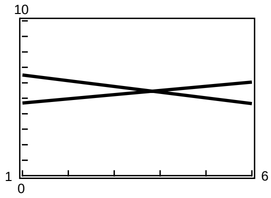
159. (a) The point of intersection represents the moment when the per capita utilizations of nectarines and cucumbers are equal.

(b) $-0.37t + 6.88 = 0.27t + 4.42$

$2.46 = 0.64t$

$t = 3.84375$ or during 1993

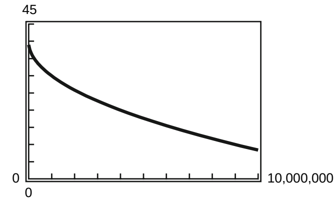
(c)



Intersection point: $(3.84375, 5.45781)$

161. $p = 40 - \sqrt{0.0001x + 1}, 0 \le x$

(a)



(b) If $p = 12.95$, then $x \approx 7{,}307{,}025$ books

163. False, they could have an infinite number of intersections.

Section P.5 Solving Inequalities Algebraically and Graphically

- You should know the properties of inequalities.
  - (a) Transitive: $a < b$ and $b < c$ implies $a < c$.
  - (b) Addition: $a < b$ and $c < d$ implies $a + c < b + d$.
  - (c) Adding or Subtracting a Constant: $a \pm c < b \pm c$ if $a < b$.
  - (d) Multiplying or Dividing by a Constant: For $a < b$,
    1. If $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
    2. If $c < 0$,then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
- You should know that

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}.$$

- You should be able to solve absolute value inequalities.
  - (a) $|x| < a$ if and only if $-a < x < a$.
  - (b) $|x| > a$ if and only if $x < -a$ or $x > a$.
- You should be able to solve polynomial inequalities.
  - (a) Find the critical numbers.
    1. Values that make the expression zero
    2. Values that make the expression undefined
  - (b) Test one value in each interval on the real number line resulting from the critical numbers.
  - (c) Determine the solution intervals.
- You should be able to solve rational and other types of inequalities.

Solutions to Odd-Numbered Problems

1. $x < 3$
Matches (d).

3. $-3 < x \le 4$
Matches (c).

5. (a) $x = 3$

$5(3) - 12 \stackrel{?}{>} 0$

$3 > 0$

Yes, $x = 3$ is a solution.

(b) $x = -3$

$5(-3) - 12 \stackrel{?}{>} 0$

$-27 \not> 0$

No, $x = -3$ is not a solution.

(c) $x = \frac{5}{2}$

$5\left(\frac{5}{2}\right) - 12 \stackrel{?}{>} 0$

$\frac{1}{2} > 0$

Yes, $x = \frac{5}{2}$ is a solution.

(d) $x = \frac{3}{2}$

$5\left(\frac{3}{2}\right) - 12 \stackrel{?}{>} 0$

$-\frac{9}{2} \not> 0$

No, $x = \frac{3}{2}$ is not a solution.

7. $-1 < \dfrac{3 - x}{2} \le 1$

(a) $x = 0$

$-1 \stackrel{?}{<} \dfrac{3 - 0}{2} \stackrel{?}{\le} 1$

$-1 \stackrel{?}{<} \dfrac{3}{2} \stackrel{?}{\le} 1$

No, $x = 0$ is not a solution.

(b) $x = \sqrt{5}$

$-1 \stackrel{?}{<} \dfrac{3 - \sqrt{5}}{2} \stackrel{?}{\le} 1$

$-1 \stackrel{?}{<} 0.382 \stackrel{?}{\le} 1$

Yes, $x = \sqrt{5}$ is a solution.

(c) $x = 1$

$-1 \stackrel{?}{<} \dfrac{3 - 1}{2} \stackrel{?}{\le} 1$

$-1 \stackrel{?}{<} 1 \stackrel{?}{\le} 1$

Yes, $x = 1$ is a solution.

(d) $x = 5$

$-1 \stackrel{?}{<} \dfrac{3 - 5}{2} \le 1$

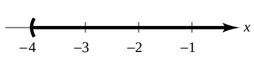
$-1 \stackrel{?}{<} -1 \stackrel{?}{\le} 1$

No, $x = 5$ is not a solution.

9. $-10x < 40$

$-\frac{1}{10}(-10) > -\frac{1}{10}(40)$

$x > -4$

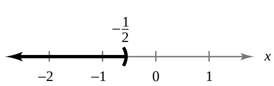


11. $4(x + 1) < 2x + 3$

$4x + 4 < 2x + 3$

$2x < -1$

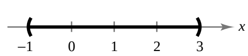
$x < -\frac{1}{2}$



13. $1 < 2x + 3 < 9$

$-2 < 2x < 6$

$-1 < x < 3$



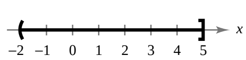
15. $-8 \le 1 - 3(x - 2) < 13$

$-8 \le 1 - 3x + 6 < 13$

$-8 \le -3x + 7 < 13$

$-15 \le -3x < 6$

$5 \ge x > -2 \Longrightarrow -2 < x \le 5$

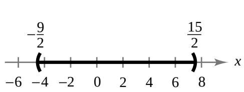


17. $-4 < \dfrac{2x - 3}{3} < 4$

$-12 < 2x - 3 < 12$

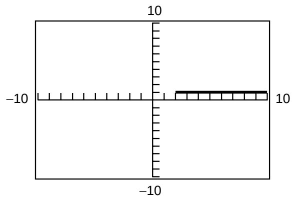
$-9 < 2x < 15$

$-\dfrac{9}{2} < x < \dfrac{15}{2}$



19. $6x > 12$

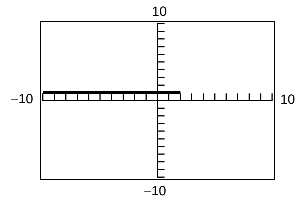
$x > 2$



21. $5 - 2x \geq 1$

$-2x \geq -4$

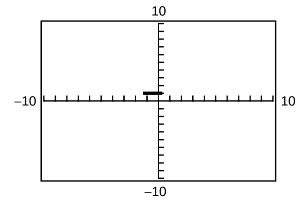
$x \leq 2$



23. $-9 < 6x - 1 < 1$

$-8 < 6x < 2$

$-\frac{4}{3} < x < \frac{1}{3}$



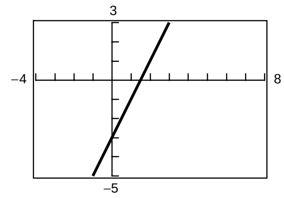
25. Using the graph, (a) $y \geq 1$ for $x \geq 2$ and (b) $y \leq 0$ for $x \leq \frac{3}{2}$.

Algebraically, (a) $y \geq 1$ (b) $y \leq 0$

$2x - 3 \geq 1$ $2x - 3 \leq 0$

$2x \geq 4$ $2x \leq 3$

$x \geq 2$ $x \leq \frac{3}{2}$



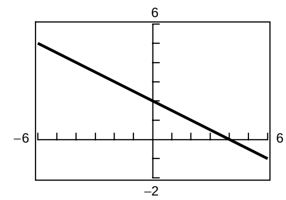
27. Using the graph, (a) $0 \leq y \leq 3$ for $-2 \leq x \leq 4$ and (b) $y \geq 0$ for $x \leq 4$

Algebraically, (a) $0 \leq y \leq 3$ (b) $y \geq 0$

$0 \leq -\frac{1}{2}x + 2 \leq 3$ $-\frac{1}{2}x + 2 \geq 0$

$-2 \leq -\frac{1}{2}x \leq 1$ $2 \geq \frac{1}{2}x$

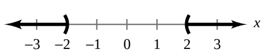
$4 \geq x \geq -2$ $4 \geq x$



29. $|5x| > 10$

$5x < -10$ or $5x > 10$

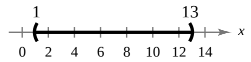
$x < -2$ or $x > 2$



31. $|x - 7| < 6$

$-6 < x - 7 < 6$

$1 < x < 13$

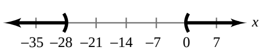


33. $|x + 14| + 3 > 17$

$|x + 14| > 14$

$x + 14 < -14$ or $x + 14 > 14$

$x < -28$ or $x > 0$



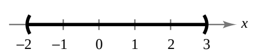
35. $|1 - 2x| < 5$

$-5 < 1 - 2x < 5$

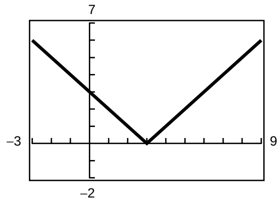
$-6 < -2x < 4$

$3 > x > -2$

$-2 < x < 3$



37. $y = |x - 3|$



(a) Graphically, $y \le 2$ for $1 \le x \le 5$

and (b) $y \ge 4$ for $x \le -1$ or $x \ge 7$

Algebraically,

(a) $y \le 2$

$|x - 3| \le 2$

$-2 \le x - 3 \le 2$

$1 \le x \le 5$

(b) $y \ge 4$

$|x - 3| \ge 4$

$x - 3 \le -4$ or $x - 3 \ge 4$

$x \le -1$ $\qquad$ $x \ge 7$

39. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than 3 units from 0.

$|x - 0| \le 3$

$|x| \le 3$

41. The graph shows all real numbers at least 3 units from 7.

$|x - 7| \ge 3$

43. All real numbers within 10 units of 12

$|x - 12| \le 10$

45. $(x + 2)^2 < 25$

$x^2 + 4x + 4 < 25$

$x^2 + 4x - 21 < 0$

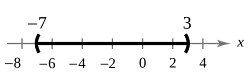
$(x + 7)(x - 3) < 0$

Critical numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x + 7)(x - 3) < 0$?

Solution set: $(-7, 3)$



47. $x^2 + 4x + 4 \ge 9$

$x^2 + 4x - 5 \ge 0$

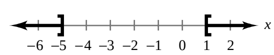
$(x + 5)(x - 1) \ge 0$

Critical numbers: $x = -5, x = 1$

Test intervals: $(-\infty, -5), (-5, 1), (1, \infty)$

Test: Is $(x + 5)(x - 1) \ge 0$?

Solution set: $(-\infty, -5] \cup [1, \infty)$



49. $x^3 - 4x \ge 0$

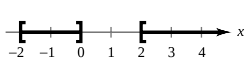
$x(x + 2)(x - 2) \ge 0$

Critical number: $x = 0, x = \pm 2$

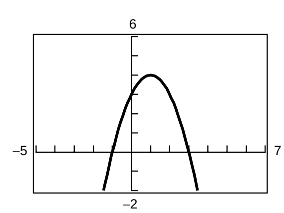
Test intervals: $(-\infty, -2), (-2, 0), (0, 2), (2, \infty)$

Test: Is $x(x + 2)(x - 2) \ge 0$?

Solution set: $[-2, 0] \cup [2, \infty)$



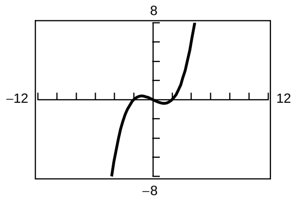
51. $y = -x^2 + 2x + 3$



(a) $y \le 0$ when $x \le -1$ or $x \ge 3$.

(b) $y \ge 3$ when $0 \le x \le 2$.

53. $y = \frac{1}{8}x^3 - \frac{1}{2}x$



(a) $y \geq 0$ when $-2 \leq x \leq 0, 2 \leq x$.

(b) $y \leq 6$ when $x \leq 4$.

55. $\frac{1}{x} - x > 0$

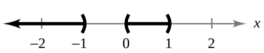
$\frac{1 - x^2}{x} > 0$

Critical numbers: $x = 0, x = \pm 1$

Test intervals: $(-\infty - 1), (-1, 0), (0, 1), (1, \infty)$

Test: Is $\frac{1 - x^2}{x} > 0$?

Solution set: $(-\infty, -1) \cup (0, 1)$



57. $\frac{x + 6}{x + 1} - 2 < 0$

$\frac{x + 6 - 2(x + 1)}{x + 1} < 0$

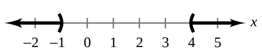
$\frac{4 - x}{x + 1} < 0$

Critical numbers: $x = -1, x = 4$

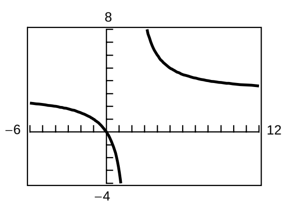
Test intervals: $(-\infty, -1), (-1, 4), (4, \infty)$

Test: Is $\frac{4 - x}{x + 1} < 0$?

Solution set: $(-\infty, -1) \cup (4, \infty)$



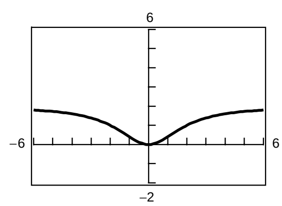
59. $y = \frac{3x}{x - 2}$



(a) $y \leq 0$ when $0 \leq x < 2$.

(b) $y \geq 6$ when $2 < x \leq 4$.

61. $y = \frac{2x^2}{x^2 + 4}$



(a) $y \geq 1$ when $x \leq -2$ or $x \geq 2$.

This can also be expressed as $|x| \geq 2$.

(b) $y \leq 2$ for all real numbers x.

This can also be expressed as $-\infty < x < \infty$.

63. $\sqrt{x - 5}$

Need $x - 5 \geq 0$

$x \geq 5$

Domain: $[5, \infty)$

65. $\sqrt[3]{6 - x}$

Domain: all real x

67. $\sqrt[4]{6x + 15}$

Need $6x + 15 \geq 0$

$6x > -15$

$x \geq -\frac{5}{2}$

Domain: $\left[-\frac{5}{2}, \infty\right)$