6. $-5 < 2x - 1 \leq 1$

(a) $x = -\frac{1}{2}$

$-5 \overset{?}{<} 2\left(-\frac{1}{2}\right) - 1 \overset{?}{\leq} 1$

$-5 \overset{?}{<} -1 - 1 \overset{?}{\leq} 1$

$-5 \overset{?}{<} -2 \overset{?}{\leq} 1$

Yes, $x = -1\frac{1}{2}$ is a solution.

(b) $x = -\frac{5}{2}$

$-5 \overset{?}{<} 2\left(-\frac{5}{2}\right) - 1 \overset{?}{\leq} 1$

$-5 \overset{?}{<} -6 \overset{?}{\leq} 1$

No, $x = -\frac{5}{2}$ is not a solution.

(c) $x = \frac{4}{3}$

$-5 \overset{?}{<} 2\left(\frac{4}{3}\right) - 1 \overset{?}{\leq} 1$

$-5 \overset{?}{<} \frac{8}{3} - 1 \overset{?}{\leq} 1$

$-5 \overset{?}{<} \frac{5}{3} \overset{?}{\leq} 1$

No, $x = \frac{4}{3}$ is not a solution.

(d) $x = 0$

$-5 \overset{?}{<} 2(0) - 1 \overset{?}{\leq} 1$

$-5 < -1 \overset{?}{\leq} 1$

Yes, $x = 0$ is a solution.

8. $|x - 10| \geq 3$

(a) $x = 13$

$|13 - 10| \overset{?}{\geq} 3$

$3 \geq 3$

Yes, $x = 13$ is a solution.

(b) $x = -1$

$|-1 - 10| \overset{?}{\geq} 3$

$|-11| \overset{?}{\geq} 3$

$11 \geq 3$

Yes, $x = -1$ is a solution.

(c) $x = 14$

$|14 - 10| \overset{?}{\geq} 3$

$4 \geq 3$

Yes, $x = 14$ is a solution.

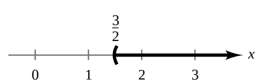
(d) $x = 9$

$|9 - 10| \overset{?}{\geq} 3$

$1 \overset{?}{\geq} 3$

No, $x = 9$ is not a solution.

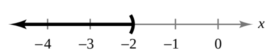
10. $2x > 3 \Longrightarrow x > \frac{3}{2}$



12. $2x + 7 < 3$

$2x < -4$

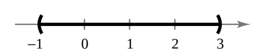
$x < -2$



14. $-2 < 3x + 1 < 10$

$-3 < 3x < 9$

$-1 < x < 3$

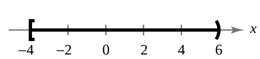


16. $0 \leq 2(x + 4) < 20$

$0 \leq 2x + 8 < 20$

$-8 \leq 2x < 12$

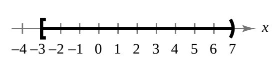
$-4 \leq x < 6$



18. $0 \leq \dfrac{x + 3}{2} < 5$

$0 \leq x + 3 < 10$

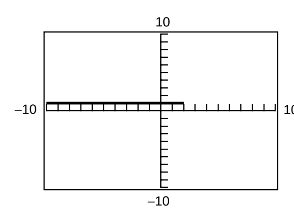
$-3 \leq x < 7$



20. $3x - 1 \leq 5$

$3x \leq 6$

$x \leq 2$

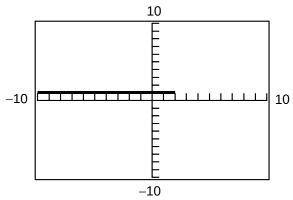


22. $3(x + 1) < x + 7$

$3x + 3 < x + 7$

$2x < 4$

$x < 2$

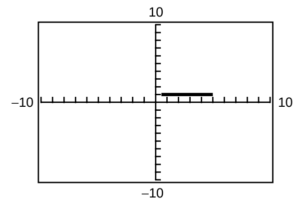


24. $-10 < 4(x - 3) \le 8$

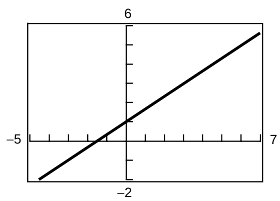
$-10 < 4x - 12 \le 8$

$2 < 4x \le 20$

$\frac{1}{2} < x \le 5$



26.



Using the graph, (a) $y \le 5$ for $x \le 6$, and (b) $y \ge 0$ for $x \ge -\frac{3}{2}$

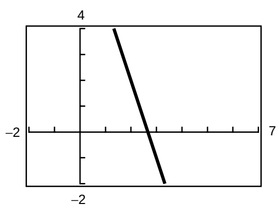
Algebraically, (a) $y \le 5$ (b) $y \ge 0$

$\frac{2}{3}x + 1 \le 5$ $\qquad \frac{2}{3}x + 1 \ge 0$

$\frac{2}{3}x \le 4$ $\qquad \frac{2}{3}x \ge -1$

$x \le 6$ $\qquad x \ge -\frac{3}{2}$

28.



Using the graph, (a) $-1 \le y \le 3$ for $\frac{5}{3} \le x \le 3$, and (b) $y \le 0$ for $x \ge \frac{8}{3}$.

Algebraically, (a) $-1 \le y \le 3$ (b) $y \le 0$

$-1 \le -3x + 8 \le 3$ $\qquad -3x + 8 \le 0$

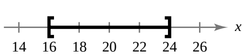
$-9 \le -3x \le -5$ $\qquad 8 \le 3x$

$3 \ge x \ge \frac{5}{3}$ $\qquad \frac{8}{3} \le x$

30. $|x - 20| \le 4$

$-4 \le -20 \le 4$

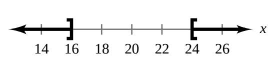
$16 \le x \le 24$



32. $|x - 20| \ge 4$

$x - 20 \ge 4$ or $x - 20 \le -4$

$x \ge 24$ or $x \le 16$

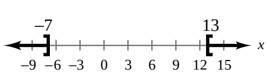


34. $\left|\frac{x - 3}{2}\right| \ge 5$

$|x - 3| \ge 10$

$x - 3 \ge 10$ or $x - 3 \le -10$

$x \ge 13$ or $x \le -7$



36. $3|4 - 5x| \le 9$

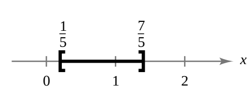
$|4 - 5x| \le 3$

$-3 \le 4 - 5x \le 3$

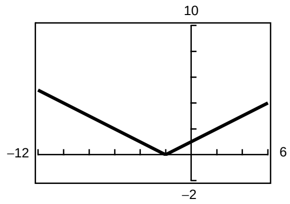
$-7 \le -5x \le -1$

$\frac{7}{5} \ge x \ge \frac{1}{5}$

$\frac{1}{5} \le x \le \frac{7}{5}$



38. $y = \left|\frac{1}{2}x + 1\right|$



Algebraically,

(a) $y \le 4$

$$\left|\tfrac{1}{2}x + 1\right| \le 4$$
$$-4 \le \tfrac{1}{2}x + 1 \le 4$$
$$-5 \le \tfrac{1}{2}x \le 3$$
$$-10 \le x \le 6$$

(b) $y \ge 1$

$$\left|\tfrac{1}{2}x + 1\right| \ge 1$$
$$\tfrac{1}{2}x + 1 \le -1 \text{ or } \tfrac{1}{2}x + 1 \ge 1$$
$$\tfrac{1}{2}x \le -2 \text{ or } \tfrac{1}{2}x \ge 0$$
$$x \le -4 \text{ or } x \ge 0$$

40. The graph shows all real numbers no more than 3 units from 0.

$$|x - 0| > 3$$
$$|x| > 3$$

42. The graph shows all real numbers no more than 4 units from -1.

$$|x + 1| \le 4$$

44. All real numbers more than 5 units from -3

$$|x + 3| > 5$$

46. $(x + 6)^2 \le 8$

$x^2 + 12x + 28 \le 0$

Zeros: $x = \dfrac{-12 \pm \sqrt{12^2 - 4(1)(28)}}{2(1)} = -6 \pm 2\sqrt{2}$

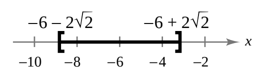
Critical numbers: $x = -6 + 2\sqrt{2}, x = -6 - 2\sqrt{2}$

Test intervals: $\left(-\infty, -6 - 2\sqrt{2}\right) \Longrightarrow x^2 + 12x + 28 > 0$

$\left(-6, -2\sqrt{2}, -6 + 2\sqrt{2}\right) \Longrightarrow x^2 + 12x + 28 < 0$

$\left(-6 + 2\sqrt{2}, \infty\right) \Longrightarrow x^2 + 12x + 28 > 0$

Solution interval: $\left[-6 - 2\sqrt{2}, -6 + 2\sqrt{2}\right]$



48. $x^2 - 6x + 9 < 16$

$x^2 - 6x - 7 < 0$

$(x + 1)(x - 7) < 0$

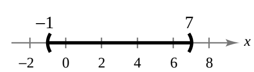
Critical numbers: $x = -1, x = 7$

Test intervals: $(-\infty, -1) \Longrightarrow (x + 1)(x - 7) > 0$

$(-1, 7) \Longrightarrow (x + 1)(x - 7) < 0$

$(7, \infty) \Longrightarrow (x + 1)(x - 7) > 0$

Solution interval: $(-1, 7)$



50. $x^4(x - 3) \le 0$

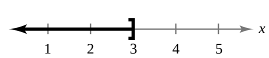
Critical numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Longrightarrow x^4(x - 3) < 0$

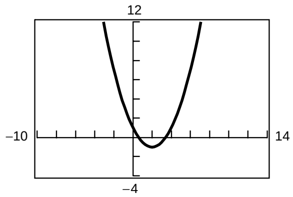
$(0, 3) \Longrightarrow x^4(x - 3) < 0$

$(3, \infty) \Longrightarrow x^4(x - 3) > 0$

Solution intervals: $(-\infty, 0] \cup [0, 3]$ or $(-\infty, 3]$



52. $y = \frac{1}{2}x^2 - 2x + 1$



(a) $y \le 1$

$\frac{1}{2}x^2 - 2x + 1 \le 1$

$x^2 - 4x \le 0$

$x(x - 4) \le 0$

$y \le 1$ when $0 \le x \le 4$.

(b) $y \ge 7$

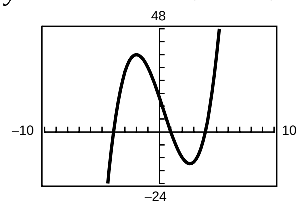
$\frac{1}{2}x^2 - 2x + 1 \ge 7$

$x^2 - 4x - 12 \ge 0$

$(x - 6)(x + 2) \ge 0$

$y \ge 7$ when $x \le -2$ or $x \ge 6$.

54. $y = x^3 - x^2 - 16x + 16$



(a) $y \le 0$

$x^3 - x^2 - 16x + 16 \le 0$

$x^2(x - 1) - 16(x - 1) \le 0$

$(x - 1)(x^2 - 16) \le 0$

$y \le 0$ when $-\infty < x \le -4,\ 1 \le x \le 4$.

(b) $y \ge 36$

$x^3 - x^2 - 16x + 16 \ge 36$

$x^3 - x^2 - 16x - 20 \ge 0$

$(x + 2)(x - 5)(x + 2) \ge 0$

$y \ge 36$ when $x = -2,\ 5 \le x < \infty$.

56. $\dfrac{1}{x} - 4 < 0$

$\dfrac{1 - 4x}{x} < 0$

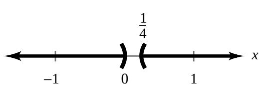
Critical numbers: $x = 0,\ x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0) \Rightarrow \dfrac{1 - 4x}{x} < 0$

$\left(0, \dfrac{1}{4}\right) \Rightarrow \dfrac{1 - 4x}{x} > 0$

$\left(\dfrac{1}{4}, \infty\right) \Rightarrow \dfrac{1 - 4x}{x} < 0$

Solution interval: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$



58. $\dfrac{x + 12}{x + 2} - 3 \ge 0$

$\dfrac{x + 12 - 3(x + 2)}{x + 2} \ge 0$

$\dfrac{6 - 2x}{x + 2} \ge 0$

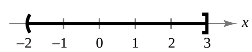
Critical numbers: $x = -2,\ x = 3$

Test intervals: $(-\infty, -2) \Rightarrow \dfrac{6 - 2x}{x + 2} < 0$

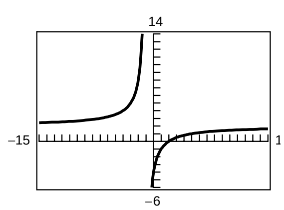
$(-2, 3) \Rightarrow \dfrac{6 - 2x}{x + 2} > 0$

$(3, \infty) \Rightarrow \dfrac{6 - 2x}{x + 2} < 0$

Solution interval: $(-2, 3]$



60. $y = \dfrac{2(x - 2)}{x + 1}$



(a) $y \le 0$

$\dfrac{2(x - 2)}{x + 1} \le 0$

$y \le 0$ when $-1 < x \le 2$.

(b) $y \ge 8$

$\dfrac{2(x - 2)}{x + 1} \ge 8$

$\dfrac{2(x - 2) - 8(x + 1)}{x + 1} \ge 0$

$\dfrac{-6x - 12}{x + 1} \ge 0$

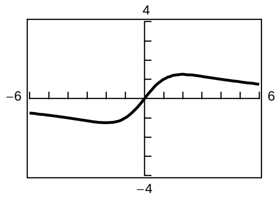
$\dfrac{-6(x + 2)}{x + 1} \ge 0$

$y \ge 8$ when $-2 \le x < -1$.

62. $y = \dfrac{5x}{x^2 + 4}$

(a)
$$y \geq 1$$
$$\frac{5x}{x^2 + 4} \geq 1$$
$$\frac{5x - (x^2 + 4)}{(x^2 + 4)} \geq 0$$
$$\frac{(x - 4)(x - 1)}{x^2 + 4} \geq 0$$

$y \geq 1$ when $1 \leq x \leq 4$.



(b)
$$y \leq 0$$
$$\frac{5x}{x^2 + 4} \leq 0$$

$y \leq 0$ when $-\infty < x \leq 0$.

64. $\sqrt{x^2 - 4}$

Need: $x^2 - 4 \geq 0$

$(x - 2)(x + 2) \geq 0$

Critical numbers: $x = \pm 2$

Testing each interval, the solution is $x \geq 2$ or $x \leq -2$.

Domain: $(-\infty, -2]$ and $[2, \infty]$

66. $\sqrt[3]{2x^2 - 8}$

Domain: all real x

68. $\sqrt[4]{4 - x^2}$

Need: $4 - x^2 \geq 0$

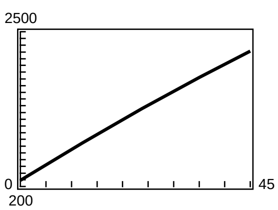
$x^2 - 4 \leq 0$

$(x - 2)(x + 2) \leq 0$

Testing each interval, the solution is $-2 \leq x \leq 2$.

Domain: $[-2, 2]$

70. (a)



(b) Graphing D together with $y_2 = 2500$, we see that $D > 2500$ when $t \approx 52.6$, or during 2002.

72. $|h - 50| \leq 30$

$-30 \leq h - 50 \leq 30$

$20 \leq h \leq 80$

h must be in the interval $[20, 80]$.

Maximum 80, minimum 20.

74. False. If $c < 0$, then $ac \geq bc$. For example, let $a = 1$, $b = 2$ and $c = -3$.

76. $|x - a| \geq 2$ means x is at 2 units from a. Matches (b).