$
$2x + 3 - x = 4$
$x = 1$
$(1, 2)$

81. $x + 4y = -24$
$x - 4y = 24$
$x + 4(-6) = -24$
$x = 0$
$(0, -6)$

$x = -24 - 4y$
$x - 4y = 24$
$(-24 - 4y) - 4y = 24$
$-24 - 8y = 24$
$-8y = 48$
$y = \dfrac{48}{-8} = -6$

82. $x - 3y = -3$
$2x + y = 8$
$x - 3(2) = -3$
$x - 6 = -3$
$x = 3$
$(3, 2)$

$x = 3y - 3$
$2(3y - 3) + y = 8$
$6y - 6 + y = 8$
$7y = 14$
$y = 2$

83. $2x + y = -1$
$-4x - 2y = -5$
$y = -2x - 1$
$-4x - 2(-2x - 1) = -5$
$-4x + 4x + 2 = -5$
$0 \neq -7$

no solution

84. $-x + 6y = -32$
$7x - 2y = 24$
$7(6y + 32) - 2y = 24$
$42y + 224 - 2y = 24$
$40y = -200$
$y = -5$

$-x = -32 - 6y$
$x = 6y + 32$
$x = 6(-5) + 32$
$x = -30 + 32$
$x = 2$
$(2, -5)$

85. $3x^3 - 6x^2 + 4x - 8 = (3x^2)x - (3x^2)2 + 4(x) - 4(2)$
$= 3x^2(x - 2) + 4(x - 2)$
$= (3x^2 + 4)(x - 2)$

86. $2x^3 - 5x^2 + 16x - 40 = 2x(x^2) - 5(x^2) + 2x(8) - 5(8)$
$= x^2(2x - 5) + 8(2x - 5)$
$= (x^2 + 8)(2x - 5)$

87. $7x^3 + 4x^2 + 35x + 20 = 7x(x^2) + 4(x^2) + 5(7x) + 5(4)$
$= x^2(7x + 4) + 5(7x + 4)$
$= (x^2 + 5)(7x + 4)$

88. $4x^3 - 3x^2 + 8x - 6 = x^2(4x) - x^2(3) + 2(4x) - 2(3)$
$= x^2(4x - 3) + 2(4x - 3)$
$= (x^2 + 2)(4x - 3)$

Quiz 2 (p. 508)

1. $\log_2 8$

$2^x = 8$

$2^3 = 8$ so $x = 3$

2. $\log_5 625$

$5^x = 625$

$5^4 = 625$ so $x = 4$

3. $\log_8 512$

$8^x = 512$

$8^3 = 512$

so $x = 3$

4. $y = \ln(x + 3)$

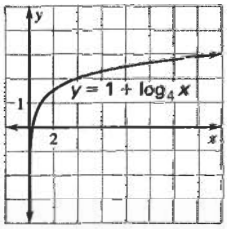
$x = \ln(y + 3)$

$e^x = y + 3$

$y = e^x - 3$

5. $y = 1 + \log_4 x$

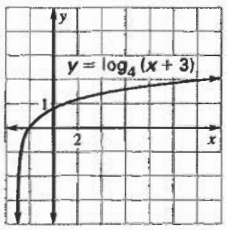
x	y
1	0
$\frac{1}{4}$	-1
4	1



Domain: $x > 0$
Range: all real numbers

6. $y = \log_4 (x + 3)$

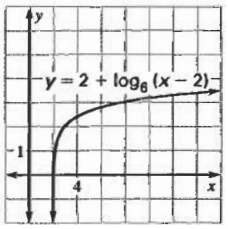
x	y
1	1
-2	0
-2.75	-1



Domain: $x > -3$
Range: all real numbers

7. $y = 2 + \log_6(x - 2)$

x	y
3	2
8	3
$2\frac{1}{16}$	1



Domain: $x > 2$
Range: all real numbers

8. $\log_3(3 \cdot 27) = \log_3 3 + \log_3 27 = 1 + 3 = 4$

9. $\log_2 \frac{1}{2}$ $\quad 2^x = \frac{1}{2}$

$2^{-1} = \frac{1}{2}$ so $x = -1$

10. $\ln e^2 = 2 \ln e = 2(1) = 2$

11. $\log_4 x^{1/2}y^4 = \log_4 x^{1/2} + \log_4 y^4$

$= \frac{1}{2}\log_4 x + 4 \log_4 y$

12. $2 \log_6 14 + 3 \log_6 x - \log_6 7$

$= \log_6(14)^2 + \log_6 x^3 - \log_6 7$

$= \log_6 \frac{196x^3}{7}$

$= \log_6 28x^3$

13. $\log_4 22 = \frac{\log 22}{\log 4} = 2.230$

14. $3e^x - 1 = 14$

$3e^x = 15$

$e^x = 5$

$\ln e^x = \ln 5$

$x \ln e = \ln 5$

$x = \ln 5$

15. $3 \log_2 x = 28$

$\log_2 x = \frac{28}{3}$

$2^{28/3} = x$

$x = 2^{28/3}$

16. $\ln(2x + 7) = \ln(x - 4)$

$2x + 7 = x - 4$

$x = -11$

no solution

17. $M = 0.291 \ln E + 1.17$

$8.5 = 0.291 \ln E + 1.17$

$7.33 = 0.291 \ln E$

$\frac{7.33}{0.291} = \ln E$

$25.189 \approx \ln E$

$e^{25.189} \approx e^{\ln E}$

$8.699 \times 10^{10} \approx E$

$E \approx 86{,}990{,}000{,}000$

$E \approx 87$ billion ergs

Lesson 8.7

8.7 Guided Practice (p. 513)

1. exponential model

2. 2 points: exponential function

2 points: power function

3. No, since 0 is not in the domain of $f(x) = \ln x$.

4. (1, 3), (2, 36)

$3 = ab^1$

$36 = ab^2$

$y = ab^x$

$a = \frac{3}{b^1}$

$36 = \left(\frac{3}{b^1}\right)b^2$

$36 = 3b$

$12 = b$

$a = \frac{3}{12} = \frac{1}{4}$

$y = \frac{1}{4} \cdot 12^x$

5. (2, 2), (4, 18)

$2 = ab^2$

$18 = ab^4$

$y = ab^x$

$a = \frac{2}{b^2}$

$18 = \left(\frac{2}{b^2}\right)b^4$

$18 = 2b^2$

$9 = b^2$

$3 = b$

$a = \frac{2}{(3)^2} = \frac{2}{9}$

$y = \frac{2}{9} \cdot 3^x$