CHAPTER P
Prerequisites

Section P.1 Graphical Representation of Data **2**

Section P.2 Graphs of Equations **7**

Section P.3 Lines in the Plane **13**

Section P.4 Solving Equations Algebraically and Graphically . . **20**

Section P.5 Solving Inequalities Algebraically and Graphically . **31**

Review Exercises **36**

CHAPTER P
Prerequisites

Section P.1 Graphical Representation of Data

- You should be able to plot points.
- You should know that the distance between (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

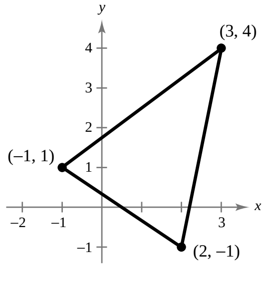
- You should know that the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

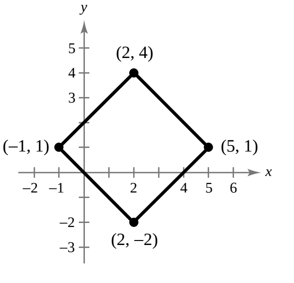
- You should know the equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$.
- You should be able to construct scatter plots, bar graphs and line graphs for a set of data.

Solutions to Odd-Numbered Exercises

1.



3.



5. A: $(2, 6)$, B: $(-6, -2)$, C: $(4, -4)$, D: $(-3, 2)$

7. A:$(0, 5)$, B:$(-3, -6)$, C:$(1, -4.5)$, D:$(-4, 2)$

9. $(-3, 4)$

11. $(-5, -5)$

13. $x > 0 \implies$ The point lies in Quadrant I or in Quadrant IV.

$y < 0 \implies$ The point lies in Quadrant III or in Quadrant IV.

$x > 0$ and $y < 0 \implies (x, y)$ lies in Quadrant IV.

15. $x = -4 \implies x$ is negative $\implies$ The point lies in Quadrant II or Quadrant III.

$y > 0 \implies$ The point lies in Quadrant I or Quadrant II.

$x = -4$ and $y > 0 \implies (x, y)$ lies in Quadrant II.

17. $y < -5 \implies y$ is negative $\implies$ The point lies in either Quadrant III or Quadrant IV.

19. Since $(x, -y)$ is in Quadrant II, we know that $x < 0$ and $-y > 0$. If $-y > 0$, then $y < 0$.

$x < 0 \implies$ The point lies in Quadrant II or in Quadrant III.

$y < 0 \implies$ The point lies in Quadrant III or in Quadrant IV.

$x < 0$ and $y < 0 \implies (x, y)$ lies in Quadrant III.

21. If $xy > 0$, then either x and y are both positive, or both negative. Hence, (x, y) lies in either Quadrant I or Quadrant III.

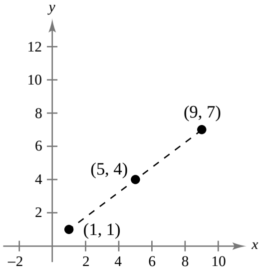
23. The x-coordinates are increased by 2, and the y-coordinates are increased by 5: $(0, 1)$, $(4, 2)$, $(1, 4)$.

25. $d = |5 - (-3)| = 8$

27. $d = |-3 - 2| = |-5| = 5$

29. $d = \sqrt{(3 - (-2))^2 + (-6 - 6)^2} = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$

31. (a)

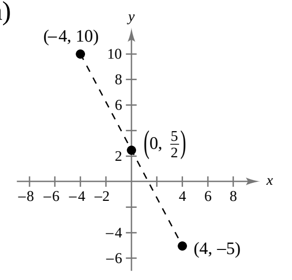


(b) $d = \sqrt{(9 - 1)^2 + (7 - 1)^2}$

$= \sqrt{64 + 36} = 10$

(c) $\left(\dfrac{9 + 1}{2}, \dfrac{7 + 1}{2}\right) = (5, 4)$

33. (a)

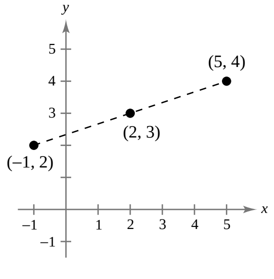


(b) $d = \sqrt{(4 + 4)^2 + (-5 - 10)^2}$

$= \sqrt{64 + 225} = 17$

(c) $\left(\dfrac{4 - 4}{2}, \dfrac{-5 + 10}{2}\right) = \left(0, \dfrac{5}{2}\right)$

35. (a)

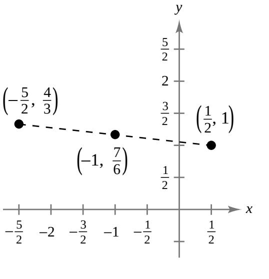


(b) $d = \sqrt{(5 + 1)^2 + (4 - 2)^2}$

$= \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

(c) $\left(\dfrac{-1 + 5}{2}, \dfrac{2 + 4}{2}\right) = (2, 3)$

37. (a)

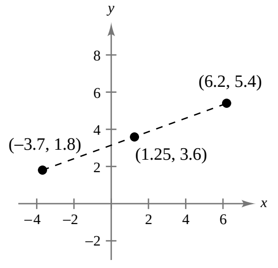


(b) $d = \sqrt{\left(\dfrac{1}{2} + \dfrac{5}{2}\right)^2 + \left(1 - \dfrac{4}{3}\right)^2}$

$= \sqrt{9 + \dfrac{1}{9}} = \dfrac{\sqrt{82}}{3}$

(c) $\left(\dfrac{-\frac{5}{2} + \frac{1}{2}}{2}, \dfrac{\frac{4}{3} + 1}{2}\right) = \left(-1, \dfrac{7}{6}\right)$

39. (a)



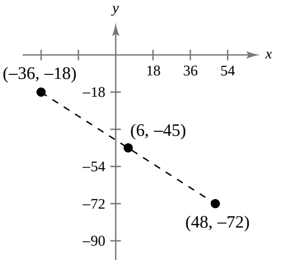
(b) $d = \sqrt{(6.2 + 3.7)^2 + (5.4 - 1.8)^2}$

$= \sqrt{98.01 + 12.96}$

$= \sqrt{110.97}$

(c) $\left(\dfrac{6.2 - 3.7}{2}, \dfrac{5.4 + 1.8}{2}\right) = (1.25, 3.6)$

41. (a)



(b) $d = \sqrt{(48 + 36)^2 + (-72 + 18)^2}$

$= \sqrt{7056 + 2916}$

$= \sqrt{9972} = 6\sqrt{277}$

(c) $\left(\dfrac{-36 + 48}{2}, \dfrac{-18 - 72}{2}\right) = (6, -45)$

43. (a) The distance between (0, 2) and (4, 2) is 4.
The distance between (4, 2) and (4, 5) is 3.
The distance between (0, 2) and (4, 5) is

$\sqrt{(4 - 0)^2 + (5 - 2)^2} = \sqrt{16 + 9} = \sqrt{25} = 5.$

(b) $4^2 + 3^2 = 16 + 9 = 25 = 5^2$

45. (a) The distance between $(-1, 1)$ and (9, 1) is 10.
The distance between (9, 1) and (9, 4) is 3.
The distance between $(-1, 1)$ and (9, 4) is

$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}.$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

47. $\left(\dfrac{1996 + 2000}{2}, \dfrac{\$520{,}000 + \$740{,}000}{2}\right) = (1998, \$630{,}000)$. The sales in 1998 are \$630,000.

49. Find distances between pairs of points.

$d_1 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{5}$

$d_2 = \sqrt{(4 + 1)^2 + (0 + 5)^2} = \sqrt{50}$

$d_3 = \sqrt{(2 + 1)^2 + (1 + 5)^2} = \sqrt{45}$

$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$

Because $d_1^2 + d_2^2 = d_3^2$, the triangle is a right triangle.

51. Find distances between pairs of points.

$d_1 = \sqrt{(0 - 2)^2 + (9 - 5)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$d_2 = \sqrt{(-2 - 0)^2 + (0 - 9)^2} = \sqrt{4 + 81} = \sqrt{85}$

$d_3 = \sqrt{(0 - (-2))^2 + (-4 - 0)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$d_4 = \sqrt{(0 - 2)^2 + (-4 - 5)^2} = \sqrt{4 + 81} = \sqrt{85}$

Opposite sides have equal lengths of $2\sqrt{5}$ and $\sqrt{85}$, so the figure is a parallelogram.

53. Since $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$$2x_m = x_1 + x_2 \qquad 2y_m = y_1 + y_2$$

$$2x_m - x_1 = x_2 \qquad 2y_m - y_1 = y_2$$

Thus, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

(a) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2(4) - 1, 2(-1) - (-2)) = (7, 0)$

(b) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2(2) - (-5), 2(4) - 11) = (9, -3)$

55. $(x - 0)^2 + (y - 0)^2 = 3^2$

$x^2 + y^2 = 9$

57. $(x - 2)^2 + (y + 1)^2 = 4^2$

$(x - 2)^2 + (y + 1)^2 = 16$

59. $(x + 1)^2 + (y - 2)^2 = r^2$

$(0 + 1)^2 + (0 - 2)^2 = r^2 \implies r^2 = 5$

$(x + 1)^2 + (y - 2)^2 = 5$

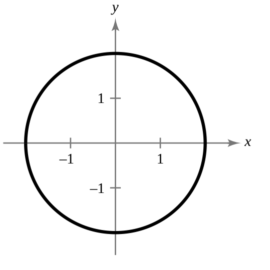
61. $r = \dfrac{1}{2}\sqrt{(6 - 0)^2 + (8 - 0)^2} = \dfrac{1}{2}\sqrt{100} = 5$

$\text{Center} = \left(\dfrac{0 + 6}{2}, \dfrac{0 + 8}{2}\right) = (3, 4)$

$(x - 3)^2 + (y - 4)^2 = 25$

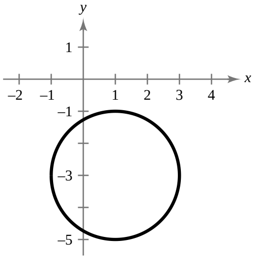
63. Center: $(0, 0)$

Radius $= 2$



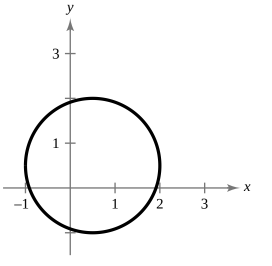
65. Center: $(1, -3)$

Radius $= 2$

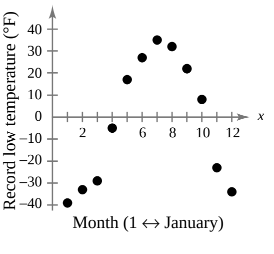


67. Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$

Radius $= \frac{3}{2}$



69.



71. The highest price was approximately \$1.66, which occured in 1996.

73. $\dfrac{1600 - 600}{600}(100) \approx 166.67\%$ from 1987 to 1999.

75. The point (65, 83) represents an entrance exam score of 65.

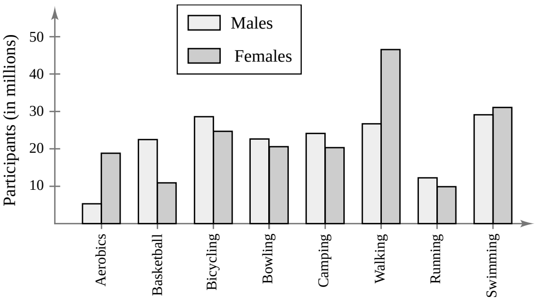
77. Corn: $\frac{45}{240}(100) = 18.75\% \approx 19\%$

Soybeans: $\frac{20}{60}(100) = 33.33\%$

Wheat: $\frac{35}{70}(100) = 50.0\%$

(Answers will vary.)

79.



81. (a) The savings decreased from 8.2% to 3.9%. The decrease is $\dfrac{8.2 - 3.9}{8.2} = 0.52$ or 52%.

(b) No. The trend limits the amount of funds available for capital improvements and investments

83. (a) Solve the equation $C = 900$:

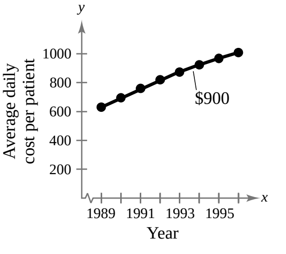
$$-2.37t^2 + 66.44t + 696.39 = 900$$
$$-2.37t^2 + 66.44t - 203.61 = 0$$

By the Quadratic Formula,

$$t = \frac{-66.44 \pm \sqrt{(66.44)^2 - 4(-2.37)(-203.61)}}{2(-2.37)}$$
$$= \frac{-66.44 \pm \sqrt{2484.0508}}{-4.74}$$

Hence, $t \approx 3.50$ and $t \approx 24.53$. Since 24.53 is not in the domain of C, the average cost C exceeded \$900 per day when $t > 3.5$ or the middle of 1993.

(b)



85. Let $(0, 0)$ represent the point of departure and let $(100, 150)$ represent the destination. Then the distance is given by

$$d = \sqrt{(100 - 0)^2 + (150 - 0)^2}$$
$$= \sqrt{10{,}000 + 22{,}500}$$
$$= \sqrt{32{,}500} = 50\sqrt{13} \approx 180.28 \text{ km.}$$

87. (a) It appears that the number of artists elected alternates between 6 and 8 per year in the 1990s. If this pattern continues, 6, 7 or 8 would be elected in 2001.

(b) Since 1986 and 1987 were the first two years that artists were elected, there was a larger number of artists chosen.

89. 1997 sales are given by the midpoint:

$$\left(\frac{1996 + 1998}{2}, \frac{1118.7 + 1371.4}{2}\right) = (1997, 1245.05)$$

The 1997 sales were approximately \$1245 million.

91. $d_1 = \sqrt{(2 - (-8))^2 + (11 - 4)^2} = \sqrt{10^2 + 7^2} = \sqrt{149}$

$d_2 = \sqrt{(-5 - (-8))^2 + (1 - 4)^2} = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$d_3 = \sqrt{(-5 - 2)^2 + (1 - 11)^2} = \sqrt{49 + 100} = \sqrt{149}$

Since $d_1 = d_3$, the triangle is isosceles. True.

93. On the x-axis, $y = 0$
On the y-axis, $x = 0$